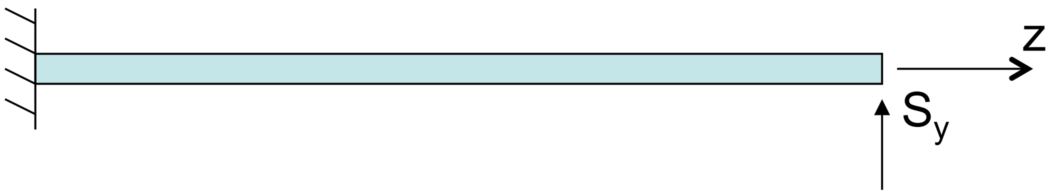
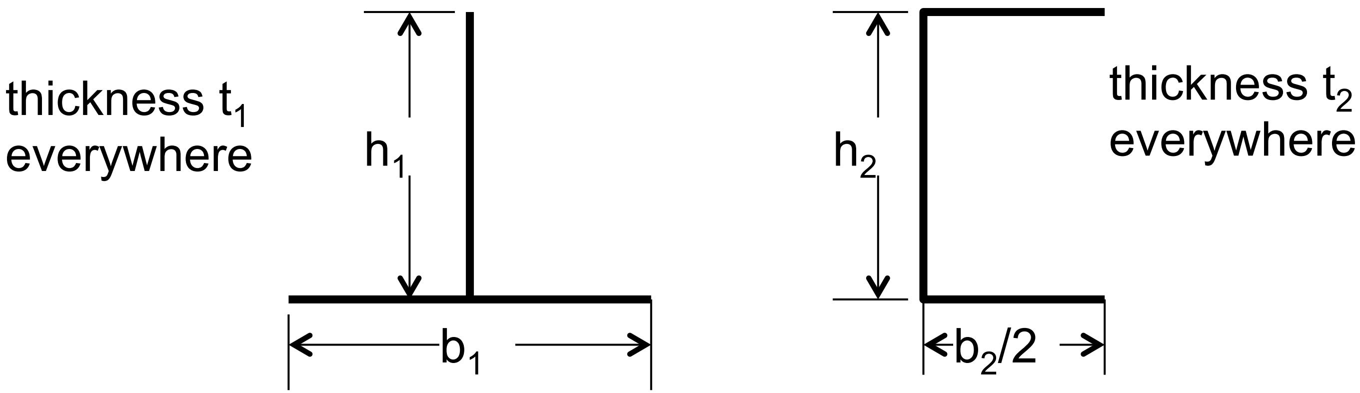
Example: Evaluation of beam cross-sections under shear and bending loads

- two different cross-sections of the exact same area are proposed for a cantilever beam of length L under shear load S_y



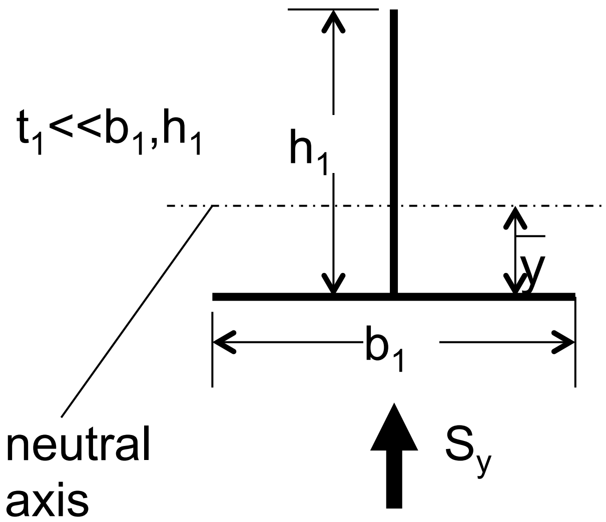
$t_1(b_1+h_1)= t_2(b_2+h_2)=A$



Example: Beam cross-sections under shear

- assume that the shear load is applied through the shear center in both cases (hence no twisting of the cross-section)

- section 1: “T” or “blade” section



- locate origin at neutral axis

- determine neutral axis location ($\bar{y}$):

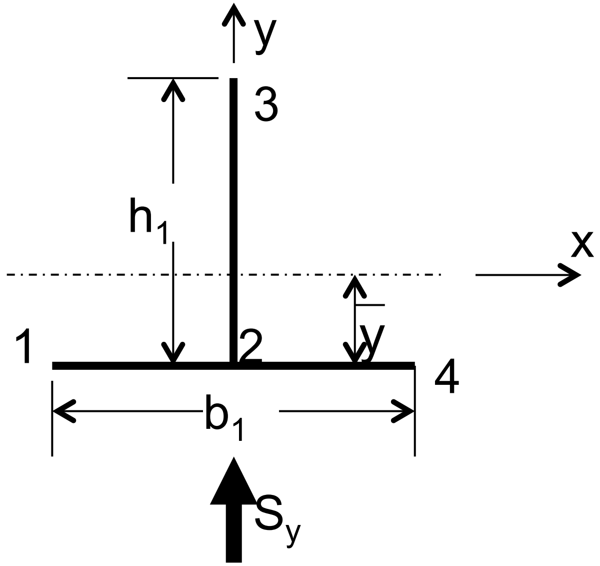
$$\bar{y} = \frac{h_1 t_1 \frac{h_1}{2} + \cancelto{\text{HOT}}{b_1 t_1 \frac{t_1}{2}}}{h_1 t_1 + b_1 t_1} = \frac{h_1^2}{2\left(h_1 + b_1\right)}$$

- determine moment of inertia I_{xx}

$$I_{xx} = \frac{t_1 h_1^3}{12} + t_1 h_1 \left(\frac{h_1}{2} - \bar{y}\right)^2 + b_1 t_1 \bar{y}^2$$

note simplifications in $\bar{y}$ and I_{xx} due to negligible t_1

Example: Beam cross-section 1 under shear



- one axis (y) is an axis of symmetry; so $I_{xy}=0$
- then, from eq (5.7)

$$q_s = -\frac{S_x}{I_{yy}}\int_0^s txds - \frac{S_y}{I_{xx}}\int_0^s tyds$$

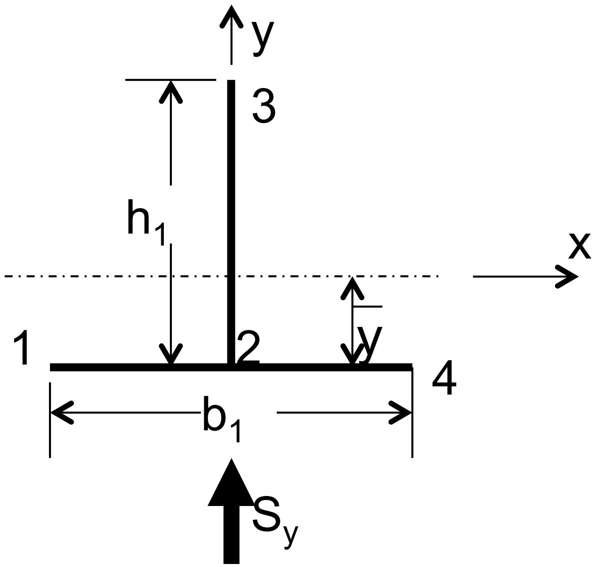
- and since $S_x=0$:

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s tyds$$

- select a convenient location for origin of s; for an open cross-section, this would be a free end such as point 1
- section 1-2: $y=-\bar{y}$; Then,

$$q_{12} = -\frac{S_y}{I_{xx}}\int_0^s t\left(-\bar{y}\right)ds = \frac{S_y}{I_{xx}} t_1 \bar{y}\, s$$

Example: Beam cross-section 1 under shear



$$q_{12} = -\frac{S_y}{I_{xx}}\int_0^s t\left(-\bar{y}\right)ds = \frac{S_y}{I_{xx}} t_1 \bar{y} s$$

- the shear flow at point 2 will consist of a contribution from q_{12} **and** q_{42}; by symmetry, $q_{12}=q_{42}$
- therefore:

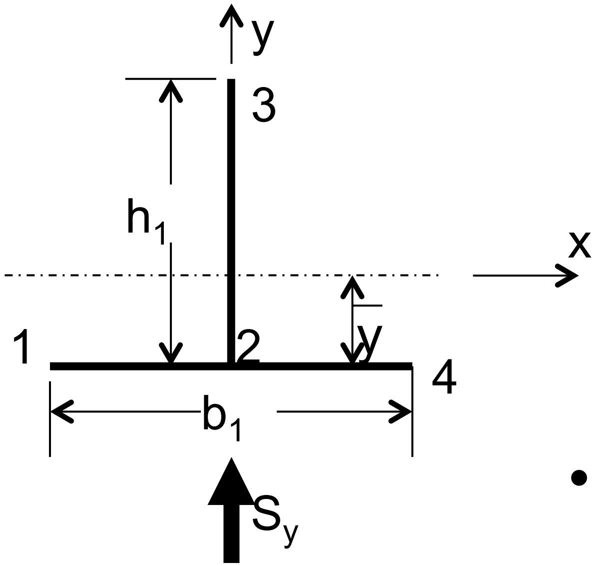
$$q_2 = 2q_{12}\left(s = \frac{b_1}{2}\right) = \frac{S_y}{I_{xx}} t_1 \bar{y} b_1$$

- section 2-3: $y=s-\bar{y}$

$$q_{23} = q_2 - \frac{S_y}{I_{xx}}\int_0^s t\left(s-\bar{y}\right)ds = \frac{S_y}{I_{xx}} t_1 \bar{y} b_1 - \frac{S_y}{I_{xx}} t_1\left(\frac{s^2}{2} - \bar{y} s\right) \Rightarrow q_{23} = \frac{S_y}{I_{xx}} t_1\left(\bar{y} b_1 + \bar{y} s - \frac{s^2}{2}\right)$$

