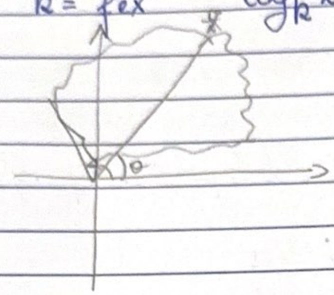
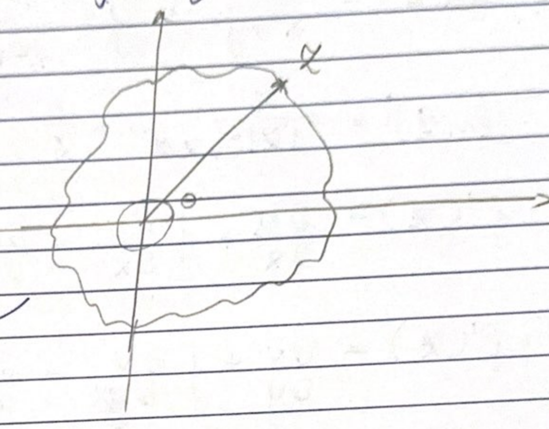
$e^z$, $\cos z$, $\sin z$,

$z \neq 0$, $\log z = \{\ln|z| + j(\arg z + 2k\pi)\}_{k\in\mathbb{Z}}$

$f: \mathbb{C}^* \to P(\mathbb{C})$

$k =$ fix $\quad \log_k z = \ln|z| + j(\arg z + 2k\pi$






ajungem la:

$\log_{k+1} z$

! nu înconjura originea.

$\log_k z \;:\; \mathbb{C}\setminus\mathbb{R}_+ \to \mathbb{C}$

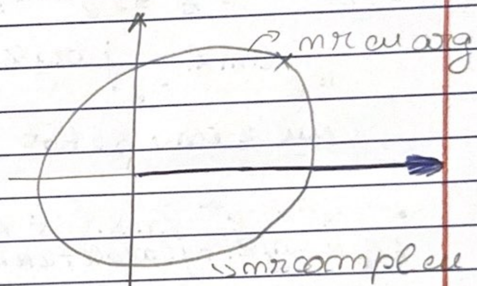
↳ $O_x$

$k =$ fixat

$\log_k z_1, z_2$

Fie $z_1 = |z_1|\, e^{j\theta_1}$

$z_2 = |z_2|\cdot e^{j\theta_2}$




are $2\pi$

$\log_k z_1 z_2 = \log_k |z_1||z_2|\cdot e^{j(\theta_1+\theta_2)}$

$\ln|z_1|\cdot|z_2| + j(\theta_1+\theta_2+2k\pi) = \overbrace{\ln|z_1| + j(\theta_1+2k\pi)}^{\log_k z_1}$

$\underbrace{+\ln|z_2| + j(\theta_2+2k\pi)}_{\log_k z_2} - 2k\pi j$

$\Rightarrow k=0 \Rightarrow \log_0 z = \ln z$ s.n. determinare principală a logaritmului

$\log_0 z = \ln z = \ln\sqrt{x^2+y^2} + j\left(\operatorname{arctg}\frac{y}{x} + p\pi\right)$

$(\ln z)'$

$\ln\sqrt{x^2+y^2} = \frac{1}{2}\ln(x^2+y^2) = u(x,y) \Rightarrow \frac{\partial u}{\partial x} = \frac{x}{x^2+y^2}$

arctg $\frac{y}{x}$

$$\frac{\partial^2 u}{\partial x^2} = \frac{x^2+y^2-2x}{(x^2+y^2)^2} - \frac{y^2-x^2}{(x^2+y^2)^2},$$

$$\frac{\partial u}{\partial y^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$

$$(\ln z)' = \frac{x}{x^2+y^2} + j\left(\frac{\frac{-y}{x^2}}{\frac{x^2+y^2}{x^2}}\right.$$

$$(\ln z)' = \frac{\bar{z}}{|z|^2, z\bar{z}} = \frac{1}{z}$$ (Dem det principala)

- $f'(z) = \frac{\partial u}{\partial x} + j\frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} - j\frac{\partial u}{\partial y}$

- $f'(z) = \frac{\partial v}{\partial y} + j\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} - j\frac{\partial u}{\partial y}$

- $e^{\ln z} \overset{?}{=} z \Rightarrow e^{\ln z} = e^{\ln|z| + j\arg z} = e^{\ln|z|} \cdot e^{j\arg z} = |z| e^{j\arg z} = z.$

(nu e inv pe tot domeniu.

