SEMINAR 4 - MSi

5/ data tre

$x - 2y = 1.$

$2x + y = 3.$

* dintre curbele transformate

$f(z) = e^z$

$m_1 = \frac{1}{2}, \quad m_2 = -2. \Rightarrow \measuredangle = \frac{\pi}{2}$

$tg\,\alpha. = \frac{m_1 - m_2}{1 + m_1 m_2}.$

$\mathcal{C}_1$

$\mathcal{C}_2$

$5x = \cancel{7} \Rightarrow x = \frac{7}{5}; \quad y = -\frac{1}{5}. \qquad \Rightarrow \left(\frac{7}{5}; -\frac{1}{5}\right)$

$z = \frac{7}{5} - \frac{j}{5}$

$\Rightarrow (1+2y, y) \quad (1+2t, t)$

$1 + 2t + jt = 1.$

$d_1. \quad (1, 0)$

$(3; 1)$

$z_1(t) = (1-t)A + tB.$

$z_1(t) = (1-t)A + t(3+j)$

$z_1(t) = 2t + 1 + tj$

$d_2 \quad (1, 1) \quad \cancel{(0, 3)} \quad (2; -1$

$z_2(t) = (1-t)(1+j) + t \cdot \cancel{3j}\,(2-j)$

$z_2(t) = 1 - jt - t + 2t - tj.$

$z_2(t) = 1 - 2jt + t. + j.$

$= 1 + t + j(1 - 2t)$

$(\mathcal{C}_1)\ w_1(t) = e^{1+2t+t.j.}$

$w_2 = e^{1+t+j(1-2t)}$

$1 + 2t = \frac{7}{5} \Rightarrow 2t = \frac{2}{5} \Rightarrow t = \frac{1}{5}$

$w_1'(t) = e^{1+2t+2j} * (2+j)$

$w_2' = e^{1+t+j(1-2t)} \circ (1 - 2j)$

produs scalar dintre w_1 şi w_2.

$x_1 x_2 + y_1 y_2 =$

Printr-o transformare: f e olomorfă $()' \neq 0$ ca păstrează

$\sphericalangle$ dintre curbe.

$(C_1) = z = z(t) = x_1(t) + j y_1(t)$

$(C_2)\quad z = z_2(t) = x_2(t) + j y_2(t)$ $\Rightarrow \cap = t_0$

z_1 se transf în $w_1(t) = u(x_1(t), y_1(t)) + jv(x_1(t), y_1(t))$

$w_2(t) = u(x_2(t), y_2(t)) + j\, v(x_2(t), y_2(t))$

$$\cos\theta = \frac{x_1 x_2 + y_1 y_2}{|z_1(t)|\,|z_2(t)|}$$

$$w_1'(t) = \frac{\partial u}{\partial x} x_1'(t) + \frac{\partial u}{\partial y} y_1'(t) + j\left(\frac{\partial v}{\partial x} x_1' + \frac{\partial v}{\partial y} y_1'\right)$$

$$w_2'(t) = \frac{\partial u}{\partial x} x_2'(t) + \frac{\partial u}{\partial y} y_2'(t) + j\left(\frac{\partial v}{\partial x} x_2' + \frac{\partial v}{\partial y} y_2'\right)$$

$$\cos = \frac{\left(\frac{\partial u}{\partial x} x_1' + \frac{\partial u}{\partial y} y_1'\right)\left(\frac{\partial u}{\partial x} x_2' + \frac{\partial u}{\partial y} y_2'\right) + \left(\frac{\partial v}{\partial x} x_1' + \frac{\partial v}{\partial y} y_1'\right) \cdot \left(\frac{\partial v}{\partial x} x_2' + \frac{\partial v}{\partial y} y_2'\right)}{|w_1'|\,|w_2'|}$$

$$\frac{\left(\frac{\partial u}{\partial x}\right)^2 \cdot (x_1' \cdot x_2' + y_1' y_2') + \left(\frac{\partial u}{\partial y}\right)^2 (y_1' y_2' + x_1' x_2')}{|w_1'| \cdot |w_2'|} =$$

înreg

$$= \frac{(x_1')^2 (x_2')^2 + y_1' y_2'}{z_1' \, z_2'} = \cos \text{ inițial}$$

1) $f \in \mathcal{H}(\mathbb{C})$

$\exists M > 0$ a.î. $|f(z)| \leq M(1 + |z|)^p$, $p \in \mathbb{N}$

at f e un polinom de grad cel mult p.

Dacă o f e deriv de p+1 ori și deriv de ord p+1 este 0 at. ca e un polinom de gr. cel mult p.

$$f^{(p+1)}(z) = \frac{(p+1)!}{2\pi j} \int_{|w-z|=R} \frac{f(w)}{(w-z)^{p+2}}\, dw$$

$w - z = R \cdot e^{jt}$; $t \in$

$$\left|f^{(p+1)}(z)\right| = \left|\frac{(p+1)!}{2\pi j} \int_0^{2\pi} \frac{f(z + R\cdot e^{jt})}{R^{p+2} \cdot e^{jt(p+2)}} \cdot Rje^{jt}\,dt\right|$$

$$\left|f^{(p+1)}(z)\right| \le \frac{(p+1)!}{2\pi} \cdot \int_0^{2\pi} \frac{\left|f(z+Re^{jt})\right| R\,dt}{R^{p+2}} \le$$

$$\le \frac{(p+1)!}{2\pi} \int_0^{2\pi} \cdot \frac{M(1 + |z+Re^{jt}|)^p}{R^{p+1}}$$

$$\le \frac{(p+1)!\,M}{2\pi R^{p+1}} \int_0^{2\pi} (1+|z|+R)^p\,dt$$

$$= \frac{(p+1)!\,M\,(1+|z|+R)^p}{R^{p+1}}$$

$$\left|f^{(p+1)}(z)\right| \le \lim_{R\to\infty} \frac{(p+1)!\,M(1+|z|+R)^p}{R^{p+1}} = 0$$

$\Rightarrow f^{(p+1)}(z) = 0$

$\Rightarrow f \in \Pi_p$.