- check: if q_{23} is correct then at point 3 must have $q_{23}=0$

Example: Beam cross-section 1 under shear



$$q_{23} = \frac{S_y}{I_{xx}} t_1 \left(\bar{y}\, b_1 + \bar{y}\, s - \frac{s^2}{2} \right)$$

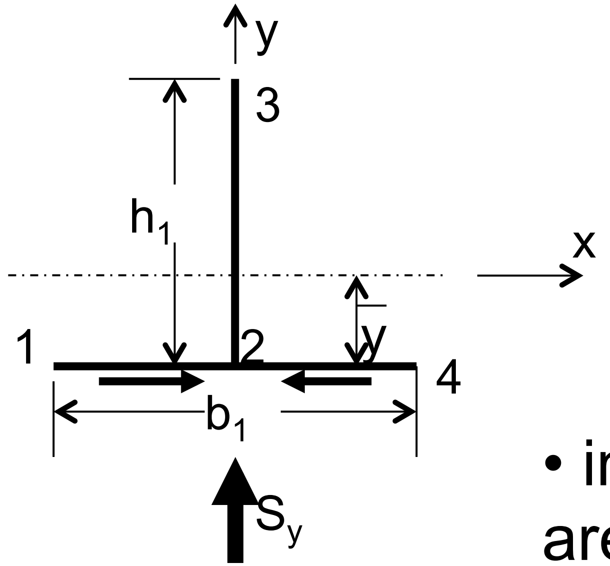
$$\bar{y} = \frac{h_1^2}{2(h_1 + b_1)}$$

• check:

$$q_{23}(s = h_1) = \frac{S_y t_1}{I_{xx}} \left[\frac{h_1^2}{2(h_1 + b_1)} b_1 + \frac{h_1^3}{2(h_1 + b_1)} - \frac{h_1^2}{2} \right] = \frac{S_y t_1}{2 I_{xx}(h_1 + b_1)} \left(h_1^2 b_1 + h_1^3 - h_1^2 (h_1 + b_1) \right) = 0$$

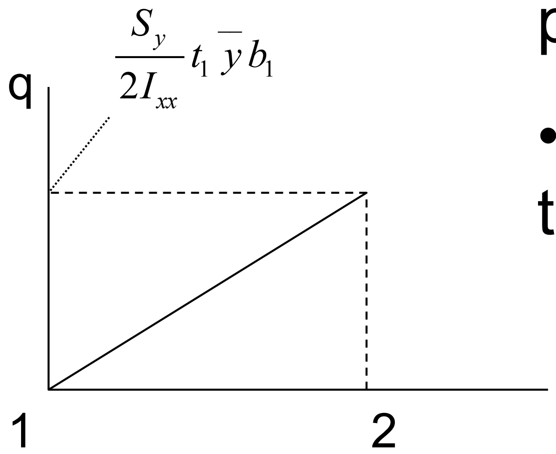
• in order to compare the two cross-sections, need to find the critical (= most positive or most negative) value of the shear flow in the cross-section

Example: Beam cross-section 1 under shear



$$q_{12} = -\frac{S_y}{I_{xx}}\int_0^s t\left(-\bar{y}\right)ds = \frac{S_y}{I_{xx}} t_1 \bar{y} s$$

$$q_{23} = \frac{S_y}{I_{xx}} t_1 \left(\bar{y} b_1 + \bar{y} s - \frac{s^2}{2}\right)$$

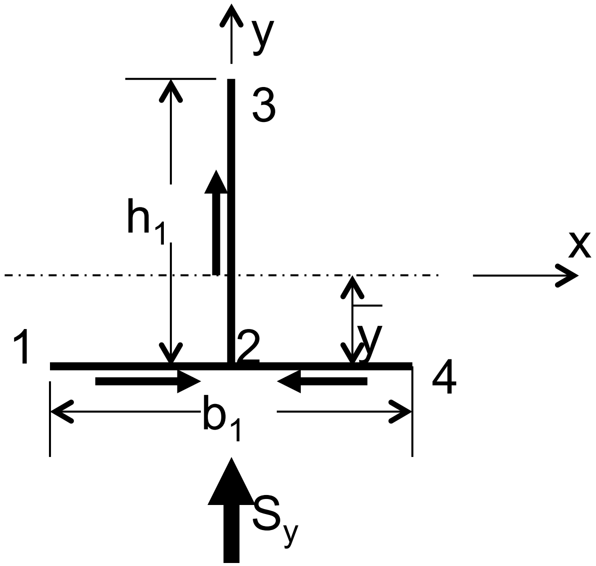


- in sections 1-2 and 4-2 the shear flows are the same, varying linearly; they start from zero and reach a max value at point 2

- the shear flow at point 2 is the sum of the contributions from q_{12} and q_{42}:

$$q_2 = \frac{S_y}{I_{xx}} t_1 \bar{y} b_1$$

Example: Beam cross-section 1 under shear

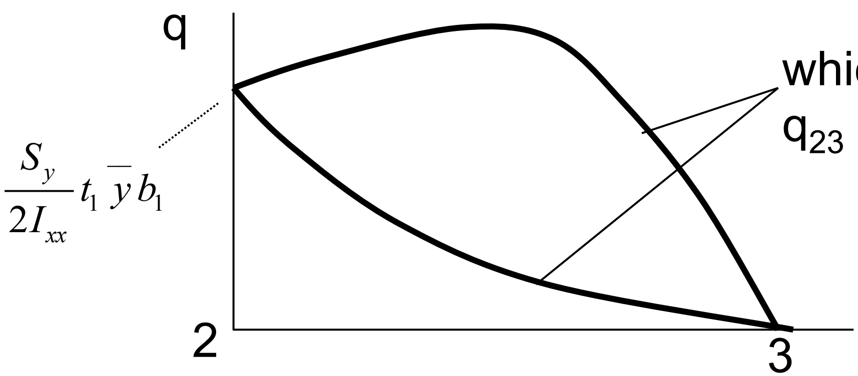


$$q_{12} = -\frac{S_y}{I_{xx}}\int_0^s t\left(-\bar{y}\right)ds = \frac{S_y}{I_{xx}}t_1\,\bar{y}\,s$$

$$q_2 = \frac{S_y}{I_{xx}}t_1\,\bar{y}\,b_1$$

$$q_{23} = \frac{S_y}{I_{xx}}t_1\left(\bar{y}\,b_1 + \bar{y}\,s - \frac{s^2}{2}\right)$$

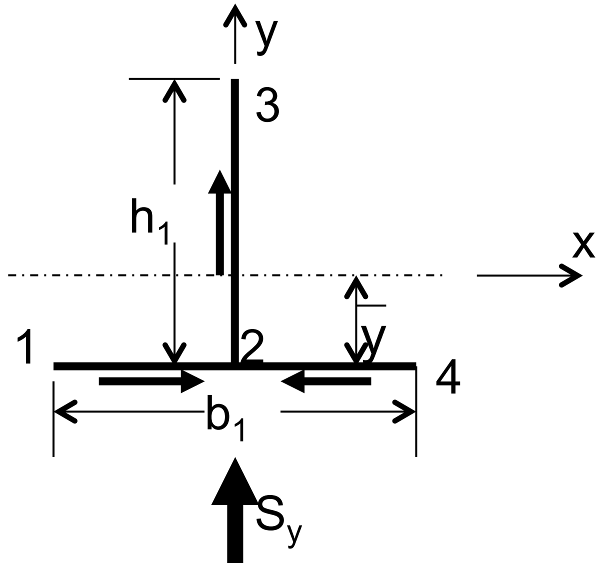
- The question is: Does q_{23} attain a higher value in section 2-3 or does it decrease monotonically to 0?



which of the two shapes does q_{23} follow?

if it is the lower, then q_2 is the max value; if it is the higher, the max value occurs somewhere between 2 and 3

Example: Beam cross-section 1 under shear



$$q_{23} = \frac{S_y}{I_{xx}} t_1 \left(\bar{y} b_1 + \bar{y} s - \frac{s^2}{2} \right)$$

- one way is to check if q_{23} has a minimum or a maximum between 0 and h_1:

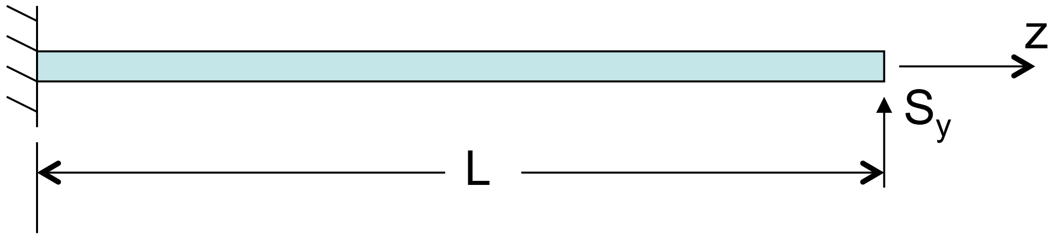
$$\frac{dq_{23}}{ds} = 0 \Rightarrow \bar{y} - s = 0 \Rightarrow s = \bar{y}$$

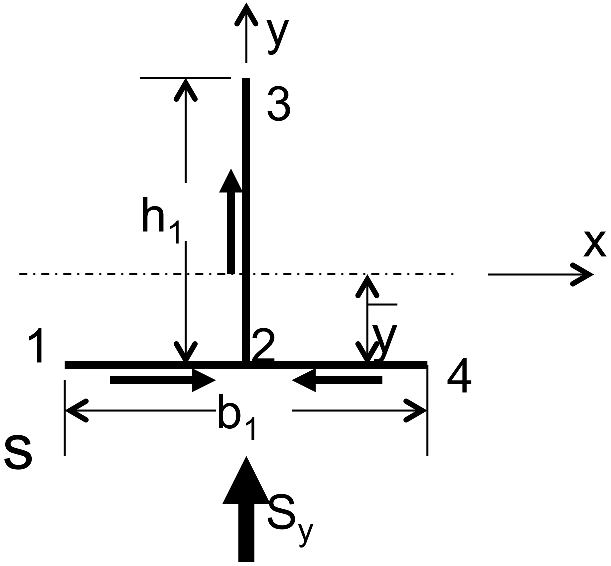
- so a max or a min occurs at $s=\bar{y}$; by substituting $\bar{y}$ for s in q_{23} (or taking the second derivative) we can find that there is a maximum there