- $x^\alpha = e^{\alpha \ln x}. (\forall x > 0), z \neq 0, \alpha \in \mathbb{C}, z^\alpha = e^{\alpha \log z} = \{e^{\alpha(\ln|z| + j(\arg z + 2k\pi))} \mid k \in \mathbb{Z}\}$

$$j^j = e^{j\log j} = e^{j(0 + j(\frac{\pi}{2} + 2k\pi))} = e^{-\frac{\pi}{2} + 2k\pi}, k \in \mathbb{Z}.$$

$$\sqrt[m]{z} = \{|z|^{\frac{1}{m}} e^{j\left(\frac{\arg z + 2k\pi}{m}\right)}\}_{k \in \mathbb{Z}}$$

$e^{j\frac{\arg z}{m}}, e^{\frac{2k\pi j}{m}}$; $card(z^\alpha) = q$, de $\alpha = \frac{p}{q}$; $(p, q) = 1$
$p, q \in \mathbb{Z}$

$z^m \leftarrow$ ia o valoare de $m \in \mathbb{Z}$

- $e^{m\log z} = e^{m(\ln|z| + j(\arg z + 2k\pi))} = |z|^m \cdot e^{jm\arg z}$

- $\underbrace{e^{2km\pi j}}_{\cos(2k\pi) + \sin(2k\pi)} = z^m$

$$\operatorname{ch} z = \frac{e^z + e^{-z}}{2} \qquad \operatorname{sh} z = \frac{e^z - e^{-z}}{2}$$

$$ch(jx) = \frac{e^{jx}+e^{-jx}}{2} = \cos x.$$

$$sh(jx) = \frac{e^{jx}-e^{-jx}}{2j} j = j \sin x$$

~~$\sin(jx) = j\,\sin(x)$~~

~~$ch(j$~~ $\cos(jx) = ch\,x.$

$$\sin(jx) = \frac{e^{-x}-e^{x}}{2j} \overset{\cdot j}{=} j\frac{(e^{x}-e^{-x})}{2} = j\,sh\,x$$

$$Ch^2 x - sh^2 x = \cos^2 x + \sin^2 x = 1.$$

$$\boxed{ch^2 x - sh^2 x = 1}$$

$$ch\,z_1 \cdot ch\,z_2 = \cos jz_1 \cdot \cos jz_2 = \frac{\cos j(z_1+z_2) + \cos j(z_1-z_2)}{2}$$

$$= ch(z_1+z_2) + ch(z_1-z_2)$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad ch\,x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}; \quad sh\,x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

## INTEGRALA COMPLEXĂ

$(\mathscr{C})\ z = z(t) = x(t) + y(t)$; $t \in I$.

- Curbă netedă dacă $x, y$ - sunt funct. deriv și $x'^2(t) + y'^2(t) > 0$

$t_0$
$$\frac{x - x(t_0)}{x'(t_0)} = \frac{y - y(t_0)}{y'(t_0)}$$

Curbă netedă pe porțiuni

$(\mathscr{C})$: $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ $t \in [a, b]$ s.n. netedă pe porțiuni dacă $\exists$ punctele $a < t_1 < t_2 \ldots < t_m < b$ a.î. pe fiecare din intervalele $[t_k, t_{k+1}]$ curba să fie derivabilă, și să îndeplinească restu

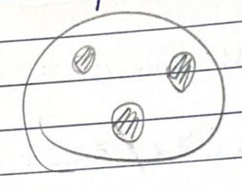
• Curbă simplă: dacă nu are pct. multiple
$M(x(t_1), y(t_1))$ s.n pct multiplu dc $\exists\, t_2 \neq t_1$.
$t_2 \in [a, b]$ a.î. $x(t_1) = x(t_2)$
$y(t_1) = y(t_2)$

Domeniu $D \subset \mathbb{C}$ cu prop că $D$ este deschisă și conexă

→ Coroană circulară
$C(r, R, z_0) = \{ z \in \mathbb{C} \mid r < |z - z_0| < R \}$

Alg • O mult s.n convexă dacă $\forall$ iau 2 pct ale sale segm de dr det de cele 2 pct e inclus în mult.

