Question 1

An engineer runs a computational model to predict the maximum point load that can be placed on the center of a thin, circular disk of an aluminium-alloy before failure occurs. He validates his model by comparing the numerical solution to an experiment where ten circular disks were placed under a point load on the center of the disk; this point load was then continuously increased until the disk failed.

The ten outcomes of this experiment all deviated slightly from the predicted solution. These variations can best be described as

1. Model error
2. Aleatory uncertainty
3. Discretisation error
4. Iteration error
5. Epistemic uncertainty
6. Round-off error

Question 2

Part a)

The inclusion of an artificial dissipation term in the numerical method can best be described as

1. Model error
2. Aleatory uncertainty
3. Discretisation error
4. Iteration error
5. Epistemic uncertainty
6. Round-off error

Part b)

The error resulting from using an upwind scheme do deal with dispersion can best be described as

1. Model error
2. Aleatory uncertainty
3. Discretisation error
4. Iteration error
5. Epistemic uncertainty
6. Round-off error

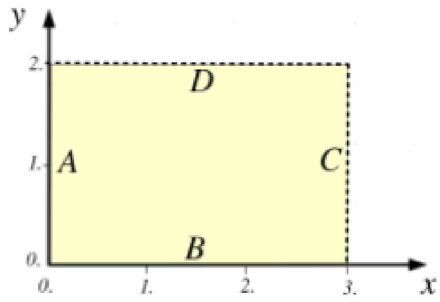
Question 3

A true boundary can have ...

1. Both a physical and numerical condition.
2. Only a physical condition.
3. Only a numerical condition.

Question 4

Part a)



Consider the domain shown above, where the following governing equation holds:

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$$

This equation can be characterized as

- Elliptic
- Parabolic
- Hyperbolic

Part b)

What is the correct number of boundary conditions that may be imposed on the boundaries (multiple answers possible)?

1. A: 1, B: 2, C: 1, D: 0
2. A: 2, B: 1, C: 0, D: 1
3. A: 1, B: 1, C: 1, D: 1
4. A: 1, B: 1, C: 0, D: 0
5. A: 1, B: 0, C: 1, D: 0

Part c)

Consider a square domain, with boundaries $x = 0$ being boundary A, $y = 0$ being boundary B, $x = 1$ being boundary C and $y = 1$ being boundary D (i.e. the same domain as shown above, but rescaled to make it a 1×1 square. Consider the following governing equation:

$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 0$$

This equation has characteristic lines along with

$$y = Ax + \text{constant}$$

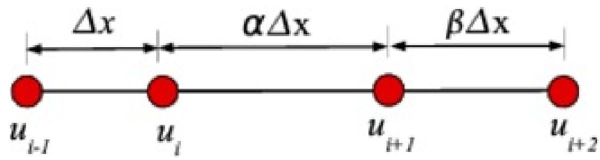
What is/are the possible value(s) of A?

Part d)

Continuing part c), what is the correct number of boundary conditions that may be imposed on the boundaries (multiple answers possible)?

1. A: 1, B: 2, C: 1, D: 0
2. A: 2, B: 1, C: 0, D: 1
3. A: 1, B: 1, C: 1, D: 1
4. A: 3, B: 2, C: 1, D: 0
5. A: 1, B: 0, C: 1, D: 2
6. A: 0, B: 2, C: 0, D: 2
7. A: 1, B: 1, C: 0, D: 0
8. A: 1, B: 0, C: 1, D: 0

Question 5



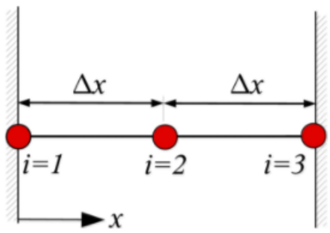
Consider the stencils shown above. Take $\alpha = 2$ and $\beta = 3$, and one aims $\frac{\partial^2 u}{\partial x^2}$ at u_i.

	$a \cdot u_{i-1}$	$b \cdot u_i$	$c \cdot u_{i+1}$	$d \cdot u_{i+2}$	
u_i	A_2	A_3	A_4	A_5	A_6
B_1	B_2	B_3	B_4	B_5	B_6
C_1	C_2	C_3	C_4	C_5	C_6
D_1	D_2	D_3	D_4	D_5	D_6
E_1	E_2	E_3	E_4	E_5	E_6

What should be the following entries?

- A_2
- B_3
- C_4
- D_6
- E_5

Question 6



Consider we want to approximate the spatial derivative $\frac{\partial^2 u}{\partial x^2}$ at the middle node $i = 2$ by using the nodes at $i = 1$, $i = 2$ and $i = 3$. What is the leading term in the truncation error? [Hint: use row reduction rather than matrix inversion to find the coefficients.]