- therefore, the max shear flow for the cross section is

$$q_{max} = q_{23max} = \frac{S_y}{I_{xx}} t_1 \left(\bar{y} b_1 + \bar{y}\,\bar{y} - \frac{\bar{y}^2}{2} \right) \Rightarrow \boxed{q_{max} = \frac{S_y}{I_{xx}} t_1 \bar{y} \left(b_1 + \frac{\bar{y}}{2} \right)}$$

Example: Beam cross-section 1 under shear





- in addition, there are direct stresses caused by the shear load due to bending

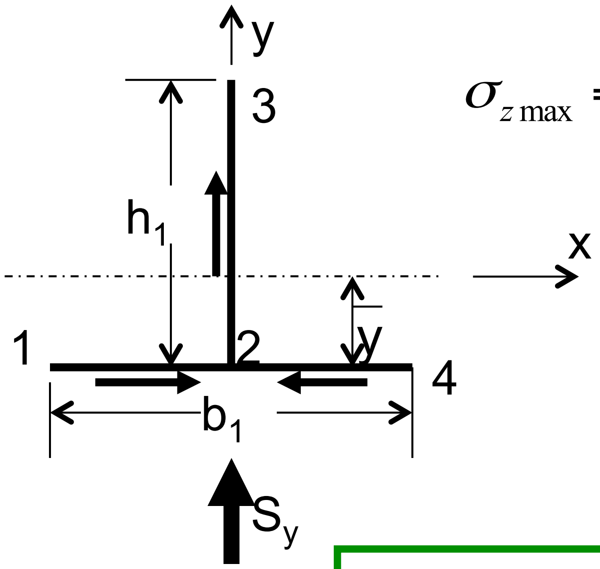
- the maximum bending moment (at the root of the beam) is

$$M_{x\max} = S_y L$$

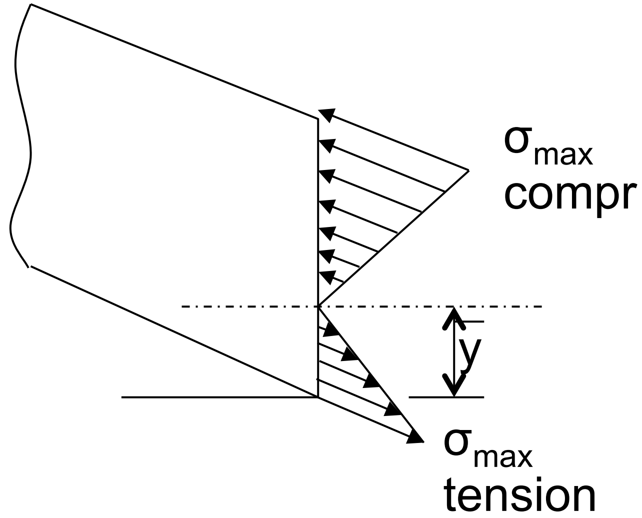
- and the corresponding maximum normal stress is (see eq 2.6 or 2.7)

$$\sigma_{z\max} = \frac{M_{x\max} y_{\max}}{I_{xx}}$$

Example: Beam cross-section 1 under shear



$$\sigma_{z\max} = \frac{M_{x\max}\, y_{\max}}{I_{xx}}$$

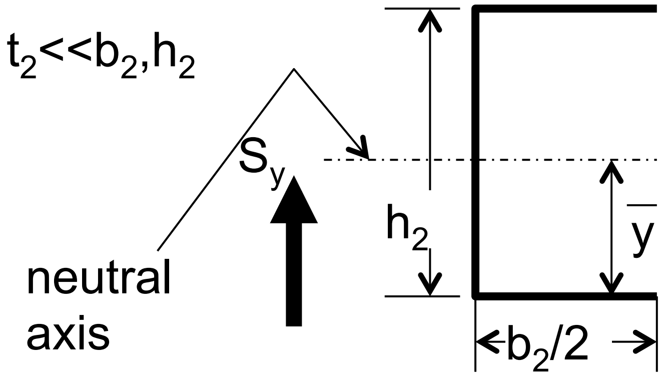


- therefore,

$$\boxed{\sigma_{\max compr} = \frac{S_y L (h_1 - \bar{y})}{I_{xx}} \qquad \sigma_{\max tension} = \frac{S_y L \bar{y}}{I_{xx}}}$$

Example: Beam cross-section 2 under shear

- section 2: “C” or “channel” section



- locate origin at neutral axis
- determine neutral axis location ($\bar{y}$):

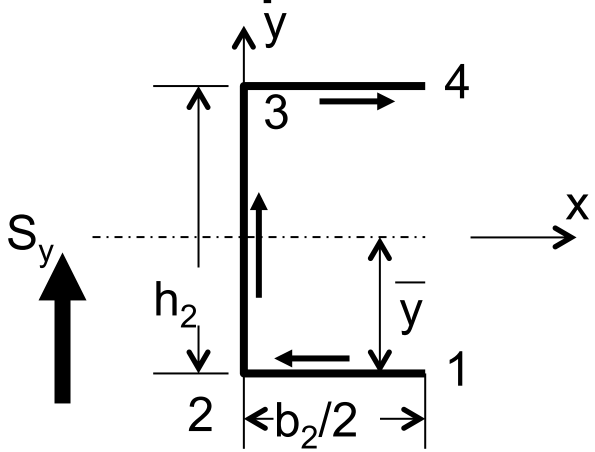
$$\bar{y} = \frac{h_2 t_2 \frac{h_2}{2} + t_2 \frac{b_2}{2} h_2}{h_2 t_2 + 2 t_2 \frac{b_2}{2}} = \frac{h_2}{2}$$

- determine moment of inertia I_{xx}

$$I_{xx} = \frac{t_2 h_2^3}{12} + 2 \frac{b_2}{2} t_2 \left(\frac{h_2}{2} \right)^2 \Rightarrow I_{xx} = \frac{t_2 h_2^2}{4} \left(\frac{h_2}{3} + b_2 \right)$$

note: (a) S_y is acting through the shear center (b) simplifications in y and I_{xx} due to negligible t_2

Example: Beam cross-section 2 under shear



- one axis (x) is an axis of symmetry; so $I_{xy}=0$
- then, from eq (5.7)

$$q_s = -\frac{S_x}{I_{yy}}\int_0^s txds - \frac{S_y}{I_{xx}}\int_0^s tyds$$

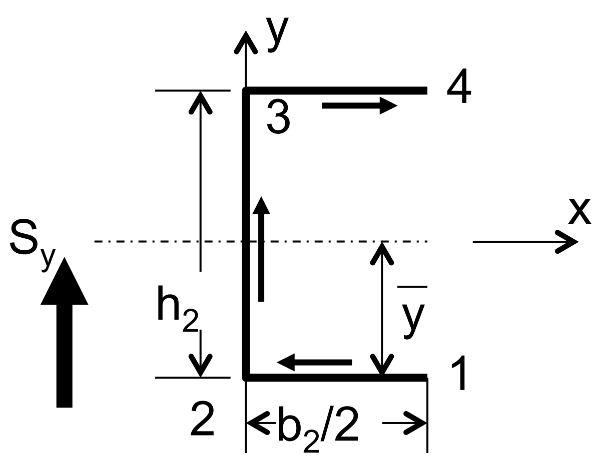
- and since $S_x=0$:

$$q_s = -\frac{S_y}{I_{xx}}\int_0^s tyds$$

- select a convenient location for origin of s; for an open cross-section, this would be a free end such as point 1
- section 1-2: $y=-\bar{y}$; Then,

$$q_{12} = -\frac{S_y}{I_{xx}}\int_0^s t\left(-\bar{y}\right)ds = \frac{S_y}{I_{xx}} t_2 \frac{h_2}{2} s$$

Example: Beam cross-section 2 under shear



$$q_{12} = -\frac{S_y}{I_{xx}}\int_0^s t\left(-\bar{y}\right)ds = \frac{S_y}{I_{xx}}t_2\frac{h_2}{2}s$$

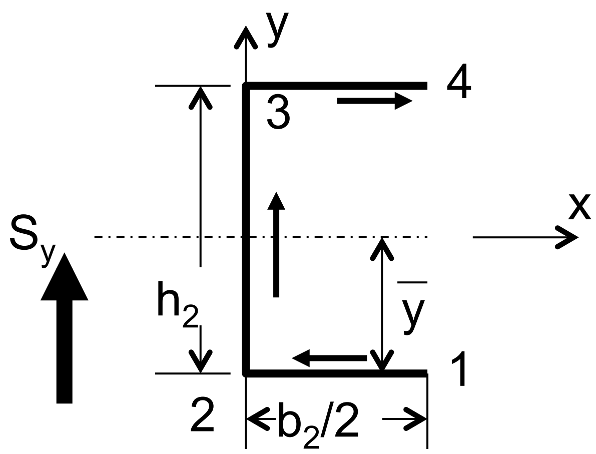
- at point 2:

$$q_2 = \frac{S_y}{I_{xx}}t_2\frac{b_2h_2}{4}$$

- section 2-3: $y=s-\bar{y}$

$$q_{23} = q_2 - \frac{S_y}{I_{xx}}\int_0^s t\left(s-\bar{y}\right)ds = \frac{S_y}{I_{xx}}t_2\frac{b_2h_2}{4} - \frac{S_y}{I_{xx}}t_2\left(\frac{s^2}{2}-\bar{y}\,s\right) \Rightarrow q_{23} = \frac{S_y}{I_{xx}}t_2\left(\frac{h_2b_2}{4}+\bar{y}\,s-\frac{s^2}{2}\right)$$

