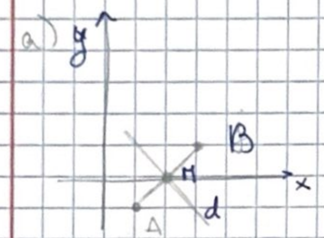
MSI - SEMINAR 1

1. a) Să se scrie ec med segm. $A(1-j)$, $B(3+j)$
b) ec dr care trece prin $A_1(5j)$, $A_2(1-5j)$
c) să se desenexe mult cu prop $4 \le |z - j| \le 5$
d) desen mult $|z - 1 + i| + |z| = 4$.

a)



panta $AB = \frac{1+1}{3-1} = 1$.

d - med AB => $m_d = -1$

M - mij. AB => $M\left(\frac{1-j+3+j}{2}\right)$ =>
$M = 1 ~~A_1~~ - A_2$

M (2)

=> $d: x - 2 = -1(y - 0)$ => $d: y = 2 - x$

b) ~~$m_{A_1A_2} = \frac{-5-5}{}$~~ med - locul geom cu prop că $d(pct, A) = d(pct, B)$

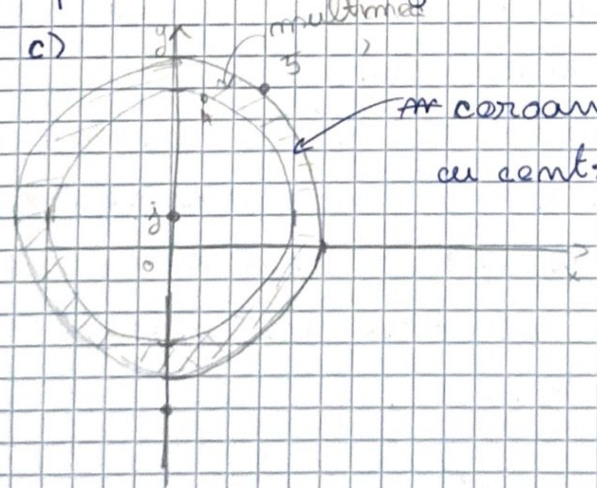
$d(z, A) \qquad d(z, B)$

a) $|z - 1 + j| = |z - 3 - j|$ => ec med. dacă scăpăm de ||

b) $z(t) = (1-t) \cdot 5j + t \cdot (1 - 5j)$; $t \in \mathbb{R}$.

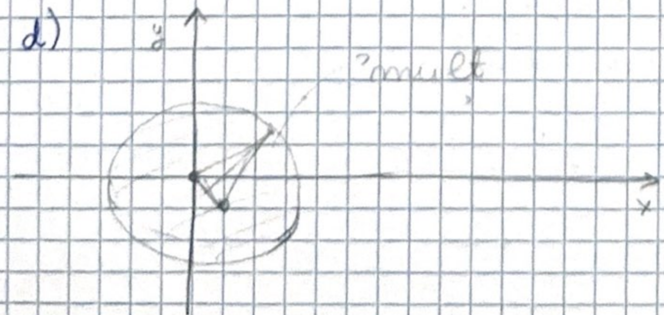
pt $t \in [0, 1]$ => segment

c)



coroană circulară cu centru în j și de raze 4 și 5

d)



elipsă - 2 focare

$d(A) + d(B) = 4$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

semiaxele elipsei

(2) $z_1, \dots, z_n \quad |z_1| = |z_2| = \dots = |z_n| = 1$

Det $a \in \mathbb{R}$ a.î. $w = \dfrac{(1+z_1)(1+z_2)\cdots(1+z_n)}{a + z_1 z_2 \cdots z_n} \in \mathbb{R}$

