RESEARCH ARTICLE

Optimized Data Partitioning for Parallel Sorting on Heterogeneous Distributed Systems using Linear Programming

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ABSTRACT
Do not read this as its just a placeholder. This research addresses the optimization of data partitioning for parallel computations in heterogeneous distributed systems, with the dual objective of minimizing financial expenditure and execution time. We develop a Mixed Integer Programming (MIP) model that incorporates critical system constraints including CPU speed, memory capacity, memory latency, network latency, and budget limitations. Our model replaces the heuristic partitioning phase of a parallel quicksort algorithm, which serves as our performance benchmark. Utilizing synthetic datasets representing both workload and node specifications, our simulations focus on pre-execution data partitioning decisions. The goal is to provide a framework for financial and time optimization within cloud computing environments, where efficient resource allocation and budget management are essential. Results demonstrate the effectiveness of our MIP-based approach in achieving improved load distribution and reduced runtime compared to traditional heuristics.

KEYWORDS
Data Partitioning; Linear Programming; Heterogeneous Distributed Systems; Cost Optimization; Makespan Minimization

1. Introduction

The widespread adoption of cloud computing and serverless architectures has increased the importance of efficient resource allocation in distributed systems. This research addresses the challenge of statically partitioning data across heterogeneous computing environments, where nodes vary in processing speed, memory capacity, network latency, and monetary implications.

We formulate data partitioning as a Linear Programming (LP) problem aimed at minimizing execution time (makespan) while adhering to defined constraints. Our approach replaces traditional heuristic-based partitioning in parallel sorting algorithms with a novel LP model that captures comprehensive system characteristics to produce balanced, cost-aware data distributions. Our linear formulation allows polynomial-time solvability, providing optimal partitioning under linear constraints without incurring computational intractability.

1.1. Background and Motivation

Distributed computing enables parallel processing of large-scale datasets across multiple nodes. However, in heterogeneous systems with diverse computational capabilities, simplistic data partitioning strategies often result in load imbalances and inefficient resource utilization[48]. Common approaches like uniform distribution or partitioning based solely on heuristics typically fail to consider system compositions in cloud environments.

Cloud computing introduces new complexities to data partitioning. These environments offer flexibility with pay-as-you-go pricing models where computing resources are available on demand[5]. This necessitates re-evaluating traditional partitioning methodologies to explicitly consider economic factors alongside performance metrics. Effective strategies must balance high performance with budget limitations—minimizing both computational time and overall expenditure for cloud resource utilization.

While Monga and Lodhi[53] demonstrated benefits of heterogeneity-aware allocation, their model primarily considered CPU speed alone. Our work introduces a more sophisticated LP-based model that integrates multiple system attributes to achieve globally optimized data distributions, providing a more realistic and budget-conscious approach to optimized data processing.

1.2. Cost Considerations in Heterogeneous Distributed Systems

In this context, cost refers to the pricing of cloud compute nodes. While higher-priced instances often imply better performance, this is not guaranteed, as pricing and performance can vary across vendors. Each node has attributes including processing speed, memory capacity, network characteristics, and pricing models inspired by real cloud offerings, reflecting typical variations in heterogeneous cloud deployments.

Allocating data across heterogeneous resources inherently involves performance-cost tradeoffs[48]. Balancing processing time with financial implications when assigning datasets to different machines is critical. Effective data partitioning can significantly reduce both runtime and monetary cost, while suboptimal partitioning leads to inflated expenses or resource limitations like memory overflows.

This paper specifically addresses statically partitioning large synthetic datasets before executing parallel sorting across simulated heterogeneous environments. Our primary objective is to minimize the overall execution time (makespan) while ensuring we satisfy the defined constraints. As a secondary objective, we aim to identify the most cost-effective solution with an equivalent makespan, using a lexicographic optimization approach. The controlled synthetic environment enables repeatable experiments independent of real-world cloud variability.