$z_1, z_2 \in D.\ \forall\, t \in [0, 1]$.
$(1-t)z_1 + t z_2 \in D$



Am • Mulțime conexă
• Domeniu conex
$f: [a, b] \to \mathbb{C}$
$f(t) = f_1(t) + j f_2(t)$

• $\int_a^b f(t)\,dt = \int_a^b f_1(t)\,dt + j \int_a^b f_2(t)\,dt$

fie $f: D \to \mathbb{C}$ - o funct comp de var compe
$(\mathcal{C}) \subset D$    $(\mathcal{C}): z = z(t),\ t \in [a, b]$
↳ curbă simplă

Def Spunem că funct $f$ e integrabilă pe curba $\mathcal{C}$ dacă există:
$$\int_a^b f(z(t))\, z'(t)\,dt = \int_{\mathcal{C}} f(z)\,dz.$$

fie $f(z) = u(x, y) + i\, v(x, y)$
at $f(z(t)) \cdot z'(t) = [u(x(t), y(t)) + v(x(t), y(t))] \cdot (x'(t) + j y'(t))$
$z(t) = x(t) + j y(t)$

$= u(x(t), y(t)) \cdot x'(t) - v(x(t), y(t)) \cdot y'(t) + (u(x(t), y(t)) \cdot y' + x' \cdot v(x(t), y(t)))$

$$\int_{\mathscr{C}} f(z)dz = \int_a^b [u(x(t)y(t))x'(t) - v(x(t)y(t))y'(t)]dt + j\int_a^b [u(x(t), y(t))y'(t) + x'(t)\cdot v(x(t), y(t))dt.$$

$$= \int_{\mathscr{C}} udx - vdy + j\int_{\mathscr{C}} v(x, y)dx + u(x, y)dy$$

- Dacă curba $\mathscr{C}$ este netedă pe porțiuni $t_0 = a, t_{m+1} = b, k = \overline{0, m}$ at prin def

$$\int_{\mathscr{C}} f(z)dz = \sum_{k=0}^{m} \int_{\mathscr{C}_k} f(z)dz.$$

$\mathscr{C}_k \to t_k, t_{k+1} \in$ Curba

- $(\mathscr{C}): |z-a| = r > 0$ $f(z) = (z-a)^m, m \in \mathbb{Z}$

$$\int_{\mathscr{C}} (z-a)^m dz = \int_0^{2\pi} r^m e^{jmt} \cdot rj \cdot e^{jt} dt$$

$(z-a)'$

$z - a = r \cdot e^{jt}$ ; $t \in [0, 2\pi)$

$$= r^{m+1} j \int_0^{2\pi} e^{jt(m+1)} dt$$

$$= r^{m+1} \begin{cases} 0, & m \neq -1 \\ 2\pi j, & m = -1. \end{cases}$$

- $$\int_{|z|=5} |z|^k dz = 5^k \int_{|z|=5} dz = 5^k \int_0^{2\pi} 5je^{jt} dt = 0$$

$m = 1 \Rightarrow 0$

TEOREMA LUI CAUCHY

Fie $f$ o funct olomorfă pe un domeniu $D$ și cu val în $\mathbb{B}$. $\mathscr{C}$ o curbă închisă, simplă $f \in \mathcal{H}(D)$ situată în $D$. At:

$$\mathscr{C} \subset D \Rightarrow \int_{\mathscr{C}} f(z)dz = 0$$

$$\int_{\mathscr{C}} f(z)dz = \int_{\mathscr{C}} udx - vdy + j\int_{\mathscr{C}} vdx + udy.$$

u și v sunt f deriv cu deriv continuă.

$$\oint_{\gamma} P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) \partial x\, \partial y$$

P, Q funct diferentiabile $\Rightarrow P = u,\ Q = v$

$$\Rightarrow - \underbrace{\iint_D \left(-\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dx\,dy}_{=0\ (C.R)} + j \underbrace{\iint_D \left(+\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) dx\,dy}_{0}$$

$= 0$

~~f(x, y)~~

TEOREMĂ

Dc f. e o funct olomorfă în domeniul D at.

$\exists$ o funcț F olomorfă cu prop.

$$F'(z) = f(z)$$