$w = \overline{w}$

$$\overline{w} = \frac{\overline{(1+z_1)}\,\overline{(1+z_2)}\cdots\overline{(1+z_n)}}{a + \overline{z_1} \cdot \overline{z_2} \cdots \overline{z_n}}$$

$$\overline{w} = \frac{(1+\overline{z_1})(1+\overline{z_2})\cdots(1+\overline{z_n})}{a + \overline{z_1} \cdot \overline{z_2} \cdots \overline{z_n}}$$

$$|z_k| = \sqrt{z_k \cdot \overline{z_k}} \Rightarrow z_k \cdot \overline{z_k} = 1 \Rightarrow \overline{z_k} = \frac{1}{z_k}$$

$$\overline{w} = \frac{\left(1+\frac{1}{z_1}\right)\left(1+\frac{1}{z_2}\right)\cdots\left(1+\frac{1}{z_n}\right)}{a + \dfrac{1}{z_1 z_2 \cdots z_n}}$$

$$\overline{w} = \frac{\left(\frac{z_1+1}{\cancel{z_1}}\right)\cdot\left(\frac{z_2+1}{\cancel{z_2}}\right)\cdots\left(\frac{z_n+1}{\cancel{z_n}}\right)}{\dfrac{a \cdot z_1 z_2 \cdots z_n + 1}{\cancel{z_1 z_2 z_3 \cdots z_n}}}$$

$$\overline{w} = \frac{(1+z_1)(1+z_2)\cdots(1+z_n)}{1 + a z_1 z_2 \cdots z_n} \Rightarrow w = \overline{w} \Rightarrow$$

$$\Rightarrow \frac{\cancel{(1+z_1)\cdots(1+z_n)}}{a + z_1 z_2 \cdots z_n} = \frac{\cancel{(1+z_1)}\cdots(1+z_n)}{1 + a z_1 z_2 \cdots z_n} \overset{*}{\Rightarrow}$$

$\Rightarrow \cancel{a+1=}\ a + z_1 \cdots z_n = 1 + a z_1 z_2 \cdots z_n$

$\Rightarrow 1 - z_1 \cdots z_n = a(1 - z_1 \cdots z_n) \Rightarrow (a-1)(1 - z_1 \cdots z_n) = 0$

* I Dc. $\exists z_k = -1 \Rightarrow a \in \mathbb{R}$.

II a) $a = 1$. (Dc $\nexists z_k = -1$)

b) $z_1 \cdots z_n = 1 \Rightarrow a \in \mathbb{R}$

(3) $z_1, z_2 \dots z_n \in \mathbb{C}^*$ cu prop

$p = |z_1| \dots |z_n|$

$$\left|\frac{z_k^n}{z_1 \dots z_n}\right| \in \mathbb{N}^* \ \forall k = \overline{1,n}$$

Să se arate că nr complexe z_k sunt conciclice (pe același cerc.

$\cancel{|z_k| = a_k}$

$$\left|\frac{z_k^n}{z_1 \dots z_n}\right| = a_k \cdot \cancel{a_k \in \mathbb{N}^*}, \ k = \overline{1,n}$$

$$a_k = \frac{|z_k^n| \dots}{|z_1| \cdot |z_2| \dots |z_n|} \quad \Big| \ a_1 a_2 \dots a_n.$$

$$a_1 \cdot a_2 \dots a_n = \frac{|z_1^n|}{|z_1| \dots |z_n|} \cdot \dots \cdot \frac{|z_n^n|}{|z_1| \dots |z_n|}$$

$$\left.\begin{array}{l} \Rightarrow a_1 \cdot a_2 \cdot \dots \cdot a_n = 1. \\ a_k \in \mathbb{N}^* \end{array}\right\} \Rightarrow a_1 = a_2 = \dots = a_n = 1.$$

$a_k = 1.$

$$\Rightarrow 1 = \frac{|z_k^n|}{p} \Rightarrow |z_k| = \sqrt[n]{p}. \quad \forall k = \overline{1,n}.$$

$\hookrightarrow$ aceeași nr.

$\Rightarrow z_k$ conciclice.

(4) a) z_1, z_2, z_3

$|z_1| = |z_2| = |z_3| = r > 0$ (nr dif de 0)

Să se calc în funcție de r.

$$\left|\frac{z_1 + z_2 + z_3}{z_1 z_2 + z_1 z_3 + z_2 z_3}\right| \leftarrow \begin{pmatrix}4\\1\end{pmatrix} \ \begin{pmatrix}4\\3\end{pmatrix}$$

se poate și $\dfrac{\binom{8}{5}}{\binom{8}{1}}$

b) generalizare $|z_1| = |z_2| = |z_3| = |z_4|$

$$\left|\frac{z_1 + z_2 + z_3 + z_4}{z_1 z_2 z_3 + z_1 z_2 z_4 + z_1 z_3 z_4 + z_2 z_3 z_4}\right| \ \text{calc.}$$