Linear programming offers a powerful and tractable mathematical framework for such optimization problems, especially when:

- Data volumes are represented as continuous variables;
- Constraints and objectives remain linear;
- No integer or combinatorial decisions are involved;

This allows the use of efficient solvers that provide globally optimal solutions in polynomial time [57]. By leveraging LP, we avoid the complexity of NP-hard integer-based scheduling, while still capturing key system characteristics through continuous modeling of load, speed, and cost.

1.3. Research Contributions

This paper contributes the following:

(1) A linear programming (LP) formulation for initial data partitioning in parallel quicksort on heterogeneous systems;
(2) An integration of the LP model into an existing heterogeneous sorting framework;
(3) A performance evaluation using synthetic datasets with extended metrics;
(4) A reproducible experimental setup and discussion of directions for future work.

2. Related Work

Heterogeneous scheduling and data allocation have been studied in various contexts. For instance,[48] considered dataset allocation across geo-distributed clouds with two objectives (processing time and cost). They formulated a linear program to place data blocks on VMs to minimize a weighted sum of time and cost, demonstrating Pareto trade-offs. Similarly,[49] used a linear model for data assignment in a hybrid heterogeneous processing environment. These works focus on resource assignment rather than within-job data partitioning, but they highlight the utility of mathematical programming for cloud cost-performance optimization.

Classic scheduling theory demonstrates that minimizing makespan on unrelated machines is NP-hard when tasks must be assigned discretely[50]. However, in our case, we model data volumes as continuous variables without requiring integer decision variables. This places our problem within the class of linear programs, which are solvable in polynomial time using interior-point methods or simplex-based solvers [57]. Thus, our static data partitioning model, while inspired by hard scheduling problems, does not inherit their computational complexity due to the absence of discrete allocation and the use of continuous variables.

In the domain of parallel sorting, many algorithms assume a homogeneous machine model. For instance, Parallel Sorting by Regular Sampling (PSRS) chooses pivots to create equally-sized partitions[51], and typical benchmarks use randomly-generated data with uniform or other distributions. These benchmarks (e.g., uniform 32-bit integer inputs) guide our synthetic data choices. However, PSRS and related methods do not account for cost or heterogeneous speeds.

In big-data systems like Spark, dynamic partitioning and scheduling algorithms have been proposed. For example,[52] developed a dynamic partitioning strategy for intermediate Spark data to mitigate skew, and a greedy scheduling method that considers node speed. They find that balanced partitioning significantly lowers completion time. Our work differs by focusing on static initial partitioning with explicit cost metrics, rather than in-job rebalancing. To the best of our knowledge, prior work has not explicitly applied linear programming to cost-aware static data partitioning in heterogeneous cloud sort workloads.

We build upon the model proposed by Monga and Lodhi[53], which partitions data across heterogeneous nodes based on CPU performance to balance load during parallel sorting. However, their approach does not consider critical real-world factors such as network latency or resource cost—key limitations in cloud and serverless settings. Moreover, their method performs initial local sorting and sampling before defining data ranges, which can lead to uneven memory usage and load imbalance during the redistribution phase.

3. Problem Formulation

We now formalize the partitioning problem based on the system characteristics introduced in Section 1.3. To ensure full control over system parameters and facilitate reproducibility, we operate within a synthetic environment. Each node is defined by parameters such as processing speed, memory capacity, network latency, bandwidth, and cost, inspired by real-world cloud configurations (e.g., AWS, GCP). This abstraction enables rigorous evaluation of partitioning strategies without relying on live infrastructure.

3.1. System Model

We partition the dataset into contiguous blocks of size d, assigning each block to a unique node. Consider a heterogeneous cluster with N nodes, where each node i is characterized by the following parameters:

- d_i: Assigned data volume (in units of d)
- r_i: Processing rate (in d/t — data units per time unit)
- m_i: Maximum data volume the node can handle at once (in units of d)
- ℓ_i: Network latency (in time units t)
- b_i: Network bandwidth (in d/t — data units per time unit)
- u_i: Usage billing rate (in c/t — cost units per time unit)

The units above are illustrative and can be adapted depending on context. For instance, processing speed r may be measured in records/ms, memory m in megabytes (MB), and cost u in USD/hour. In this work, we use normalized or synthetic units to model relative performance and cost differences between nodes, without binding the system to a specific infrastructure or currency.