Example: Beam cross-section 2 under shear



$$q_{23} = \frac{S_y}{I_{xx}} t_2 \left(\frac{h_2 b_2}{4} + \bar{y}\, s - \frac{s^2}{2} \right)$$

- check: q_4 must be zero
- note that q_3 is given by

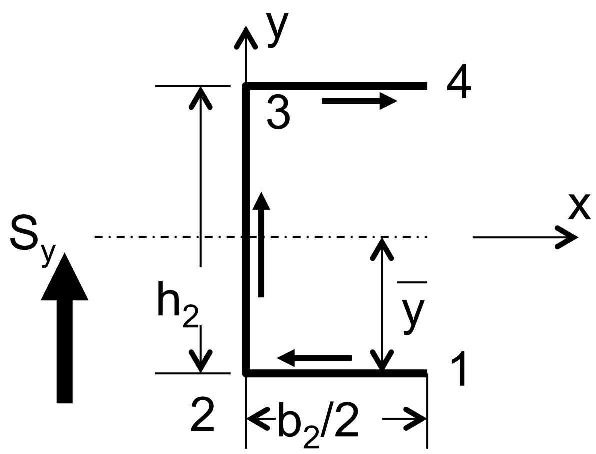
$$q_3 = q_{23}(s = h_2) = \frac{S_y}{I_{xx}} t_2 \left(\frac{h_2 b_2}{4} + \bar{y}\, h_2 - \frac{h_2^2}{2} \right)$$

- but, we showed earlier that $\bar{y} = \dfrac{h_2}{2}$ Substituting:

$$q_3 = \frac{S_y}{I_{xx}} t_2 \left(\frac{h_2}{4} b_2 + \frac{h_2}{2} h_2 - \frac{h_2^2}{2} \right) \Rightarrow q_3 = \frac{S_y}{I_{xx}} t_2 \frac{h_2 b_2}{4}$$

which is exactly the same as q_2, as it should because of symmetry

Example: Beam cross-section 2 under shear



- section 3-4: $y=+\bar{y}=h_2/2$
- then

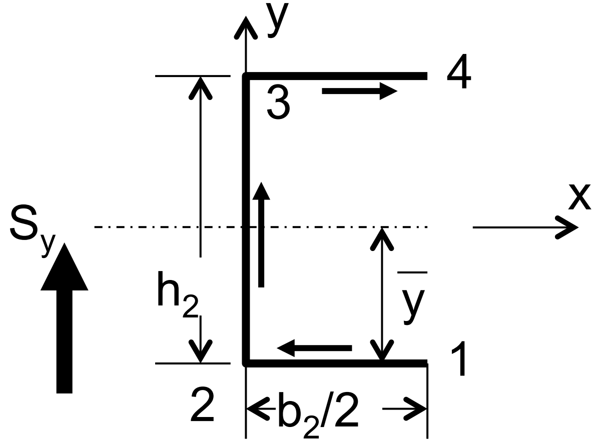
$$q_{34} = q_3 - \frac{S_y}{I_{xx}} \int_0^s t\,\bar{y}ds = \frac{S_y}{I_{xx}} t_2 \frac{h_2 b_2}{4} - \frac{S_y}{I_{xx}} t_2 \frac{h_2}{2} s$$

- so

$$q_4 = q_{34}\left(s = \frac{b_2}{2}\right) = \frac{S_y}{I_{xx}} t_2 \frac{h_2 b_2}{4} - \frac{S_y}{I_{xx}} t_2 \frac{h_2}{2}\frac{b_2}{2} = 0$$

- the max shear flow in the cross-section will be either q_2 or the highest value of q in section 2-3

Example: Beam cross-section 2 under shear



$$q_2 = \frac{S_y}{I_{xx}} t_2 \frac{b_2 h_2}{4}$$

$$q_{23} = \frac{S_y}{I_{xx}} t_2 \left(\frac{h_2 b_2}{4} + \bar{y}\, s - \frac{s^2}{2} \right)$$

• as for the case of the "T" cross-section,

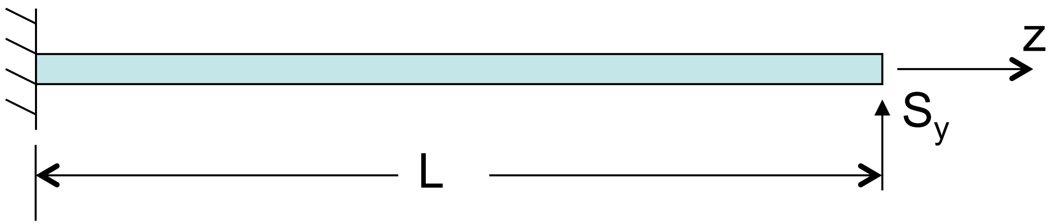
$$\frac{dq_{23}}{ds} = 0 \Rightarrow \bar{y} - s = 0 \Rightarrow s = \bar{y}$$

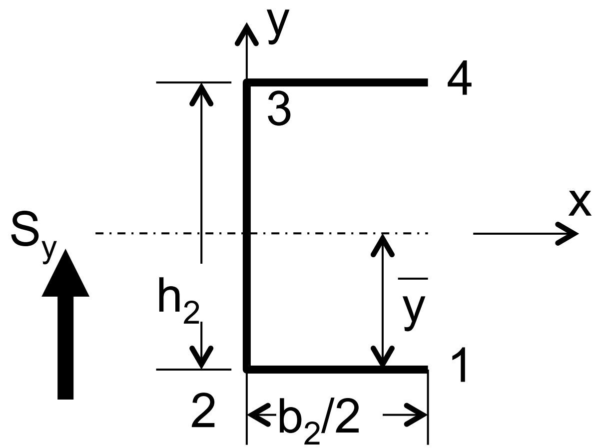
• and using the fact that $\bar{y}=h_2/2$:

$$q_{23\max} = \frac{S_y}{I_{xx}} t_2 \left(\frac{h_2 b_2}{4} + \frac{h_2}{2}\frac{h_2}{2} - \frac{h_2^{\,2}}{8} \right) \Rightarrow \boxed{q_{23\max} = \frac{S_y}{I_{xx}} \frac{t_2 h_2}{4} \left(b_2 + \frac{h_2}{2} \right)}$$

which is greater than q_2 and thus is the highest shear flow in the cross-section

Example: Beam cross-section 2 under shear



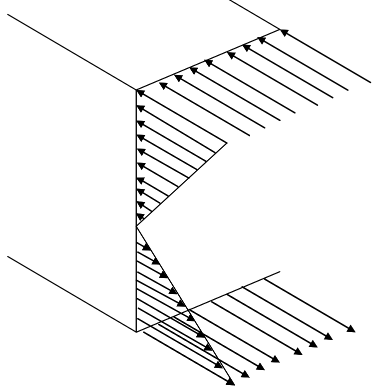


- as for the "T" cross-section, there are also normal stresses caused by the moment S_yL

- the equations are the same as for the "T" stiffener

$$\sigma_{\max compr} = \frac{S_y L (h_2 - \overline{y})}{I_{xx}}$$

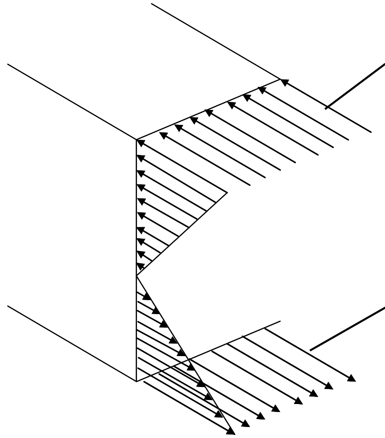
$$\sigma_{\max tension} = \frac{S_y L \overline{y}}{I_{xx}}$$



Example: Beam cross-section 2 under shear

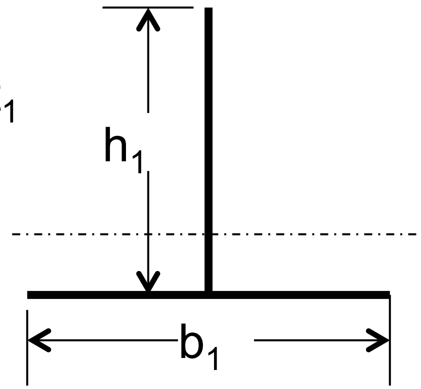
- which, using the fact that $\bar{y}=h_2/2$, simplify to:

$$\boxed{\sigma_{\max compr} = \sigma_{\max tension} = \frac{S_y L h_2}{2I_{xx}}}$$



Example: Evaluation of beam cross-sections under shear and bending loads

thickness t_1



$$I_{xx} = \frac{t_1 h_1^3}{4}\left(\frac{1}{3} + \frac{b_1}{h_1 + b_1}\right)$$

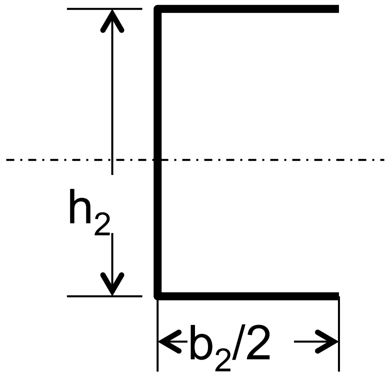
$$q_{\max} = \frac{3}{2}\frac{S_y}{h_1\left(h_1 + 4b_1\right)}\frac{\left(h_1 + 2b_1\right)^2}{\left(h_1 + b_1\right)}$$

$$\tau_{\max} = \frac{3}{2}\frac{S_y}{h_1 t_1\left(h_1 + 4b_1\right)}\frac{\left(h_1 + 2b_1\right)^2}{\left(h_1 + b_1\right)}$$

$$\sigma_{\max compr} = \frac{6S_y L}{t_1 h_1^2}\frac{\left(h_1 + 2b_1\right)}{\left(h_1 + 4b_1\right)}$$

$$\sigma_{\max tension} = \frac{6S_y L}{t_1 h_1}\frac{1}{\left(h_1 + 4b_1\right)}$$

thickness t_2



$$I_{xx} = \frac{t_2 h_2^2}{4}\left(\frac{h_2}{3} + b_2\right)$$

$$q_{\max} = \frac{3}{2}\frac{S_y\left(h_2 + 2b_2\right)}{h_2\left(h_2 + 3b_2\right)}$$

$$\tau_{\max} = \frac{3}{2}\frac{S_y\left(h_2 + 2b_2\right)}{h_2 t_2\left(h_2 + 3b_2\right)}$$

