$\frac{1}{2^{2n}}$

1) b. Dem. eg.

$$ch^{2n}x = \frac{1}{2^{2n}}\left(\sum_{k=0}^{n-1} 2\binom{2n}{k} ch\, 2(n-k)x + \binom{2n}{n}\right)$$

$$ch\, x = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

$$(a+b)^n = \sum_{k=0}^{n} T_{k+1}$$

$$T_{k+1} = \binom{n}{k} a^{n-k} b^{k}$$

$$\cos^{2n}x = \left(\frac{(\cos x + j\sin x) + (\cos x - j\sin x)}{2}\right)^{2n}$$

cos -kx + j sin -k

$$= \frac{1}{2^{2n}}\sum_{k=0}^{2n}\binom{2n}{k}(\cos x + j\sin x)^{2n-k}\cdot(\cos x - j\sin x)^{k}$$

$$= \frac{1}{2^{2n}}\sum_{k=0}^{2n}\binom{2n}{k}\left(\cos(2n-2k)x + j\sin(2n-2k)x\right)$$

↳ dă 0

$$\Rightarrow \cos^{2n}x = \frac{1}{2^{2n}}\sum_{k=0}^{2n}\binom{2n}{k}\cos 2(n-k)x$$

$2n,\ 1 = 2n-1$

$$= \frac{1}{2^{2n}}\left[\sum_{k=0}^{n-1}\binom{2n}{k}\cos 2(n-k)x + \underbrace{\binom{2n}{n}\cos 2\cdot 0\cdot x}_{0} + \sum_{k=n+1}^{2n}\binom{2n}{k}\right]$$

constante

$k-n+1=p$

$\binom{n}{k} = \binom{n}{n-k}$ → combinări complementare

- primul = ultimul => .... la fel la rest

$k = n+1 \Rightarrow p = 0.$

$k = 2n \Rightarrow p = n-1$

Suma 2. $= \sum_{p=0}^{n-1}\binom{2n}{p+n+1}\cdot\cos(2(n-n-1-p)x)$

$$S_2 = \sum_{p=0}^{n-1}\binom{2n}{n-p-1}\cdot\cos 2(p+1)x$$

Dc. $x := jx$.

$$\Rightarrow ch^{2n}x = \frac{1}{2^{2n}}\left(2\sum_{k=0}^{n-1}\binom{2n}{k} ch((2n-k)x) + \binom{2n}{n}\right)$$

Alea cu sinus la fel

2) $\forall\, t \in \mathbb{R} \quad \{ z \mid |z| = \frac{1}{\sqrt{1+t^2}} \}$

$$a_n = \frac{\sum_{k=1}^{n} k^{jt}}{n} \qquad (x^{\alpha} = e^{\alpha \ln x})$$

Dem. $\lim_{n \to \infty} a_n \nexists$ dar $\exists \lim_{n\to\infty} n^{-jt} a_n \in A$

a_n - bine def.

- la tablă ai poză, sumă Riemann

3) $f(z) = f(z, \bar{z})$

$f(z) = \cancel{u(\,,\,) + v(\,,\,)}$

$u\left(\frac{z+\bar z}{2}, \frac{z-\bar z}{2j}\right)$; $v\left(\frac{z+\bar z}{2}, \frac{z - \bar z}{2j}\right)$

$f(z) = |z^2|$ - nu e doar deriv, doar în 0.

$$\frac{\partial f}{\partial \bar z} = \frac{\partial u}{\partial x} \cdot \underbrace{\frac{1}{2}}_{\left(\frac{z+\bar z}{2}\right)'_{\bar z}} + \frac{\partial u}{\partial y}\left(\frac{+j}{2j}\right) + j\left(\frac{\partial v}{\partial x} \cdot \frac{1}{2} + \frac{\partial v}{\partial y} \cdot \frac{-1}{2j}\right)$$

$$= \frac{1}{2}\left[\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} + j\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]$$

Dc deriv $\Leftrightarrow$ deriv $= 0$

$\Leftrightarrow$ Îndepl cond C.R.