3.2. Derived Parameters

We derive additional parameters from the node characteristics to facilitate the MIP formulation:

3.2.1. Processing Time

$$x_i = \frac{d_i}{r_i} \tag{1}$$

where x_i is the time required by node i to process its assigned data volume d_i.

3.2.2. Transfer Time

$$y_i = \ell_i + \frac{d_i}{b_i} \tag{2}$$

where y_i is the communication overhead for node i, computed from latency ℓ_i and bandwidth b_i.

3.2.3. Total Cost

$$w = \sum_{i=1}^{N} u_i \cdot x_i \tag{3}$$

where w denotes the cumulative cost of utilizing all selected nodes.

3.3. Optimization Objectives

The primary objective of our MIP model is to minimize the total execution time (makespan) of the parallel sorting process. This ensures that the overall completion time is minimized, taking into account both computation and communication overheads. Formally, the primary objective is defined as:

$$\min z = \max_{i=1}^{N}(x_i + y_i) \tag{4}$$

where z represents the maximum time taken by any node i to complete its assigned data processing and communication tasks. This formulation captures the makespan of the entire parallel sorting operation, ensuring that the slowest node determines the overall completion time. In addition to minimizing makespan, we aim to choose the most cost-effective solution among equivalent combinations. To achieve this, we introduce a secondary objective that minimizes the total financial expenditure. This is incorporated into the model using a lexicographic optimization approach:

$$\min \ z + \varepsilon \cdot w \tag{5}$$

In this formulation, w represents the cumulative cost of utilizing the selected nodes, and ε is set to a negligible value (e.g., 10^{-6}) to prioritize makespan minimization while breaking ties in favor of cost efficiency. This approach ensures that among solutions with equivalent makespan, the one with the lowest cost is selected.

3.4. Constraint Definitions

The proposed LP model is subject to several constraints that reflect system limitations and ensure a feasible allocation of data across nodes. These constraints incorporate resource boundaries (e.g., memory and budget), performance considerations (e.g., makespan), and completeness of the partitioning scheme.

3.4.1. Makespan Constraint

$$z \geq \frac{d_i}{r_i} + \ell_i + \frac{d_i}{b_i} \quad \forall i \in \{1, \ldots, N\} \tag{6}$$

This constraint ensures that the total execution time, represented by the variable z, is at least as large as the time taken by any individual node i to both process its assigned data and transfer it to the next stage. It combines computation time $\frac{d_i}{r_i}$ with communication latency ℓ_i and data transfer time $\frac{d_i}{b_i}$.

3.4.2. Memory Constraint

$$d_i \leq m_i \quad \forall i \in \{1, \ldots, N\} \tag{7}$$

Each node has a limited memory capacity m_i, and this constraint ensures that the volume of data d_i assigned to node i does not exceed its available memory.

3.4.3. Coverage Constraint

$$\sum_{i=1}^{N} d_i = D \tag{8}$$

This constraint enforces full data allocation: the entire dataset of size D must be partitioned and distributed among the N available nodes without omission or duplication.

3.4.4. Non-negativity Constraint

$$d_i \geq 0 \quad \forall i \in \{1, \ldots, N\} \tag{9}$$

This standard constraint ensures that each node receives a non-negative volume of data, reflecting the physical impossibility of assigning negative quantities.

3.4.5. Budget Constraint (Optional)

$$0 \leq \sum_{i=1}^{N} u_i \cdot d_i \leq B \tag{10}$$

In addition to node-specific parameters, we optionally define a user-defined input B, representing the maximum allowable expenditure. This parameter must be a non-negative number, with a default value of 0. Including this constraint models scenarios where financial limitations must be respected.

4. Heterogeneous PSRS (Baseline Algorithm)

This is the model of Monga and Lodhi [53] which we will use as a baseline. The algorithm is designed to sort a list of items using multiple workers, each with different speeds.