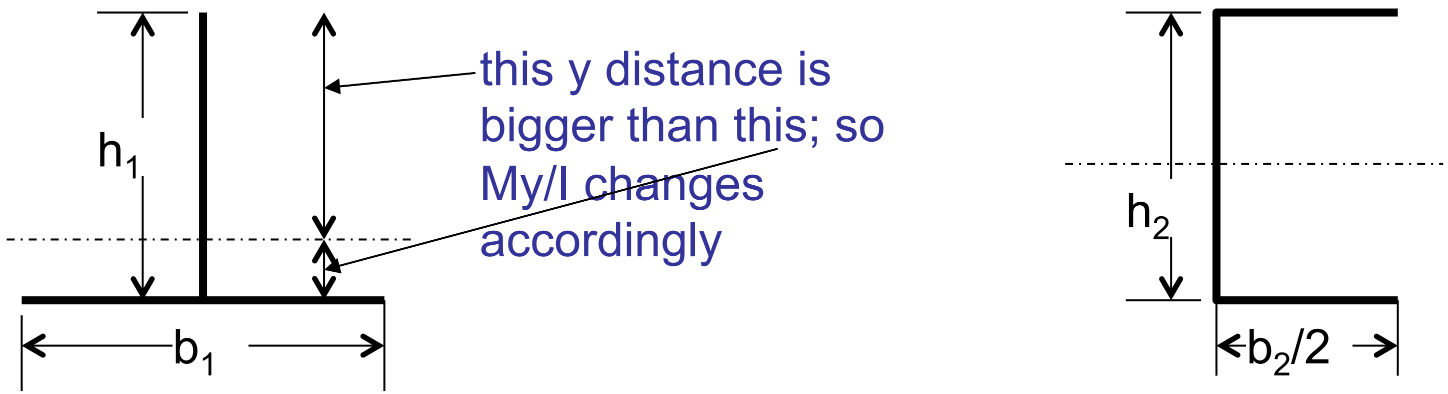
$$\sigma_{\max compr} = \frac{6S_y L}{t_2 h_2}\frac{1}{\left(h_2 + 3b_2\right)}$$

$$\sigma_{\max tension} = \frac{6S_y L}{t_2 h_2}\frac{1}{\left(h_2 + 3b_2\right)}$$

Example: Evaluation of beam cross-sections under shear and bending loads

• note that the magnitude of the max compressive stress for the "T" section is bigger than the max tensile stress

• for the "C" section, the magnitudes of max compressive and tensile stresses are the same



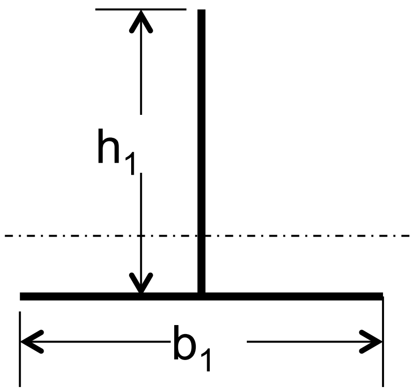
Example: Relative magnitude of shear and normal stresses

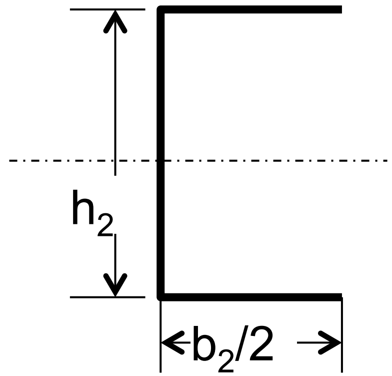
- in general, to determine failure one would have to find where in each cross-section, the shear and normal stresses combine to give the worst von Mises stress

- it turns out that for most beams (L>>h,b>>t) the shear stresses are very small compared to the normal stresses and it suffices to do the strength check using only the max normal stress (tensile or compressive depending on the relative values of the tensile or compressive yield strengths of the material)

Example: Relative magnitude of shear and normal stresses

• a way to check this is to take the ratio of maximum shear stress to maximum normal stress for each cross-section; after simplification we get:



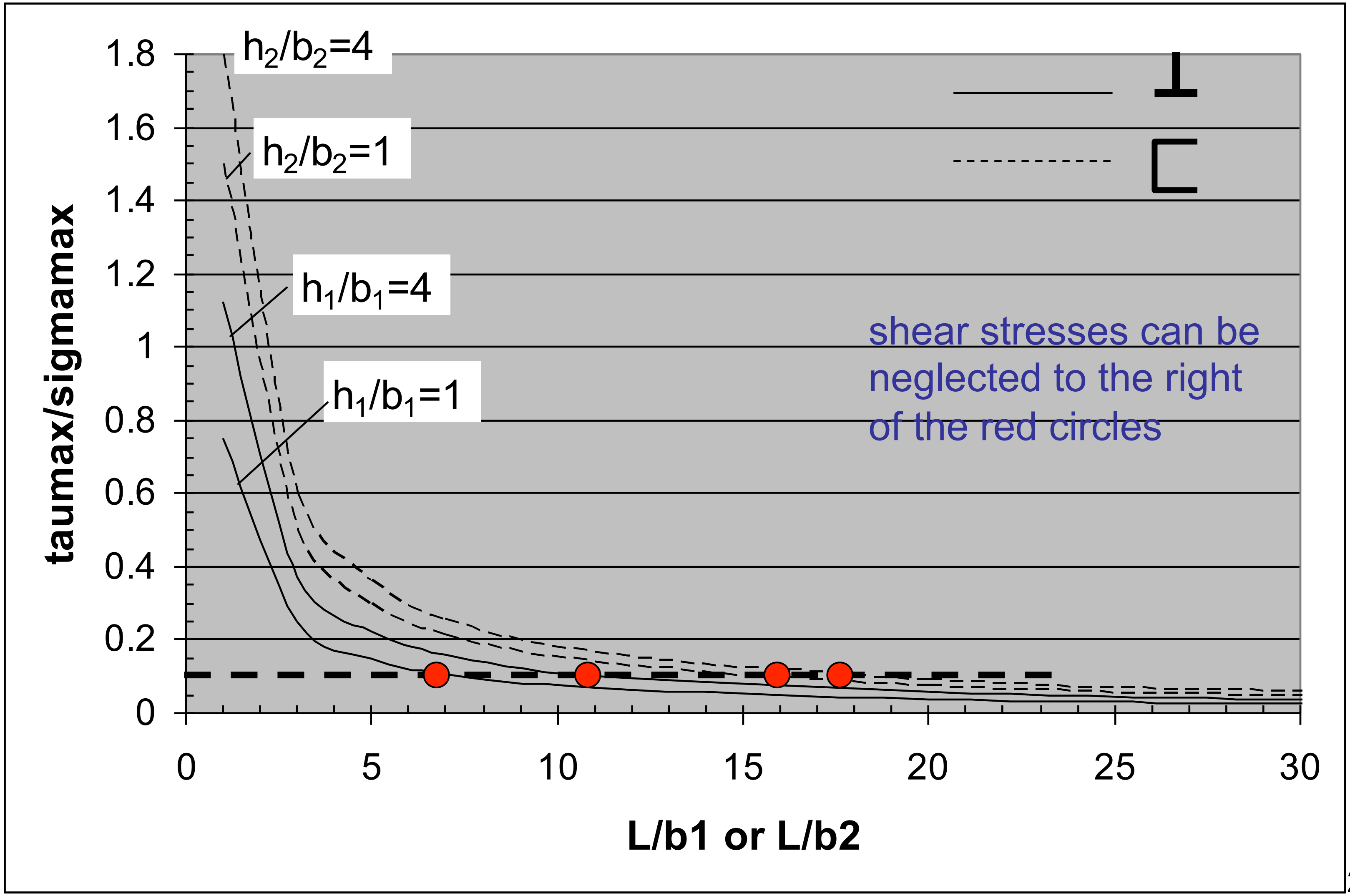


$$\frac{\tau_{\max}}{\sigma_{\max}} = \frac{1}{4}\frac{\left(h_1 + 2b_1\right)^2}{\left(h_1 + b_1\right)L} = \frac{1}{4}\frac{\left(\frac{h_1}{b_1} + 2\right)^2}{\left(\frac{h_1}{b_1} + 1\right)\frac{L}{b_1}}$$

