MATEMATICI SPECIALE ÎN INGINERIE - CURS 1.

NUMERE COMPLEXE

$\mathbb{R}^n$, $(\mathbb{R}^2, +, \cdot)$ $\quad (x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$

$(x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2 - y_1 y_2, x_1 y_2 + y_1 x_2)$

$(\mathbb{R}^2, +, \cdot)$ - corp comutativ ; en+ $(0,0)$, en$\cdot$ $(1,0)$

Lema

$\mathbb{R}_1 = \{(x, 0) \mid x \in \mathbb{R}\}$, $(\mathbb{R}_1, +, \cdot)$ - subcorp $(\mathbb{R}^2, +, \cdot)$

-> arătăm că e corp (stabilă, en $\in \mathbb{R}_1$, $\forall$ elem inv cu $\cdot$)

$(x,0) \cdot (\frac{1}{x}, 0) = (1, 0)$

Lemă => $\mathbb{R}_1 \approx \mathbb{R}$ -> izomorf.

dem. $\varphi: \mathbb{R}_1 \to \mathbb{R}$ sau $\mathbb{R} \to \mathbb{R}_1$ bij

$\hookrightarrow \varphi((x,0)) = x$

$\forall (x,0) \overset{\varphi}{=} x$

$(x,y) = (x,0) + (0,y) = (x,0) + (0,1) \cdot (y,0)$

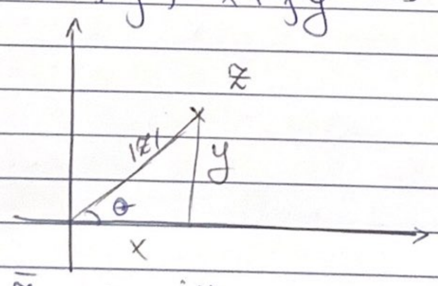
$\overset{\varphi}{=} x + (0,1) y$ $\qquad (0,1) = i / j$

$j^2 = (0,1)(0,1) = (-1, 0)$

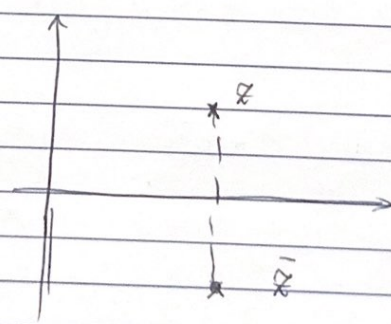
$(x,y) = x + jy$ - formă algebrică a nr complex

(x,y) - afixul pct de coordonate

$(x,y) = x + jy \Rightarrow (\mathbb{C}, +, \cdot) = (\mathbb{R}^2, +, \cdot)$



$|z| = \sqrt{x^2 + y^2}$



$\bar{z} = x - jy$

$\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$

$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2$

$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

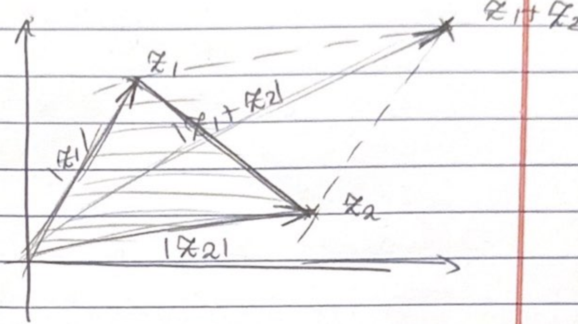
$x = \operatorname{Re} z = \frac{z + \bar{z}}{2}$ partea Reala

$y = \operatorname{Im} z = \frac{z - \bar{z}}{2j}$ part Img

$|z| = \sqrt{z \cdot \bar{z}}$

$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$

$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$



! $|z_1 + z_2| \leq |z_1| + |z_2|$

$d(z_1, z_2) = |z_1 - z_2|$

! $\boxed{|z_1 - z_2| \geq \big||z_1| - |z_2|\big|}$

Forma trigonomometrică

$x = |z| \cos \theta$

$y = |z| \sin \theta$

$z = |z|(\cos \theta + j \sin \theta)$

θ- argument z ($\arg z$), $\theta \in [0, 2\pi)$ sau $\theta \in [-\pi, \pi)$

$z = x + jy = |z|(\cos\theta + i \sin\theta)$

$x = |z| \cos \theta \Rightarrow \cos\theta = \frac{x}{|z|}$ pt $z = 0 \Rightarrow$

$\sin\theta = \frac{y}{|z|}$

$\theta = \operatorname{arctg} \frac{y}{x} + k\pi$ ~~$\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$~~ $k \in \{0, 1, -1\}$

$1 + j = z$ $\arg z = \frac{\pi}{4}$

$-1 - j = z$ în pot i tre la

$z_1 = r_1(\cos \theta_1 + i \sin \theta_2)$

$z_2 = r_2(\cos \theta_2 + j \sin \theta_2)$

$z_1 \cdot z_2 = r_1 r_2 (\cos(\theta_1 + \theta_2) + j \sin(\theta_1 + \theta_2))$

$z_2 \neq 0$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}(\cos(\theta_1 - \theta_2) + j \sin(\theta_1 - \theta_2))$