4.1. Pseudocode

Algorithm 1: Heterogeneous PSRS (Baseline Algorithm)

Inputs:

- *data*: Input data to be sorted
- n: Total number of data items
- p: Number of processors
- $perf[0 \dots p-1]$: Relative performance of each processor

Output:

- Sorted data

Phase 1: Local Sorting and Sampling
for *each processor* i **do**
  $size_i = \frac{n \cdot perf[i]}{\sum perf}$
  Assign $size_i$ items to processor i
  Processor i performs sequential quicksort on its local data
  Select $L = (p-1) \cdot perf[i]$ samples from local sorted data
  Send samples to the designated coordinator
Phase 2: Pivot Selection
Designated coordinator gathers and sorts all samples
Selects $p-1$ global pivots and broadcasts them to all processors
Phase 3: Data Redistribution
for *each processor* **do**
  Partition local data using global pivots
  Send partitioned data to corresponding processors
  Retain the portion corresponding to its final range
Phase 4: Final Sorting and Merge
Each processor sorts its final local data
Coordinator concatenates all sorted segments for final output

4.2. Limitations

The Heterogeneous PSRS algorithm by Monga and Lodhi[53] effectively distributes sorting tasks across processors with varying speeds but has notable inefficiencies. Its local sorting phase precedes global partitioning, often leading to imbalanced data redistribution, where some nodes receive disproportionately larger portions. This imbalance increases memory usage and processing delays. Additionally, the algorithm overlooks communication costs and network latency, critical factors in distributed and cloud-based systems. These limitations highlight the need for a more adaptive approach that integrates performance and cost considerations when assigning data ranges.

5. LP-Guided Sorting (Proposed Algorithm)

We propose a modified algorithm that replaces the sampling and redistribution stages with a LP formulation that statically partitions the global key range across heterogeneous distributed nodes.

5.1. Bucket Assignment

Given the total dataset size D and a heterogeneous set of N nodes with parameters $(r_i, m_i, \ell_i, b_i, u_i)$, we solve the MIP formulation (Section **??**) to compute optimal partition sizes d_i such that:

- Processing time is balanced according to each node's capability
- Communication and processing costs are minimized
- Constraints on memory, load, and budget are respected

Once d_i values are determined, each node is assigned a fixed key range. For example (illustrative only):

Worker	**Percentage of Total Data**
A (4x speed)	50%
B (3x speed)	30%
C (2x speed)	20%

5.2. Merge Strategies

Aspect	**Option A: Block Merge (Centralized Bucketing)**	**Option B: Coordinated Min-Max Pop (Asynchronous Routing)**
Data Handling	Each node returns a fully sorted block corresponding to a unique key range	Coordinator continuously requests the current min and max from each node's local sorted data
Merge Strategy	Coordinator performs a two-ended merge by popping global min and max from the node outputs	Dynamically merges results by selecting the global min and max across all nodes
Efficiency	Efficient for well-partitioned data; avoids re-querying nodes	Suitable when node outputs are fragmented or partially overlapping
Memory Usage	Requires all blocks to be received and held in memory before merging	Lower memory usage as data is processed dynamically
Coordination Overhead	Minimal; relies on pre-sorted blocks	Higher coordination overhead; increases latency and total merge time

Table 1. Comparison of Merge Strategies

5.3. Pseudocode

Algorithm 2: LP-Guided Hybrid (Proposed Algorithm)

Inputs:

- D: Total dataset
- N: Number of nodes
- r_i: Processing rate of node i
- m_i: Memory capacity of node i
- b_i: Network bandwidth of node i
- ℓ_i: Network latency of node i
- u_i: Usage cost rate of node i

Output:

- Sorted dataset

Phase 1: Static Partitioning
Solve LP to determine proportion p_i of data that node i should process, where $\sum p_i = 1$
Compute data volume per node: $d_i = p_i \cdot |D|$
Phase 2: Block Distribution
For each node i, calculate how many items it should receive by multiplying its proportion p_i with the total number of items $|D|$, then rounding down: $\text{floor}(p_i \cdot |D|)$
Set starting index $c_0 = 0$ and compute cut points c_i by adding up the sizes
for *each node i from 1 to N* **do**
  Distribute subarray $D[c_{i-1} : c_i]$ to node i
Phase 3: Local Sorting
for *each node i* **do**
  Sort received data using sequential quicksort
Phase 4: Final Merge
Option A: Centralized Block
Let $S_1, S_2, \ldots, S_N$ be the sorted blocks received from each node
Initialize a double-ended queue: `merged_sorted` ← empty deque
while *any S_i is non-empty* **do**
  Identify the global minimum and maximum from heads and tails of non-empty S_i
  Pop the global min and append to the **left**, max to the **right** of `merged_sorted`
return *`merged_sorted`*

Option B: Coordinated Min-Max Pop
Initialize a double-ended queue: `merged_sorted` ← empty deque
Let each node expose head and tail of its local sorted buffer on request
while *total number of popped elements* $< |D|$ **do**
  foreach *node i* **do**
    Send h_i and t_i (head and tail) to coordinator
  Coordinator selects global min and max from all h_i, t_i
  Source nodes pop their respective elements
  Append global min to **left**, global max to **right** of `merged_sorted`
return *`merged_sorted`*

6. Theoretical Complexity Analysis

A rigorous complexity analysis highlights the theoretical advantages of our LP-guided approach over the baseline PSRS algorithm, particularly in heterogeneous distributed nodes.

6.1. Baseline Algorithm

6.1.1. Time Complexity

The original Heterogeneous PSRS algorithm involves four distinct phases, each contributing to the overall time complexity:

6.1.1.1. Phase 1: Local Sorting and Sampling. Each of the N nodes sorts its local data of size $O(n/N)$ on average, taking $O\left(\frac{n}{N}\log\frac{n}{N}\right)$ time using quicksort. Sampling takes $O(N)$ time per node. The communication of samples to the coordinator takes $O(N^2)$ in the worst case if all nodes send their $O(N)$ samples sequentially. Thus, this phase has an average time complexity of $O\left(\frac{n}{N}\log\frac{n}{N} + N^2\right)$.

6.1.1.2. Phase 2: Pivot Selection. The coordinator sorts $O(N^2)$ samples, which takes $O(N^2 \log N^2) = O(N^2 \log N)$ time. Broadcasting the $N-1$ pivots to all N nodes takes $O(N)$ time in parallel or $O(N^2)$ sequentially.

6.1.1.3. Phase 3: Data Redistribution. Each node partitions its $O(n/N)$ local data using $N-1$ pivots, which takes $O(n/N \cdot N) = O(n)$ in the worst case if the pivots are poorly chosen. The redistribution of data among nodes can take up to $O(n)$ in the worst-case scenario where one node receives almost all the data.

6.1.1.4. Phase 4: Final Sorting and Merge. Each node sorts its received data, which can be up to $O(n)$ in the worst case, leading to $O(n \log n)$ for a single node if load balancing is poor. The coordinator then merges the N sorted segments, which takes $O(n)$ time.

6.1.1.5. Overall Complexity. The PSRS algorithm executes in four distinct phases: local sorting and sampling, pivot selection, data redistribution, and final sorting or merging. Combining these phases, the average-case time complexity of the original PSRS algorithm is dominated by the local sorting and pivot selection, yielding

$$\mathcal{O}\left(\underbrace{\frac{n}{N}\log\frac{n}{N}}_{\text{local sorting}} + \underbrace{N^2 \log N}_{\text{pivot selection}} + \underbrace{n}_{\text{redistribution}}\right) \tag{11}$$

Under ideal load balancing and when $N \ll n$, this simplifies to:

$$\mathcal{O}\left(\underbrace{\frac{n \log n}{N}}_{\text{local sorting}} + \underbrace{N^2 \log N}_{\text{pivot selection}}\right) \quad (\text{since } \log \frac{n}{N} \approx \log n \text{ for } N \ll n) \tag{12}$$