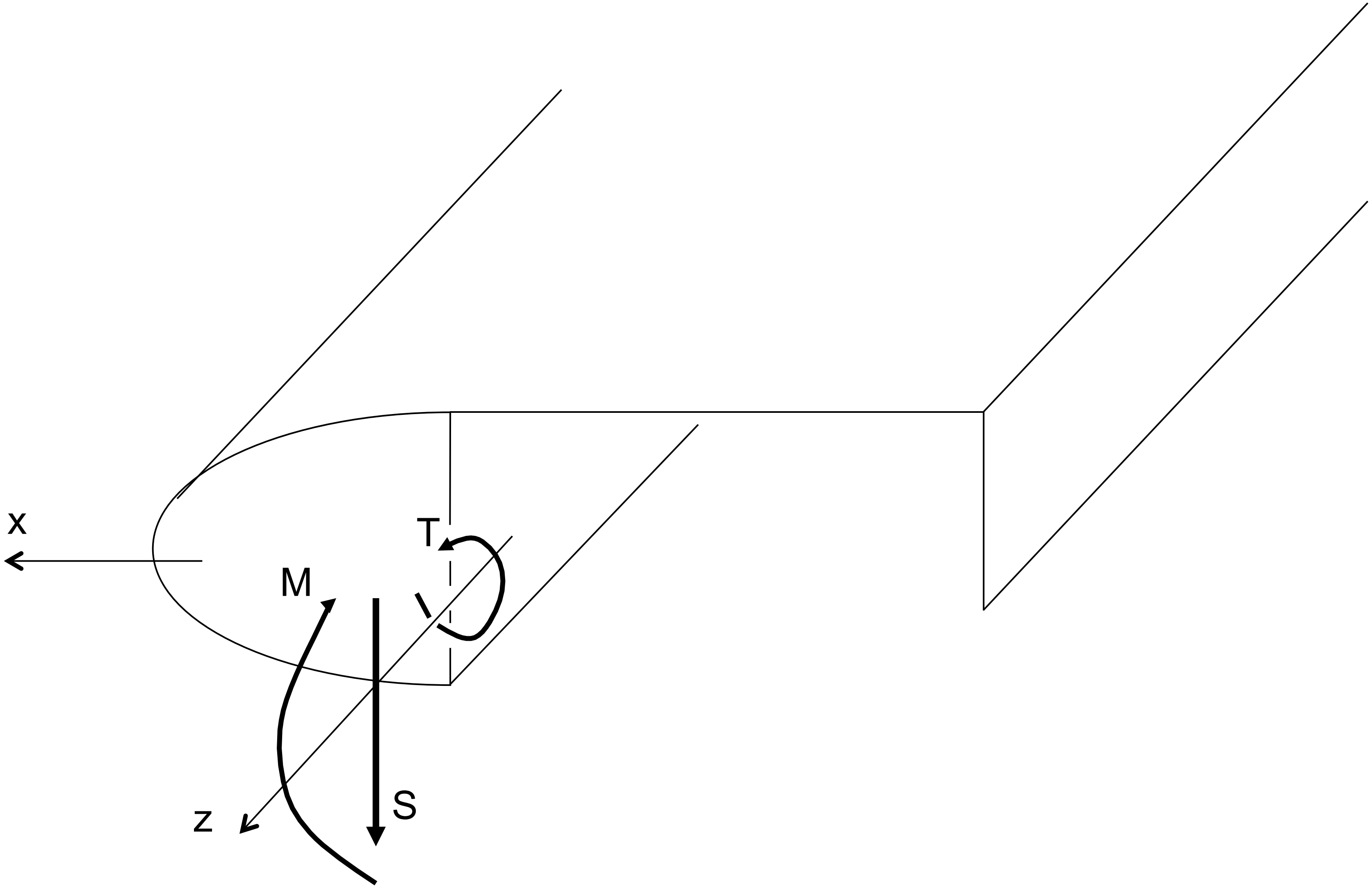
$$\frac{\tau_{\max}}{\sigma_{\max}} = \frac{1}{4}\frac{\left(h_2 + 2b_2\right)}{L} = \frac{1}{4}\frac{\left(\frac{h_2}{b_2} + 2\right)}{\frac{L}{b_2}}$$

note that σ_{maxten} is used because it is more "conservative"

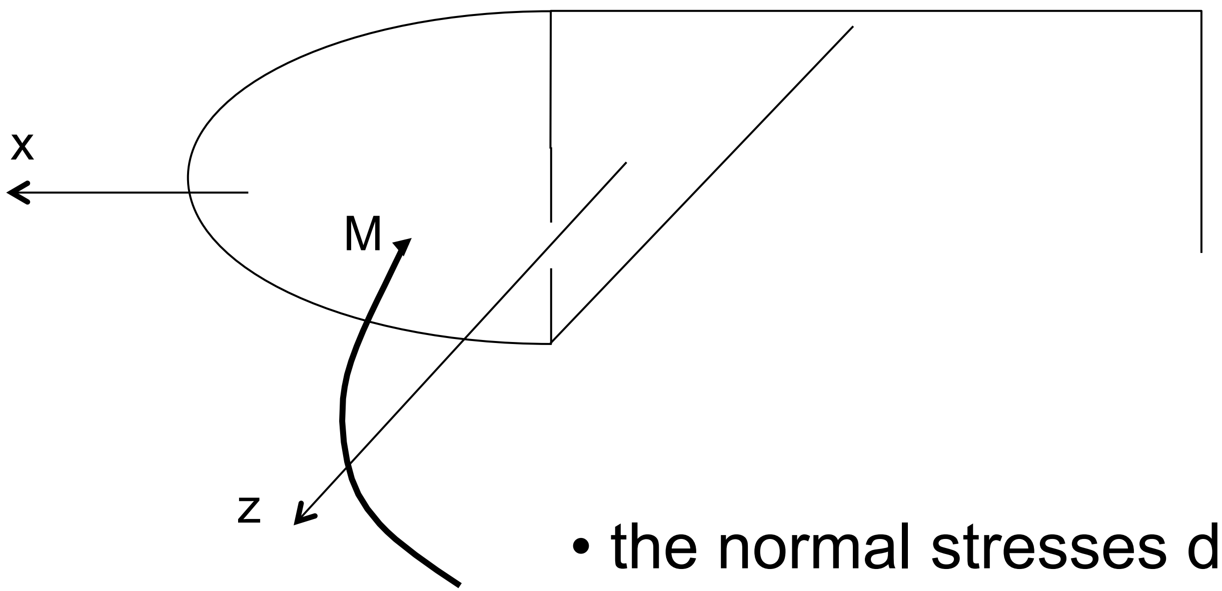
Example: Relative magnitude of shear and normal stresses



Combined closed and open section beams



Combined open and closed sections under bending



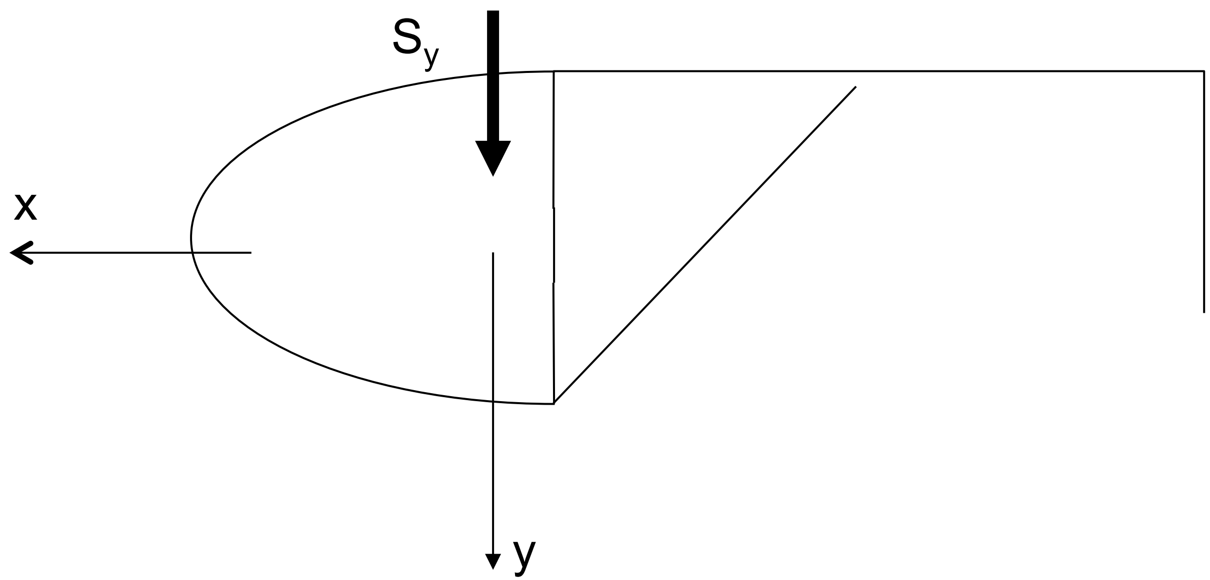
- the normal stresses do not change if there is an open or closed section

- normal stresses are still given by eq. (2.5)

$$\sigma_z = \frac{I_{xx}M_y - I_{xy}M_x}{I_{xx}I_{yy} - I_{xy}^2}x + \frac{I_{yy}M_x - I_{xy}M_y}{I_{xx}I_{yy} - I_{xy}^2}y$$

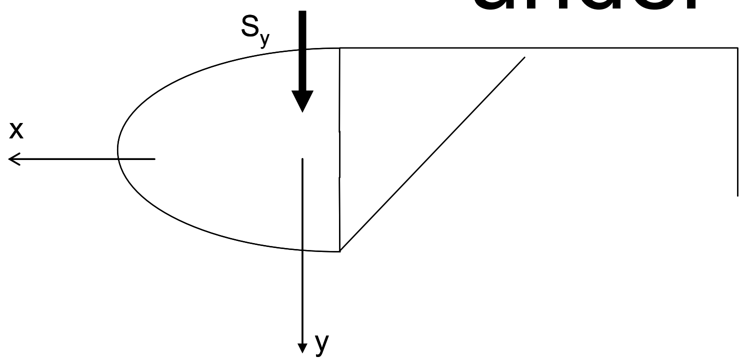
determine neutral axis, I_{xx}, etc for the entire cross-section and apply eq (2.5); alternatively, find which portion of M is taken by which member and do simpler analysis