25 cam

$(\cos\theta \pm j\sin\theta)^n = (\cos n\theta \pm j\sin n\theta)$

Spații metrice

• (X, d), $d : X \times X \to \mathbb{R}$

d - distanță dacă

a) $d(x,y) \geq 0$ a') $d(x,y) = 0 \Leftrightarrow x = y$

b) $d(x,y) = d(y,x)$

c) $d(x,y) \leq d(x,u) + d(u,y)$ $\forall x,y,u$

$|x_n \to x| \Leftrightarrow \to 0$ (care e dist eucl dintre x_n, x)

$\Rightarrow x_n \in X$: $x_n \to x \Leftrightarrow d(x_n, x) \xrightarrow[n\to\infty]{} 0$

• $(\mathbb{C}, |\,|)$ $d(z_1, z_2) = |z_1 - z_2|$

$d(x,y) = \frac{|x \cdot y|}{1 + |x \cdot y|}$ ← e metrică

$d(x,0) = \frac{|x|}{1+|x|} < 1 \Rightarrow$ mărginit

Spațiu metric complet

(X,d) s.n sp met comp dc $\forall$ șir fundamental e conv și are lim în X

$x_n \in X$ s.n fundamental dc $\forall \varepsilon > 0$ $\exists N(\varepsilon)$ a.î. $\forall n > N(\varepsilon)$, $\forall p \in \mathbb{N}$

$\boxed{d(x_{n+p}, x_n) < \varepsilon}$

$d(x_{n+p}, x_n) < \alpha_n$ $\forall p \in \mathbb{N}$ a.t. putem vedea nr n.

$\alpha_n \to 0$

$p \to \infty$ $d(x, x_n) \leq \alpha_n < \varepsilon$

Teoremă

TEMĂ dem.

$(\mathbb{C}, |\,|)$ e un sp. metric complet

$z_n \in \mathbb{C}$, $z_n = x_n + j y_n$

$\boxed{T}$ z_n e conv $\Leftrightarrow$ x_n și y_n sunt conv și are loc

$\lim_{n\to\infty} z_n = \lim_{n\to\infty} x_n + j \lim_{n\to\infty} y_n$

$z = x + jy$

$|z_n - z| = \sqrt{(x_n - x)^2 + (y_n - y)^2} \to 0 \Leftrightarrow$

$(x_n - x) \to 0$ și $(y_n - y) \to 0$

ex. $z_n = \sum_{k=1}^{n} \frac{\cos k}{(k^2+j)}$ să se stud conv z_n și dacă conv, det n a.î. $|z_n - z| < \frac{1}{10}$

$$|z_{n+p} - z_n| = \left| \sum_{k=n+1}^{n+p} \frac{\cos k}{(k^2+j)} \right| \le \sum_{k=n+1}^{n+p} \frac{1}{|k^2+j|}$$

$$= \sum_{k=n+1}^{n+p} \frac{1}{\sqrt{k^4+1}} \le \sum_{k=n+1}^{n+p} \frac{1}{k(k-1)}$$

$$|z_{n+p} - z_n| \le \sum_{k=n+1}^{n+p} \left(\frac{1}{k-1} - \frac{1}{k}\right) = \frac{1}{n} - \frac{1}{n+p} < \frac{1}{n} < \varepsilon \text{ de}$$

$\Leftrightarrow n > \frac{1}{\varepsilon}$

=> fundamental

$\varepsilon > 0$

$\cancel{z_{n+p}}$ $|z - z_n| \le \frac{1}{n} < \frac{1}{10}$ => $n > 10$

- $(\mathbb{C}, +, \cdot)$ are dimensiunea 2 peste
- $\alpha, 1$ $\quad Im\, \alpha \neq 0$

$a_1 \cdot 1 + a_2 \alpha = 0$, $a_1, a_2 \in \mathbb{R}$

$\alpha = x_1 + j y_1$ => $(a_1 + a_2(x_1 + i y_1) = 0$

$$\begin{cases} a_1 + a_2 x_1 = 0 \Rightarrow a_1 = 0 \\ a_2 y_1 = 0 \Rightarrow a_2 = 0 \end{cases}$$

$f: \mathbb{C} \to \mathbb{C}$

$f: D \subset \mathbb{C} \to \mathbb{C}$

$\mathbb{C} = \mathbb{R}$ $\quad f: \mathbb{R} \to \mathbb{C}$ at f s.n funct. complexă de var reale

$(\mathcal{C})$ $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ $\quad t \in J$ $\quad$ ec param ale $\mathcal{C}$

A, B $\quad d \ni A, B$ $\quad M \in d$

$M = (1-t)A + tB$

$x(t) = (1-t)x_A + t x_B$ $\quad t \in \mathbb{R}$ ca să fie pe dr AB

$y(t) = (1-t)y_A + t y_B$

pt segment $[AB]$ $t \in [0, 1]$

$(\mathcal{C})\ z = z(t) \Leftrightarrow x(t) + j\,y(t),\ t \in I$

• Cerc:

$|z - a| = r$ Int cea mai mare mult ( ) e m

$|z - a| < r$

Aderența mult

$a \in \bar{A}$; (a pct aderent = $\forall$ bilă cu centru în a și ~~raz~~ $B(a, r)$ $\cap\ A \neq \emptyset$) ???

$a \in A' = \forall\ Bila(a, r) \setminus \{a\} \cap A \neq \emptyset$

↳ pct de acumulare

$\bar{\mathbb{N}} = \mathbb{N}$

$\mathbb{Q}' = \mathbb{R}$