Question 1

An engineer is running a simulation of a time-dependent partial differential equation. He increases the timestep Δt. What kind of error increases/decreases with this?

1. Model error: increases
2. Model error: decreases
3. Discretisation error: increases
4. Discretisation error: decreases
5. Iteration error: increases
6. Iteration error: decreases
7. Round-off error: increases
8. Round-off error: decreases

Question 2

An engineer runs a model of a wind turbine, as sketched in figure 1, with the cylindrical tube denoting the boundary of the domain to be evaluated. At this boundary, the velocity V is set constant. The error associated with this boundary condition can be best characterised as

1. Model error
2. Aleatory uncertainty
3. Discretisation error
4. Iteration error
5. Epistemic uncertainty
6. Round-off error

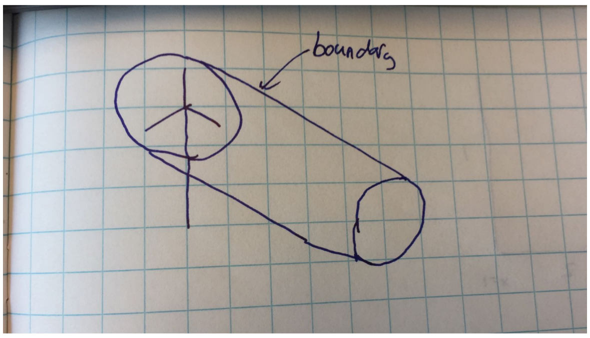


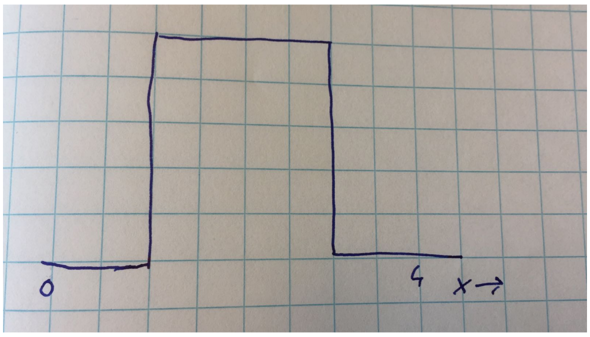
Figure 1: Wind mill.

Question 3

Consider the partial differential equation

$$\frac{\partial u}{\partial t} + \frac{1}{x}\frac{\partial u}{\partial x} = 0$$

and consider the initial condition shown in the figure below.



The following solution behaviour should be observed:

- The peak will contract as it convects to the right
- The peak will not move but its magnitude will decrease in time
- The peak will expand as it convects to the right
- The peak will contract as it convects to the left
- The peak will split into to left-going and right-going components.
- The peak will not move but its magnitude will increase in time
- The peak will expand as it convects to the left

Question 4

Consider the partial differential equation

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$$

Part a)

How many characteristics does this partial differential equation have? Enter an integer number.

Part b)

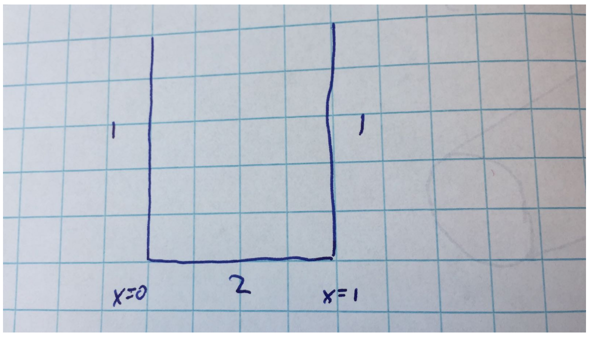
What property is constant along a characteristic? Choose one of

- The solution value is constant along a characteristic.
- The solution value is zero along a characteristic.
- A linear combination of the first derivatives of u is constant along a characteristic.
- A linear combination of the first derivatives of u is zero along a characteristic.

Part c)

The number of applied boundary conditions along each boundary is shown in the figure below. What kind of boundary condition can be applied along the $x = 0$ boundary (i.e. the left boundary)?

- Dirichlet
- Dirichlet or Neumann
- Dirichlet, Neumann or numerical
- Dirichlet or numerical
- Neumann
- Neumann or numerical
- Numerical



Question 5

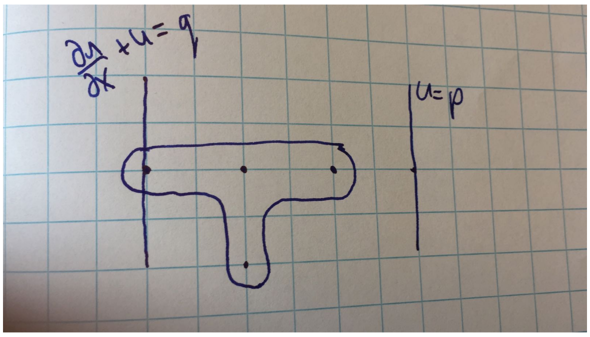
Consider the domain shown in the figure below, which consists of four nodes in spatial direction. The governing PDE is

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

and along the left boundary, the following boundary condition is applied:

$$\frac{\partial u}{\partial x} + u = q$$

whereas along the right boundary, the boundary condition $u = p$ is applied.



Application of an implicit finite difference scheme, using the drawn stencil in the figure, leads to a matrix equation of the form

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} u_1^{n+1} \\ u_2^{n+1} \\ u_3^{n+1} \\ u_4^{n+1} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

Part a)

Calculate the entry A_{11}.

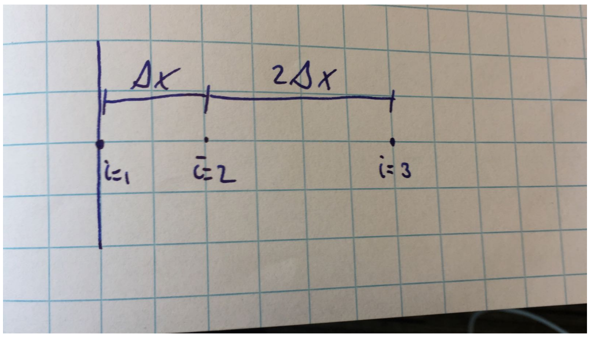
Part b)

Calculate the entry A_{21}.

Part c)

Calculate the entry A_{43}.

Question 6



Consider the stencils shown above. One aims to find a central difference scheme for $\partial u/\partial x$ at u_1.

	$a \cdot u_i$	$b \cdot u_{i+1}$	$c \cdot u_{i+2}$	
u_i	A_{11}	A_{12}	A_{13}	B_1
	A_{21}	A_{22}	A_{23}	B_2
	A_{31}	A_{32}	A_{33}	B_3
	A_{41}	A_{42}	A_{43}	B_4

Part a)

Compute the entry A_{23}.

Part b)

Compute the entry B_2.

Part c)

For now, *assume* that the values on the last row are given by $A_{41} = 0$, $A_{42} = 3$ and $A_{43} = -1$. Compute the leading term in the truncation error. Use the following notation: $\Delta x = X$, $\partial u/\partial x = D$, $\partial^2 u/\partial x^2 = E$, $\partial^3 u/\partial x^3 = F$ and $\partial^4 u/\partial x^4 = G$.