Combined open and closed sections under shear



• two cases depending on whether S_y acts through the shear center or not

Combined open and closed sections under shear



- S_y through the shear center

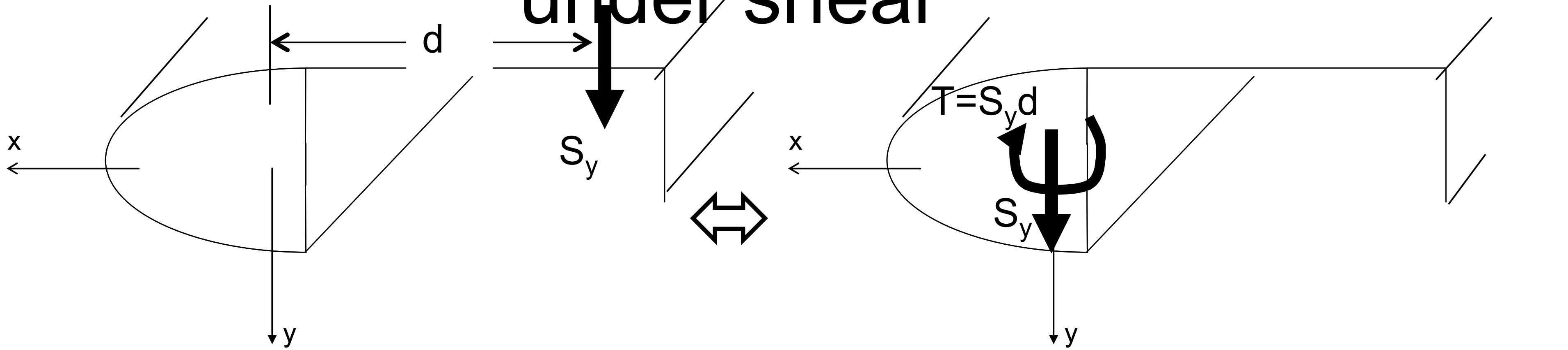
- analysis proceeds as for closed and open section beams with equations (5.6) and (5.8), (5.10)

$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s tx\,ds - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s ty\,ds \qquad (5.6)$$

$$q_s - q_{s0} = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s tx\,ds - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s ty\,ds \qquad (5.8)$$

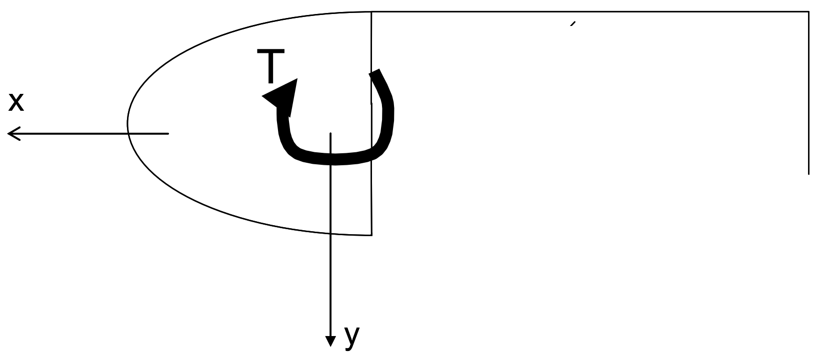
$$\oint_s p q_b\,ds + 2Aq_{s0} = 0 \qquad (5.10)$$

Combined open and closed sections under shear



- S_y not through the shear center
- break the problem in two, one with the shear load through the shear center and one with an applied torque equal to the shear load times the distance from the shear center
- superposition of the two problems gives the answer

Combined open and closed sections under torsion

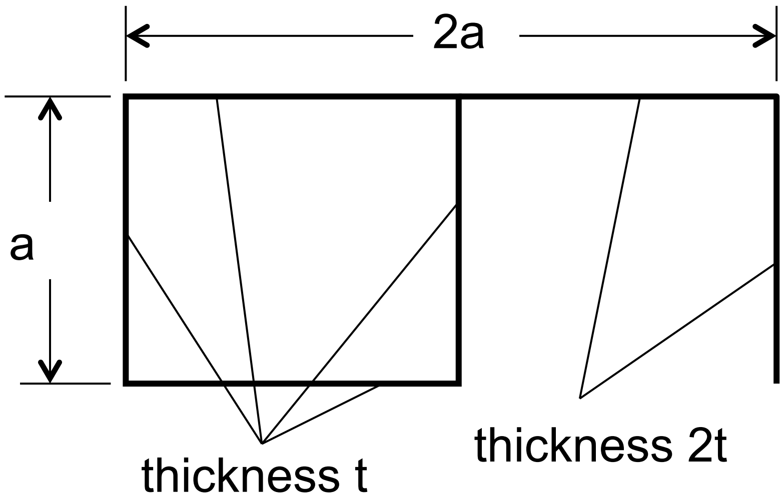


• we have shown in lecture 4 that open cross-sections have very small J

• so if we combine a closed and an open section, the contribution of the open portion to the torsional rigidity is negligible (see example below) and one can approximate J of the entire cross-section with J of the closed section

• still, shear stresses develop also in the open section and have to be checked for failure

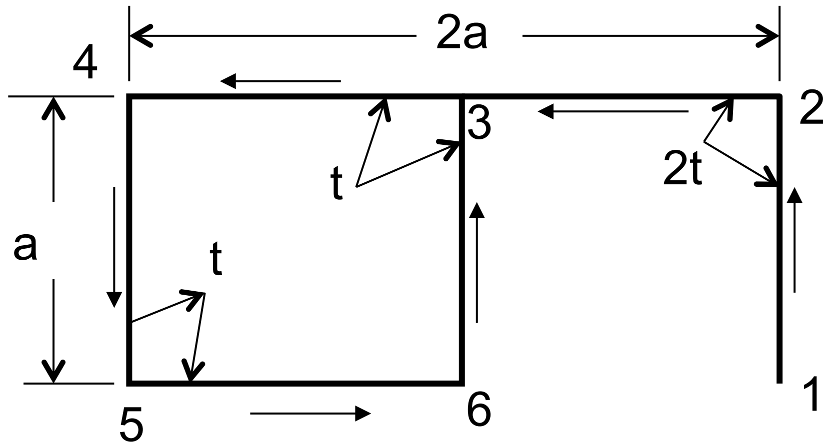
Example: Combined open and closed section under torsion



Determine the rate of twist and the maximum shear stress in open and closed sections under torque T

• first, some points are necessary to put things in perspective

Example: Combined open and closed section under torsion

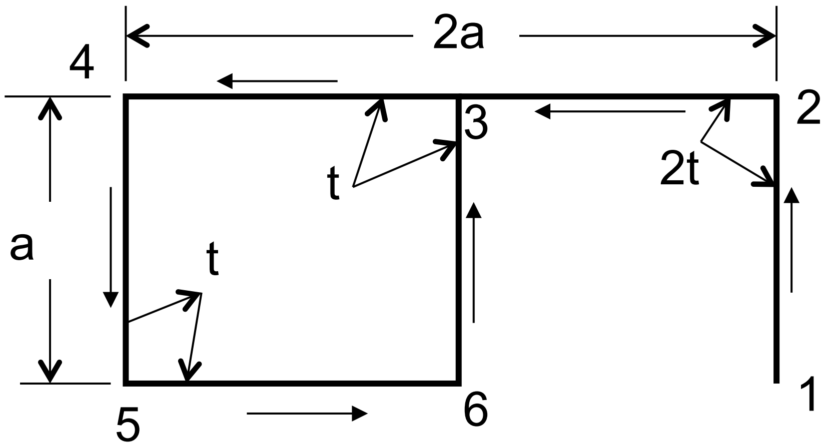


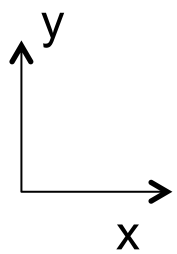
- we know from lecture 5 that the shear flow in the closed portion under pure torque is constant, so:

$$q_{34} = q_{45} = q_{56} = q_{63} = q$$

- Consider now vertical equilibrium: Since q_{45} and q_{63} have the same magnitude and opposite directions, they cancel each other. This leaves only q_{12}

Example: Combined open and closed section under torsion





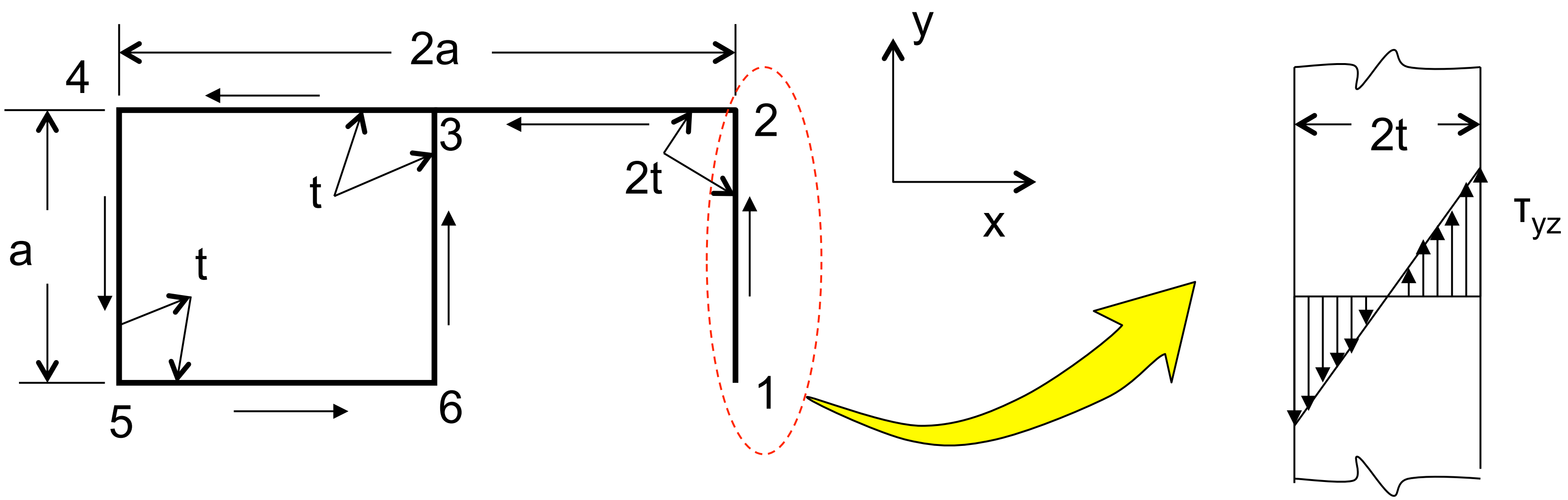
$$\sum F_y = 0 \Rightarrow \int_1^2 q_{12} ds = 0$$

- this means that if q_{12}=const then q_{12}=0; otherwise q_{12} just integrates to zero

- similarly, horizontal equilibrium shows that, since q_{34} balances q_{56}, q_{23} is either 0 or integrates to zero

so is there a shear flow from 1 to 2 and 2 to 3 or not??