However, due to dynamic pivot selection and load imbalance:

- If one node is assigned a disproportionately large portion of the data, the worst-case time complexity becomes:

$$\mathcal{O}\left(\underbrace{n \log n}_{\text{sorting dominated by the largest partition}}\right) \tag{13}$$

- If the local sorting algorithm performs poorly on specific input patterns (e.g., quicksort with highly unbalanced partitions):

$$\mathcal{O}\left(\underbrace{n^2}_{\text{quadratic worst-case sorting due to poor partitioning}}\right) \tag{14}$$

6.1.2. Space Complexity

For space complexity, the PSRS algorithm requires local storage for each node to store its assigned data, which is $O(n/N)$ on average.

6.1.2.1. Phase 1: Local Sorting and Sampling. During the local sorting and sampling phase, each node stores its assigned data locally, requiring $O(n/N)$ space. Additionally, each node generates $O(N)$ samples, which are sent to the coordinator. The coordinator stores $O(N^2)$ samples in total, as it collects $O(N)$ samples from each of the N nodes.

6.1.2.2. Phase 2: Pivot Selection. In the pivot selection phase, the coordinator sorts the collected $O(N^2)$ samples. This requires $O(N^2)$ space for storing and processing the samples. The pivots themselves, $N-1$ in number, require negligible additional space.

6.1.2.3. Phase 3: Data Redistribution. During data redistribution, nodes may need temporary buffers to hold data being sent or received. These buffers can be up to $O(n/N)$ in size per node, depending on the volume of data being exchanged.

6.1.2.4. Phase 4: Final Sorting and Merge. In the final sorting and merge phase, each node holds a sorted segment of approximately $O(n/N)$. The coordinator may require additional space to merge the sorted segments, but this is typically $O(n)$ as it processes the entire dataset.

6.1.2.5. Overall Space Complexity. The total space complexity of the PSRS algorithm is $O(n)$, as the total data is partitioned and stored across the nodes. The coor-

dinator requires additional space for the samples, $O(N^2)$, which is negligible compared to the total data size for large n.

6.2. Proposed Algorithm

For the LP-guided algorithm, the process is significantly streamlined.

6.2.1. Time Complexity

6.2.1.1. Phase 1: Static Partitioning. This is solved once as a precomputation step. The complexity of solving a Linear Programming problem is polynomial, typically handled by efficient algorithms like the interior-point method or simplex method. For practical LP solvers, the complexity is often approximated as $O(N^3)$, where N is the number of nodes, based on the number of variables (proportional to N) and constraints (also proportional to N).

Although the constraint matrix of a linear program has $O(N^2)$ entries, solving the LP involves more than just accessing these entries. Matrix operations such as factorization or inversion generally take $O(N^3)$ time [57]. While sparsity can reduce this in practice, we conservatively estimate the complexity as $O(N^3)$. This is a one-time overhead, and for a fixed number of nodes N, it becomes negligible over repeated runs.

6.2.1.2. Phase 2: Data Distribution. Since the assignment is static and precomputed, each node directly receives its designated data block without coordination overhead or routing logic. The data is assumed to be globally addressable or pre-divided based on offsets, so the coordinator does not mediate or relay data. This results in a linear time complexity for distribution $O(n)$ as each of the n items is sent directly to its assigned node.

6.2.1.3. Phase 3: Local Sorting. Each of the N nodes sorts its assigned partition of size d_i. Since the LP aims to balance the workload, we expect $d_i \approx O(n/N)$. The sorting time per node is $O\left(\frac{n}{N}\log\frac{n}{N}\right)$. With parallel execution, the overall sorting time remains $O\left(\frac{n}{N}\log\frac{n}{N}\right)$.

6.2.1.4. Phase 4: Final Merge. Both options use a coordinated merge phase where the coordinator retrieves the global minimum and maximum elements in each round. This requires scanning N candidates per iteration, with $n/2$ iterations in total (since two items are merged per round), yielding a worst-case time complexity of $O(nN)$ for the merge. This step is optional if a distributed sorted output is acceptable.