Question 7

Part a)

Consider the following PDE:

$$\frac{\partial^2 u}{\partial y^2} - x\frac{\partial^2 u}{\partial x^2} = \frac{\sqrt{y}}{u^2+1}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x}$$

on the domain $0 < x < 1$ and $0 < y < 1$. This PDE can be described as

1. Elliptic
2. Parabolic
3. Hyperbolic

Part b)

Consider the following PDE:

$$\left(\frac{\partial u}{\partial x}\right)^2 \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} + \sqrt{u}\frac{\partial^2 u}{\partial x \partial y} = \ln(u)$$

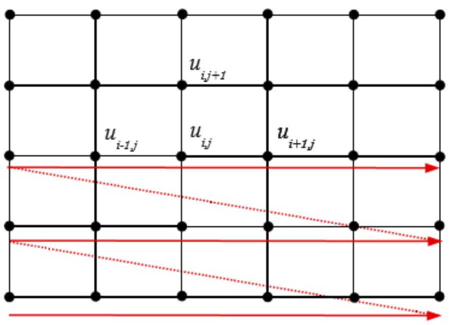
How can this PDE best be characterized as?

1. Linear
2. Quasi-linear
3. Non linear

Question 8

Part a)

Consider the 5×6 mesh shown below.

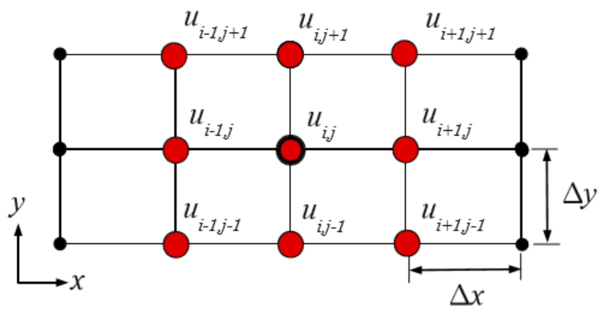


Suppose the following PDE governs:

$$\frac{\partial^2 u}{\partial x^2} + k\frac{\partial^2 u}{\partial y^2} = 0$$

with k a positive number. Someone uses the stencil shown below, using the nodes $u_{i-1,j}$, $u_{i,j-1}$, $u_{i,j}$, $u_{i,j+1}$ and $u_{i+1,j}$ to estimate $u_{i,j}$. The following finite difference formulas are found:

$$\begin{aligned}
\frac{\partial^2 u}{\partial x^2} &= \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} \\
\frac{\partial^2 u}{\partial y^2} &= \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} \\
\frac{\partial u}{\partial x} &= \frac{u_{i+1,j} - u_{i,j}}{\Delta x}
\end{aligned}$$



This leads to the algebraic system $[A]\mathbf{u} = \mathbf{b}$, where $\mathbf{u}$ is the vector of unknowns, which starts with $u_{1,1}$ and is ordered by first increasing i, aligned with x, then increasing j. $\mathbf{b}$ is a vector of equivalent dimension filled with numbers.

What is the matrix coefficient $A_{15,15}$ (i.e. the coefficient in the 15th row, 15th column of Matrix A)?

Part b)

What is the corresponding value of $\mathbf{b}$, i.e. b_{15}?

Part c)

What is the bandwidth of this discretisation method?

Part d)

Using this discretisation method, what is the total number of nodes at which boundary conditions, either numerical or physical, need to be imposed?

Part e)

Suppose one wishes to use a Neumann boundary on the bottom nodes (located at $y = 0$), i.e $\left.\frac{\partial u}{\partial x}\right|_{y=0} = \sin(x)$. What will then the following entries be?

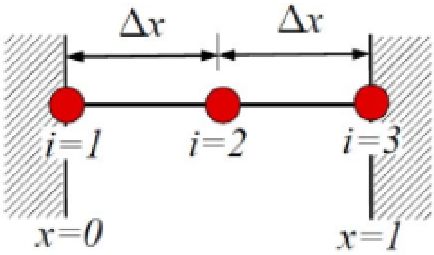
- $A_{1,1}$ (i.e. the entry in matrix $[A]$ in the first row, and first column).
- $A_{1,2}$ (i.e. the entry in matrix $[A]$ in the first row, second column).
- $A_{1,3}$ (i.e. the entry in matrix $[A]$ in the first row, third column).
- b_1 (i.e. the first entry of vector $\mathbf{b}$).

Question 9

Consider the following discretisation of a PDE using the finite-difference method

$$\frac{1}{\Delta x}\begin{bmatrix} \Delta x & 0 & 0 \\ \frac{-1}{2} & \Delta x & \frac{1}{2} \\ \frac{1}{2} & -2 & \frac{3}{2} \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

using the nodes shown in the figure below.



Part a)

What is the PDE that is being solved here?

1. $u_{xx} + u_x = 0$
2. $u_{xx} + u = 1$
3. $u_x + u = 1$
4. $u_x + u = 0$
5. $u_x x + uu_x = 0$
6. $uu_x = 1$

Part b)

What boundary condition is used at $x = 0$?

1. Dirichlet: $u(0) = 1$
2. Neumann: $u(0) = 1$
3. Dirichlet: $u(0) = 0$
4. Neumann: $u(0) = 0$
5. Dirichlet: $u_x(0) = 0$
6. Neumann: $u_x(0) = 0$
7. A numerical condition

Part c)

What boundary condition is used at $x = 1$?

1. Dirichlet: $u(1) = 1$
2. Neumann: $u(1) = 1$
3. Dirichlet: $u(1) = 0$
4. Neumann: $u(1) = 0$
5. Dirichlet: $u_x(1) = 0$
6. Neumann: $u_x(1) = 0$
7. A numerical condition