DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Aerospace Engineering

TU Delft

Exam	AE2223-II Experimental Design and Data Analysis
Total number of pages	4 (including this page)
Date & time (start & end)	11 APril 2019 13:30 - 15:30
Responsible lecturer	Prof. dr. ir. P.N.A.M. Visser

Only the work / answers written on examination paper will be assessed, unless otherwise specified under 'Additional Information'.

Total number of questions: 4 (4 open and 0 multiple choice)

Max. number of points: 100

☐ **all questions have equal weight**
☑ **questions differ in weight** (and is mentioned per question, or in an overview)

Use of tools and sources of information during examination

Not allowed:

- Mobile (smart)phone, and/or any device with internet connectivity, unless otherwise specified below.
- Answers written with pencil.
- Tools and/or sources of information, unless otherwise specified below.

Allowed:

☐ books ☐ notes ☐ dictionaries ☐ syllabus ☐ formula sheets ☐ computer
☑ calculator, model/type: non-graphic
☐ other:

Additional information (if necessary)

Exam graded by: Prof. dr. ir. P.N.A.M. Visser & Dr. R.M. Groves
Date (max. 15 working days after examination date): 26 April 2019

Talking with other students in the examination room is not allowed
Every suspicion of fraud is reported to the Board of Examiners

Question 1 (30 points)

Consider the following equation, where *Range* observations are modeled as a linear function of *Time*:

$$Range \;=\; x_1 \;+\; Time \times x_2$$

The following Table lists the observations together with their noise level:

Time (s)	*Range* (m)	Noise level (m)
1.0	1.5	0.5
2.0	4.5	0.5
3.0	6.5	0.5
4.0	7.5	0.5

It is assumed that the observation **error covariance matrix** Q_y is a diagonal matrix, where the observations have a noise level as indicated in the Table above.

a (5 pt) Provide the full matrix Q_y (i.e. fill in the numerical values).

b (10 pt) Compute the weighted least-squares solution for x_1 and x_2 (specify the units). Clearly show the different steps.

c (5 pt) Compute the formal errors for x_1 and x_2 (specify the units).

d (10 pt) Compute the observation residuals scaled by their weight using the weighted least-squares method.

Use can be made of the following formulas:

$$\hat{x} \;=\; (A^T Q_y^{-1} A)^{-1} A^T Q_y^{-1} \hat{y}$$

$$Q_x \;=\; (A^T Q_y^{-1} A)^{-1}$$

$$Q_e \;=\; Q_y - A Q_x A^T$$

Question 2 (20 points)

a (5 pt) Why is point positioning by GPS typically an overdetermined estimation problem? (max 30 words)

b (5 pt) Why is point positioning by GPS an iterative estimation proces? (max 30 words)

c (5 pt) Mention five error sources that affect the positioning precision by GPS when using terrestrial (ground based) receivers. (max 10 words per error source)

d (5 pt) Why is it expected that navigation by GPS is less precise over the polar areas? (max 30 words)

Question 3 (20 points)

a (5 pt) Why is it possible to derive gravity field information from GPS observations on board of a LEO satellite? (max 50 words)

b (5 pt) What is the biggest advantage of having a GPS receiver on board of a LEO satellite for atmospheric profiling? (max 20 words)

c (5 pt) What is the post-facto precision level that can be achieved using high-quality GPS receivers for determining the distance between two LEO satellites that are separated by a few hundreds of kilometers? (max 10 words)

d (5 pt) Why is it possible to achieve higher precision levels for determining the relative position of nearby GPS receivers than their absolute precision? (max 30 words)

Question 4 (30 points)

The (unweighted) design matrix for point positioning by GPS is given by:

$$\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & \frac{1}{2}\sqrt{2} & 1.0 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\sqrt{2} & 1.0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\sqrt{2} & 1.0 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\sqrt{2} & 1.0 \\ 0.0 & 0.0 & 1.0 & 1.0 \end{bmatrix}$$

The estimated parameters are the position in longitude, latitude and height direction, respectively, and the receiver clock multiplied by the speed of light.

a (5 pt) How many GPS satellites are in view?

b (5 pt) Compute the normal matrix.

c (5 pt) What are the values for HDOP, PDOP, VDOP, TDOP and GDOP?

d (5 pt) It is assumed that the observation error level is 0.3 m. What is then the uncertainty of the horizontal position (m)?

e (5 pt) If the observation error level is 0.1 m, then what is then the uncertainty of clock estimate (in seconds)?

f (5 pt) What is the correlation between the clock and the height?

Note: use can be made of the formulas listed at Question 1