6.2.1.5. Overall Complexity. With the updated distribution phase, the total time complexity becomes:

$$O\left(\underbrace{N^3}_{\text{LP solve}} + \underbrace{n}_{\text{distribution}} + \underbrace{\frac{n}{N}\log\frac{n}{N}}_{\text{local sort}} + \underbrace{nN}_{\text{merge}}\right) \tag{15}$$

This streamlined model avoids the intermediate routing layer and central buffers

entirely, improving both runtime and memory efficiency. When $N \ll n$, this simplifies to:

$$O\left(\underbrace{nN}_{\text{merge}} + \underbrace{\frac{n}{N}\log\frac{n}{N}}_{\text{local sort}}\right) \tag{16}$$

amortizing the $O(N^3)$ LP cost over repeated executions.

6.2.2. Space Complexity

Each node stores only its assigned partition of size d_i, where $\sum d_i = n$, resulting in a total distributed memory usage of $O(n)$.

6.2.2.1. Phase 1: Static Partitioning. The LP solver still consumes $O(N^2)$ space for storing the constraint matrix and internal buffers. This space is allocated once during initialization and reused across multiple executions.

6.2.2.2. Phase 2: Data Distribution. With direct data assignment, no intermediate data structures are needed at the coordinator. Each item is sent directly to its destination node without temporary storage. Thus, the coordinator's memory usage is reduced to only managing lightweight metadata (e.g., offsets or boundaries), which requires $O(N)$ space.

6.2.2.3. Phase 3: Local Sorting. Each node performs an in-place sort on its local partition. Since no additional structures are required beyond the input array and constant auxiliary space for the sorting algorithm (e.g., quicksort or heapsort), the space complexity remains $O\left(\frac{n}{N}\right)$ per node. The total space across all nodes is still $O(n)$.

6.2.2.4. Phase 4: Final Merge. The coordinated merge step requires the coordinator to keep track of one active element (or pointer) per node, resulting in $O(N)$ memory overhead. Optionally, if streaming is not used, the coordinator may need to buffer the entire merged result, requiring an additional $O(n)$ space.

6.2.2.5. Overall Space Usage.

- LP solver and metadata: $O(N^2)$ at initialization.
- Total data storage: $O(n)$, distributed across N nodes.
- Merge coordination: $O(N)$ for pointers; optionally $O(n)$ for buffering.

6.3. Complexity Comparison

In this section, we formally analyze and compare the time complexity of the baseline PSRS algorithm and the proposed LP-guided algorithm, incorporating both Option A and Option B for data distribution. The analysis emphasizes both average-case behavior and worst-case scenarios under heterogeneous processing environments.

Algorithm	Average Case	Worst Case
PSRS	$O\left(\frac{n \log n}{p} + p^2 \log p\right)$	$O(n \log n)$
LP-guided	$O(n + \frac{n}{N} \log \frac{n}{N} + nN)$	$O(N^3 + n + \frac{n}{N} \log \frac{n}{N} + nN)$

Table 2. Time Complexity Comparison (Average vs Worst Case)

Algorithm	Average Case	Worst Case
PSRS (Baseline)	$O(n)$	$O(n)$
LP-guided (Option A)	$O(n)$	$O(2n + N^2)$
LP-guided (Option B)	$O(n)$	$O(n + N^2)$

Table 3. Space Complexity Comparison (Average vs Worst Case)

Aspect	PSRS	LP-Guided
Load Balancing	Data-dependent and dynamic, prone to imbalance	Statically balanced using node profiles
Synchronization	Multiple barriers: sampling, pivoting, merging	Minimal, only at merge (if needed)
Communication Overhead	$O(p^2)$ (samples), $O(n)$ (data)	$O(n)$ (initial only)
Preprocessing	None	One-time $O(N^3)$ for LP
Memory Usage	Prone to imbalance due to uneven partition sizes	Evenly balanced partitions based on LP solution
Worst-case	$O(n \log n)$	Consistently balanced, better in heterogeneity

Table 4. Performance Factors Comparison

The LP-guided algorithm offers clear advantages over PSRS in terms of scalability, memory balance, and load distribution. By eliminating runtime sampling and pivot-based partitioning, LP-guided minimizes both load and memory imbalance as well as communication overhead, making it particularly suitable for heterogeneous systems.