1. f olomorfă : dez în jurul lui z_0 se reduce la dez în jurul lui 0.

$w = z - z_0$, $z = w + z_0$

$$f(w+z_0) = \sum_{n=0}^{\infty} a_n w^n, \quad |w| < R$$

$\Rightarrow w = z - z_0$, $f(z) = \sum_{n=0}^{\infty} (\cancel{a_n}\ (z - z_n)^n$.

2) $f(z) = \frac{P(z)}{Q(z)}$ $|w| < r$.

$\frac{1}{1-z} = z^n, |z| < 1$

$|w| > R\ (1 + z^\alpha)$.

$\sum_{n=0}^{\infty} \binom{\alpha}{n} z^n;\ |z| < 1.$

$\binom{\alpha}{n}\quad \frac{\alpha(\alpha-1)\dots(\alpha - n+1)}{n}$

a) $f(z) = \frac{1}{2+z}$, $z_0 = 1$.

b) $f(z) = \frac{z+j}{z^2+3z+4}$, $z_0 = j$

c) $f(z) = \dfrac{z}{(z+1)^2(z+j)}$ $\quad z_0=0$

a) $w = z-1 \Rightarrow z = w+1$

$f(w+1) = \dfrac{1}{3+w} = \dfrac{1}{3} \cdot \dfrac{1}{\frac{w}{3}+1} = \dfrac{1}{3}\sum_{n=0}^{\infty}\left(-\dfrac{w}{3}\right)^n =$

$= \sum_{n=0}^{\infty} \dfrac{(-1)^n w^n}{3^{n+1}}$ adev dc $\left|-\dfrac{w}{3}\right| < 1$, adică

$|w| < 3$

$\Rightarrow f(z) = \sum_{n=0}^{\infty} \dfrac{(-1)^n (z-1)^n}{3^{n+1}}$, $|z-1| < 3$.

$f^{(1)}(1) =$

$\dfrac{(-1)^n}{3^{n+1}} = \cancel{f(n)} \cdot \dfrac{f^{(n)}(1)}{n!}$

b) $\dfrac{z+1}{z^2+5z+4}$

$f(z) = \dfrac{z+1}{(z-1)(z-4)} = \dfrac{A}{z-1} \cdot \dfrac{B(z-1)}{z-4}$

$\dfrac{z+j}{z-1}$ pt $z=4$.

$A = -\dfrac{1+j}{3}$

$B = \dfrac{4+j}{3}$

$z - j = w$

$\dfrac{1}{z-1} = \dfrac{1}{w+j-1} = \dfrac{1}{j-1} \cdot \dfrac{1}{1+\frac{w}{j-1}} = \dfrac{1}{j-1}\sum_{n=0}^{\infty} \dfrac{w^n(-1)^n}{(j-1)^n}$

$\dfrac{1}{z-1} = \sum_{n=0}^{\infty} \dfrac{(-1)^n (z-j)^n}{(j-1)^{n+1}}$, $\left|\dfrac{w}{j-1}\right| < 1$, $|z-j| < \sqrt{2}$

c) $\dfrac{1}{z-4}$

$z - j = w$ $= \dfrac{1}{w+j+4} = \dfrac{1}{j+4} \cdot \dfrac{1}{1+\frac{w}{j+4}} = \dfrac{1}{j+4}\sum_{n=0}^{\infty} \dfrac{(-1)^n w^n}{(j-4)^{n}}$

$\dfrac{1}{z-4} = \sum_{n=0}^{\infty} \dfrac{(-1)^n (z-j)^n}{(j-4)^{n+1}}$, $|z-j| < \sqrt{17}$

c) $f(z) = \dfrac{z}{(z+1)^2\cdot(z+j)} = \dfrac{A}{(z+1)} + \dfrac{B}{(z+1)^2} + \dfrac{C}{z+j}$

$C = \dfrac{z}{(z+1)^2}\Big|_{z=-j} = \dfrac{-j}{(-j+1)^2} = \dfrac{-j}{-1-2j+1} = \dfrac{1}{2}$

$B = \dfrac{z}{z+j}\Big|_{z=-1} \Rightarrow B = \dfrac{-1}{-1+j} = \dfrac{1}{j-1}$

$\cancel{A} = \Rightarrow z^2 \quad A\,z \quad > z > z\ ;\ z \to \infty$

$\Rightarrow 0 = A + 0 + C \Rightarrow A = -C \Rightarrow A = -\dfrac{1}{2}.$

$k \geq 2$
$k \in \mathbb{N}$

$\left(\dfrac{1}{z+1}\right)^k = (1+z)^{-k} = \sum_{n=0}^{\infty} \binom{-k}{n}\cdot z^n =$

$\binom{-k}{n} = \dfrac{-k(-k-1)\cdots(-k-n+1)}{n!} = (-1)^n \dfrac{\frac{(k+n-1)!}{(k-1)!}}{n!}$

$= (-1)^n \dfrac{(k+n-1)!}{} \quad \binom{k+n-1}{n}$