Question 1

Consider the finite difference scheme

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \frac{c}{2\Delta x}\left(-3u_i^n + 4u_{i+1}^n - u_{i+1}^n\right) = 0$$

which is an approximation for $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$.

a) Is this discretisation consistent, and if it is, what is its order of accuracy in time and in space? (Multiple-choice)

b) Consider now the stability of this finite difference scheme. Compute the amplification factor ρ.

Question 2

Consider the Laplace equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

on the domain $0 \leq x \leq 1, 0 \leq y \leq 1$.

a) An engineer wants to use a manufactured solution to compute the order of accuracy. He decides to use the manufactured solution $u = e^{xy}$. Give an expression for the source term $S(x, y)$.

b) For this manufactured solution, give the expression for the boundary condition along $y = 0$, i.e., what is $u(x, 0)$ equal to?

c) The engineer decides to be a daredevil and try different manufactured solutions. Which of the following manufactured solutions is/are suitable (multiple answers possible)?

1. $x + y$
2. $\sqrt{xy}$
3. $\sin\left(\frac{x}{y-\frac{1}{2}}\right)$
4. $\frac{1}{(x+1)(y+1)}$
5. $y\cos(x)$

Question 3

An engineer obtains the solution values for various step sizes Δx shown in the table below.

Δx	f
1	47.2
$\frac{1}{2}$	43.1
$\frac{1}{4}$	40.5
$\frac{1}{8}$	40.2

a) Take the best three rows: compute the observed order of accuracy.

b) What can be said about the first three rows?

- They are suitable for Richardson extrapolation.
- They are suitable for Richardson interpolation.
- They use a finer mesh than the bottom three rows.
- They are unsuitable for Richardson extrapolation.

c) Take the best three rows: compute the best estimate of the error.

Question 4

An engineer wishes to solve the PDE

$$\frac{\partial^2 u}{\partial x^2} = f$$
$$u_x(0) = 0$$
$$u(1) = g$$

where $g = 0$.

a) Consider the weighting functions shown in figure 1; which is/are suitable weighting functions? (multiple answers possible)

- $\phi_1 = x$
- $\phi_2 = 1 - x$
- $\phi_3 = 1 - x^2$

b) Now, set $f = 1$, and weighting function ϕ_2 will not be omitted. Compute the coefficients a_1 and a_3.

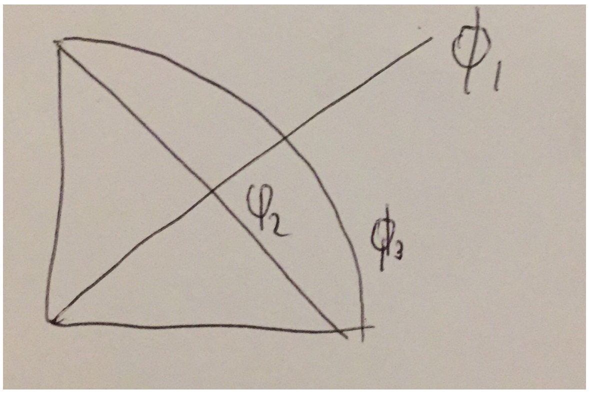

Figure 1: Weighting functions.