în funcție de R.

a) $\left|\dfrac{z_1+z_2+z_3}{z_1z_2+z_2z_3+z_1z_3}\right|$ II se pot interschimba, se face la fel

a_i = s_1, s_2, s_3

$x^2-sx+p=0$

$z^3 - s_1 z^2 + s_2 z - s_3 = 0$ ← ec gr III

$|z_1 z_2 z_3| = r^3$

$\Rightarrow = \left|\dfrac{z_1+z_2+z_3}{\dfrac{r^3}{z_3}+\dfrac{r^3}{z_1}+\dfrac{r^3}{z_2}}\right|$

$|\bar{z}|=|z|$ $\quad r^2 = z_k \cdot \bar{z}_k \Rightarrow \dfrac{1}{z_k} = \dfrac{\bar{z}_k}{r^2}$

$\left|\dfrac{z_1+z_2+z_3}{r(\bar{z}_1+\bar{z}_2+\bar{z}_3)}\right| = \dfrac{|z_1+z_2+z_3|}{r\,|\bar{z}_1+\bar{z}_2+\bar{z}_3|} = \dfrac{1}{r}$

b) $z^4 - s_1 z^3 + s_2 z^2 - s_3 z + s_4 = 0$

$\Rightarrow |z_1 z_2 z_3 z_4| = r^4$ $\quad \Rightarrow r^2 = z_k \cdot \bar{z}_k$

$\Rightarrow \dfrac{|z_1+z_2+z_3+z_4|}{\left|\dfrac{r^4}{z_1}+\dfrac{r^4}{z_2}+\dfrac{r^4}{z_3}+\dfrac{r^4}{z_3}\right|} = \dfrac{1}{r^2}$ $\quad \Rightarrow \dfrac{1}{z_k} = \dfrac{\bar{z}_k}{r^2}$

$\Rightarrow r^2 \dfrac{|z_1+z_2+z_3+z_4|}{|\bar{z}_1+\bar{z}_2+\bar{z}_3+\bar{z}_4|} = \dfrac{1}{r^2}$

$|z| = |\bar{z}|$

⑤ Să se calc lim șirului de nr complexe

$z_n = \sum\limits_{k=1}^{n} \dfrac{1}{n+2k \cdot j} = \sum\limits_{k=1}^{n} \dfrac{n-2kj}{n^2+4k^2} = \sum\limits_{k=1}^{n} \dfrac{n}{n^2+4k^2} - \dfrac{2kj}{n^2+4k^2}$

calc lim $\Rightarrow$ conv $\Rightarrow$ ~~lim. = x~~ $+ jy$

$\lim z_n = \lim x_n + j \lim y_n$

sumă Riemann

$\mathrm{Re}\, z_n = \sum\limits_{k=1}^{n} \dfrac{n}{n^2+4k^2}$ $\quad \Rightarrow z_n = \sum\limits_{k=1}^{n} \dfrac{n}{n^2+4k^2} - j \sum\limits_{k=1}^{n} \dfrac{2k}{n^2+4k^2}$

$\mathrm{Im}\, z_n = \sum\limits_{k=1}^{n} \dfrac{-2k}{n^2+4k^2}$

$\Rightarrow = \dfrac{1}{n} \sum\limits_{k=1}^{n} \dfrac{1}{1+\dfrac{4k^2}{n^2}} - j \cdot \dfrac{1}{n} \sum\limits_{k=1}^{n} \dfrac{\dfrac{2k}{n}}{1+\dfrac{4k^2}{n^2}}$

$\lim z_n \longrightarrow \int_0^1 \frac{1}{1+4x^2}dx - \int_0^1 \frac{2x}{1+4x^2}\,dx$

$\lim z_n \longrightarrow \frac{1}{2} \operatorname{arctg} 2x \Big|_0^1 - \frac{1}{4} \ln(1+4x^2)\Big|_0^1$

$z_n \to \frac{1}{2} \operatorname{arctg} 2 - \frac{1}{4} \ln(5)$

$z_n \to \frac{1}{2} \operatorname{arctg} 2 - \frac{1}{4} \ln 5$