Time complexity:

- PSRS relies on dynamic sampling and pivot selection, which can lead to inefficiencies on large systems with many nodes.
- LP-guided approaches require preprocessing but offer more consistent performance, especially in repeated or large-scale sorting tasks.

Space complexity:

- PSRS is memory-efficient on average but may suffer from *memory imbalance* if partition sizes vary significantly due to skewed data.
- LP-guided approaches incur preprocessing overhead but provide *deterministic partitioning*, leading to better memory balance and overall scalability.

Overall, LP-guided algorithms are better suited for scenarios requiring repeated sorts or large datasets on heterogeneous constraints. PSRS remains viable for simpler, homogeneous setups with fewer nodes and lower memory pressure.

7. Experimental Setup

To evaluate the proposed LP-Guided Hybrid Sorting algorithm, we developed a simulation environment in Python that emulates a distributed system comprising heterogeneous artificial nodes. Each node is modeled with distinct computational and network characteristics to reflect real-world cloud or edge environments. Our setup draws inspiration from prior work, such as Monga and Lodhi's evaluation on a heterogeneous cluster of UltraSPARC processors, while extending the model with scalable simulation and optimization capabilities.

7.1. Node Model

Each artificial node is defined by the following parameters:

- **CPU speed** r_i (operations per second): determines how quickly the node performs sorting computations.
- **Memory capacity** m_i: the maximum number of elements a node can hold during processing.
- **Bandwidth** b_i and **latency** ℓ_i: simulate data transfer times during partitioning and merging stages.
- **Usage cost** u_i: an optional cost metric to simulate real-world pricing of compute resources.

Nodes execute local sorts on subarrays and participate in global merges, coordinated either centrally or collaboratively. Resource constraints are enforced strictly throughout the simulation.

7.2. Dataset

We generated synthetic datasets D of uniformly distributed integers with sizes ranging from 10^3 to 10^6 elements. Each dataset is randomly shuffled before processing. Additionally, to simulate skewed distributions akin to Monga and Lodhi's setup, we tested with datasets containing concentrated ranges of values interspersed with random values.

7.3. Partitioning Strategy

Data is initially partitioned across nodes by solving a linear program (LP) that minimizes overall resource cost while accounting for node speeds r_i and memory limits m_i. The resulting LP solution yields optimal distribution proportions p_i for each node. Unlike static distribution in traditional models, this dynamic strategy promotes both performance and load balance under resource constraints.

7.4. Evaluation Metrics

To benchmark performance, we evaluate the system using the following metrics:

- **Total Execution Time**: Measured from initial data partitioning to completion of global merge.
- **Load Balance**: Assessed through deviation in per-node workload based on the assigned data.

- **Node Utilization**: Captures the effective usage of each node's memory and CPU.
- **Cost Efficiency**: Evaluated as $\sum u_i \cdot d_i$, where d_i is the data processed by node i.

These metrics align with those used in prior work, such as execution time comparisons under random and skewed inputs, and deviation analysis to assess load balancing.

7.5. Environment

All experiments were conducted on a simulated cluster using Python's threading and time modules. Each node runs in an independent thread, with synthetic delays introduced to simulate CPU computation time and network transfer latency.

Experiments were conducted with clusters of $N = 3, 5, 10$ nodes, where node parameters were randomly sampled as follows:

- $r_i \in [50, 300]$ operations/sec,
- $m_i \in [100, 10{,}000]$ elements,
- $b_i \in [1, 10]$ MB/s.

In contrast to the physical hardware constraints of the UltraSPARC-based cluster used by Monga and Lodhi, our virtualized testbed allows fine-grained control over system variability and scalability. Furthermore, our timing measurements include all stages of execution, from data distribution to final collection of sorted output, mirroring their holistic performance evaluation methodology.