$\Rightarrow 0 = 0$

* $f: D \to \mathbb{C}$, D sim față de Ox, $f \neq c$.

D - sim față de $Ox \Rightarrow \forall z \in D \Rightarrow \bar z \in D$.

a) Dc f e olomorfă at $\bar f(z)$ nu e olomorfă

b) $f(\bar z)$ nu e olomorfă

c) $\bar f(\bar z)$ e olomorfă

a) $\bar f(z) = u(x,y) - j v(x,y)$

$\bar f$ olomorf $\Leftrightarrow$ Verif C.R

$$\left.\begin{cases} \frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y} = \frac{\partial v}{\partial y} \Rightarrow \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial u}{\partial x} = 0 \\ \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = -\frac{\partial v}{\partial x} \Rightarrow \frac{\partial v}{\partial x} = 0 \Rightarrow \frac{\partial u}{\partial y} = 0 \end{cases}\right\} \Rightarrow f\text{-const} \Rightarrow \text{contradicție}$$

$\Rightarrow \partial u = 0 \Rightarrow u = c_1$
$\partial v = 0 \Rightarrow v = c_2$ $\Big\} \Rightarrow f = c_1 + j c_2$ - const complexă ⇒
contradicție
$\Rightarrow \bar{f}$ - deriv $\Leftrightarrow$ e constantă

b) $\bar{f}(\bar{z})$ - olomorfă

$\bar{f}(\bar{z}) = u(x, -y) \mp j v(x, -y)$

$\begin{cases} \frac{\partial u}{\partial x} = +\frac{\partial v}{\partial y} \\ -\frac{\partial u}{\partial y} = +\frac{\partial v}{\partial x} \end{cases} \Rightarrow$ derivabilă

4) Det f. olomorfe f, știind că $|f(z)| = (x^2+y^2)e^x$

$z = x + jy$

$|f(z)|^2 = z^2 \cdot \bar{z}^2 \cdot e^{(z+\bar{z})}$

$f(z) \cdot \bar{f}(\bar{z}) = z^2 \bar{z}^2 (e^{(z+\bar{z})}$

$\frac{f(z)}{z^2 e^z} = \frac{\bar{z}^2 e^{\bar{z}}}{\bar{f}(\bar{z})}$

$f(z) = z^2 \cdot e^z C$

$|z^2 e^z C| = |z|^2 e^x$

$\cancel{|z^2|} |e^z| |C| = \cancel{|z|^2} e^x$

$\cancel{e^x} \cdot |C| = \cancel{|e^x|} \Rightarrow |C| = 1$

$C = e^{j\theta}$

4.2) $\arg f(z) = x \cdot y$; $z = x + jy$

$f(z) = |f(z)| \cdot e^{j xy}$

$f(z) = |f(z)| \cdot e^{j \frac{z^2 - \bar{z}^2}{4j}}$

$f^2(z) = |f(z)|^2 e^{\frac{z^2 - \bar{z}^2}{2}}$

$f^2(z) = f(z) \cdot \bar{f}(\bar{z}) \cdot e^{\frac{z^2 - \bar{z}^2}{2}}$ $\quad | \cdot f(z)$

$f(z) = \bar{f}(\bar{z}) \cdot e^{\frac{z^2 - \bar{z}^2}{2}}$

$\frac{f(z)}{e^{\frac{z^2}{2}}} = \bar{f}(\bar{z}) \cdot e^{\frac{-\bar{z}^2}{2}} = C$

$f(z) = C \cdot e^{\frac{z^2}{e^2}}$

$C \cdot e^{\frac{z^2}{e^2}} = |C| \cdot \cancel{\left| e^{\frac{z^2}{2}} \right| = e^{jxy}}$

$\frac{z^2}{2} = \frac{(x+jy)^2}{2} = \frac{x^2-y^2}{2} + jxy$

$\Rightarrow C = |C| \rightarrow C > 0.$ - poate fi doar real

$f(z) = C \cdot e^{\frac{z^2}{2}}, \quad C \in \mathbb{R}_+$

Prob 5