[PRINT]

AE2220-II Computational Modelling (2018-2019),
Filippo Tagliacarne, 6/23/19 at 4:18:13 PM CEST

Question 1: *Score 0.5/1*

Consider the equation

$u_t = r\,u$

If the Euler implicit method with time step T is used to integrate the equation in time, the numerical amplification factor will be:

$\rho =$

Your response	Correct response
1+r*T	**1/(-T*r+1)**

Grade: 0/1.0

and the amplitude error will be $er_a =$

Your response	Correct response
abs(exp(r*T))	$\left|e^{(rT)}\right|$

Grade: 1/1.0

$- \operatorname{abs}(\rho)$

(enter a function of r and use abs(x) to denote the magnitude of x).

Total grade: 0.0×1/2 + 1.0×1/2 = 0% + 50%

Question 2: *Score 0.66/1*

Consider the following two-stage time march:

$$\tilde{u} = u^n + \frac{\Delta t}{2}\left(\frac{\mathrm{d}\,u^n}{\mathrm{d}\,t}\right)$$

$$u^{n+1} = u^n + \Delta t\left(\frac{\mathrm{d}\,\tilde{u}}{\mathrm{d}\,t}\right)$$

The $\lambda - \sigma$ relation of the time march can be expressed (use $T = \Delta t$, $L = \lambda$ and $S = \sigma$):

answer:

Your response	Correct response
S=1+T*L+(1/2)*(T*L)^2	**S = 1 + L*T + L*L*T*T/2**

Grade: 1/1.0

(enter the complete equation including an = sign)

This is an approximation for the series expansion of (enter a function of L and T)

Your response	Correct response

LT	$_e(LT)$

Grade: 0/1.0

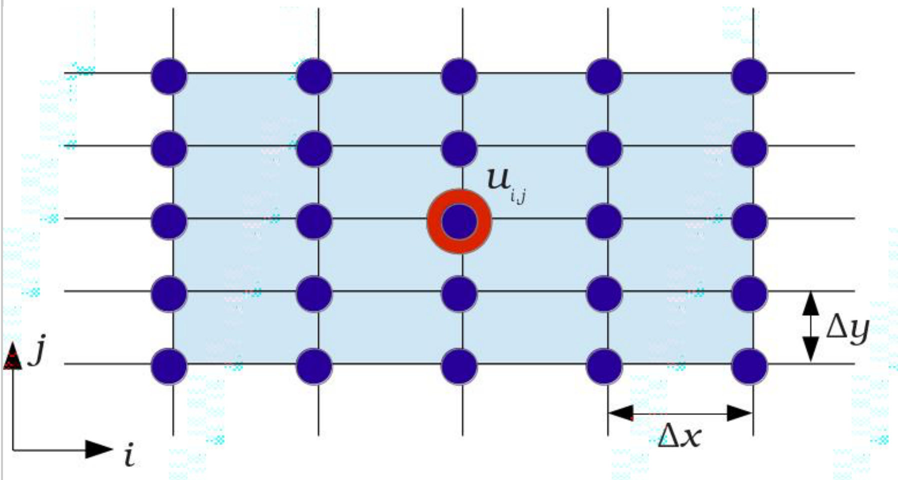
The $\lambda - \sigma$ relation above

Your response	Correct response
does not have spurious roots	**does not have spurious roots**

Grade: 1/1.0

Total grade: 1.0×1/3 + 0.0×1/3 + 1.0×1/3 = 33% + 0% + 33%

Question 3: *Score 0/1*



A finite-difference discretisation for a PDE with Dirichlet BCs uses a centered 3-point stencil in both the i and j directions on the domain shown above. If the resulting algebraic system is to be solved by a Jacobi $j = constant$ line method, and the solution vector is ordered in terms of groups of increasing i for each j, each iteration will require the solution of a smaller matrix problem of the form:

Your response	Correct response
$\begin{pmatrix} a_{11} & a_{12} & 0 & a_{14} & 0 \\ a_{21} & a_{22} & a_{23} & 0 & a_{25} \\ 0 & a_{32} & a_{33} & a_{34} & 0 \\ a_{41} & 0 & a_{43} & a_{44} & a_{45} \\ 0 & a_{52} & 0 & a_{54} & a_{55} \end{pmatrix}$	$\begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 \\ 0 & a_{32} & a_{33} & a_{34} & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & a_{54} & a_{55} \end{pmatrix}$

Grade: 0/2.0

The solution method which would be most efficient for this smaller problem is

Your response	Correct response
Gaussian elimination	**the Thomas algorithm**

Grade: 0/1.0

which scales with

Your response	Correct response
N^3	**N**

Grade: 0/1.0

(enter a function of N).

In general, a robust method for find the solution of a system with a non-sparse matrix is

Your response	Correct response
point Gauss-Seidel iteration	**Gaussian elimination**

Grade: 0/1.0

which scales with

Your response	Correct response
N^2	**N^3**

Grade: 0/1.0

(enter a function of N).

Total grade: 0.0×2/6 + 0.0×1/6 + 0.0×1/6 + 0.0×1/6 + 0.0×1/6 = 0% + 0% + 0% + 0% + 0%

Question 4: *Score 0.5/1*

(This question consists of 3 parts)

Consider the folllowing Euler Implicit upwind discretisation for the linear advection equation:

$$\frac{u_i^{m+1} - u_i^m}{\Delta t} + \frac{c\left(u_i^{m+1} - u_{i-1}^{m+1}\right)}{\Delta x} = 0$$

where m indicates the current time step.

Part 1

Using the notation $U_i = u_i^{m+1}$, $V_i = u_i^m$, and $C = \frac{c\Delta t}{\Delta x}$,
write the update formula from iteration n to $n+1$ for a point-Jacobi method used to iteratively determine the solution at the next time step, U_i .

$U_i^{n+1} =$

Your response	Correct response
V[i]^n - C*(U[i]^n - U[i-1]^n)	$\frac{V_i + C\,U_{(i-1)}{}^n}{1+C}$

Grade: 0/2.0

(use the notation U[i-1]^(n) to indicate terms such as U_{i-1}^{n} .

Note also that the values of V_i do not change during the iteration, so such terms require no "n" superscript and can be entered as just V[i] .

Finally note that all U_i^{n+1} terms must be gathered to the left-hand side)

Part 2

Assume the method is re-written as a point Gauss-Seidel method with relaxation factor Q.

If the convergence of this method with $Q = 1$ is erratic (the error both increases and decreases as the iterations proceed) one should try setting

Your response	Correct response
Q less than 1 but greater than 0	**Q less than 1 but greater than 0**

Grade: 2/2.0

Part 3

One could also re-write the method as a line Gauss-Seidel method with relaxation. This would likely result in

Your response	Correct response
an improved	**an improved**

Grade: 1/1.0

rate of convergence with

Your response	Correct response
a better	**the same**

Grade: 0/1.0

scaling of work with the size of the system, N (compared with the method considered in **Part 2**) .

Total grade: 0.0×2/6 + 1.0×2/6 + 1.0×1/6 + 0.0×1/6 = 0% + 33% + 17% + 0%