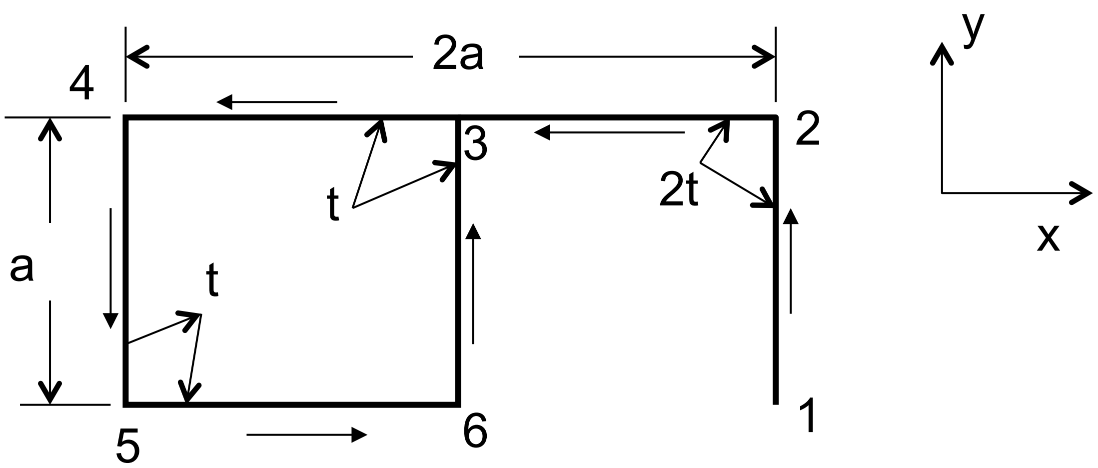
Example: Combined open and closed section under torsion



• in lecture 4 we found that for a narrow rectangular cross section, τ_{yz} is linear through the thickness and $\tau_{xz}\approx 0$ (except near the top and bottom ends 1 and 2)

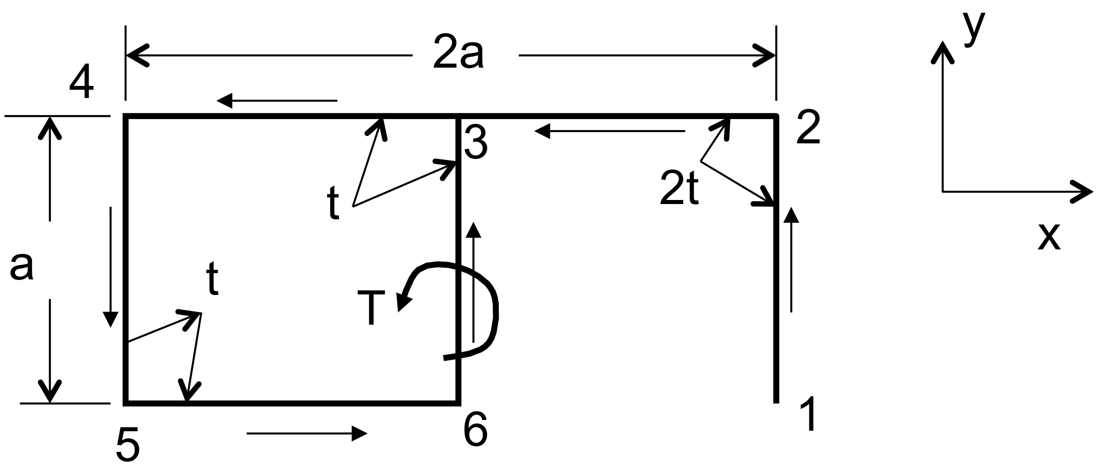
• this means that our standard definition $q=t\tau$, which assumes τ is const through the thickness, is no longer valid

Example: Combined open and closed section under torsion



- the exact same reasoning applies to segment 2-3 but now τ_{xz} varies linearly through the thickness and $\tau_{yz} \approx 0$
- so, while q in segments 1-2 and 2-3 is not well-defined, the shear stresses due to torsion do exist and should be checked for failure

Example: Combined open and closed section under torsion



• let the subscripts "c", "o", and "T" denote the closed portion, open portion and total cross-section

• then, from the basic torque equation (3.25):

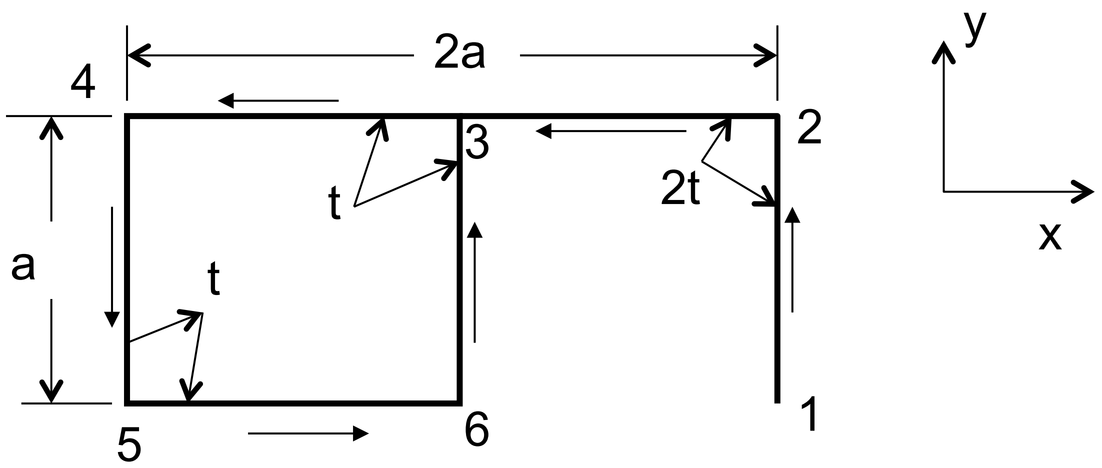
$$T_c = (GJ)_c \left(\frac{d\theta}{dz}\right)_c$$

$$T_o = (GJ)_o \left(\frac{d\theta}{dz}\right)_o$$

$$T_T = (GJ)_T \left(\frac{d\theta}{dz}\right)_T$$

but also, $T_T = T_c + T_o$

Example: Combined open and closed section under torsion



$$T_c = (GJ)_c \left(\frac{d\theta}{dz}\right)_c$$

$$T_o = (GJ)_o \left(\frac{d\theta}{dz}\right)_o$$

$$T_T = (GJ)_T \left(\frac{d\theta}{dz}\right)_T$$

T_T=Tc+To

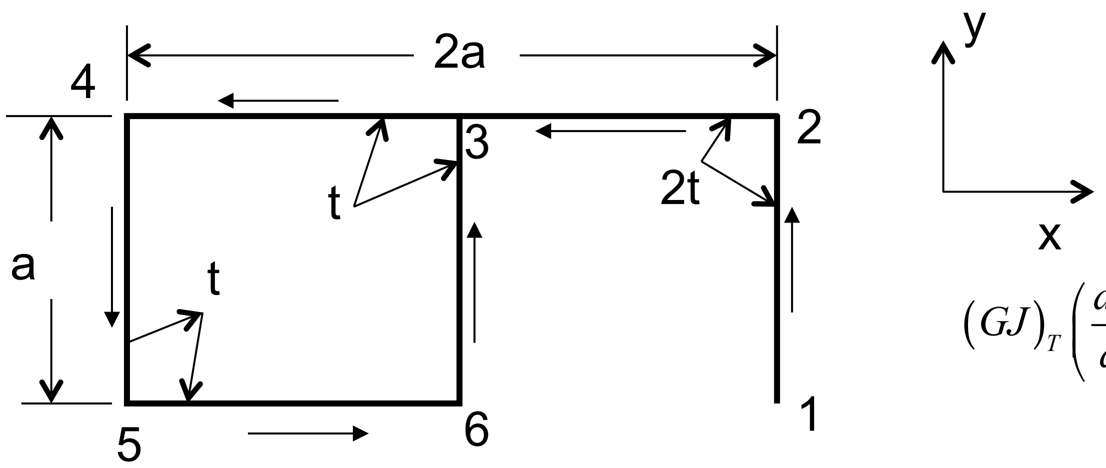
- using the first 3 to substitute in the last equation:

$$(GJ)_T \left(\frac{d\theta}{dz}\right)_T = (GJ)_c \left(\frac{d\theta}{dz}\right)_c + (GJ)_o \left(\frac{d\theta}{dz}\right)_o$$

- but the rate of twist is the same for each portion and equal to the overall rate of twist

$$\left(\frac{d\theta}{dz}\right)_T = \left(\frac{d\theta}{dz}\right)_c = \left(\frac{d\theta}{dz}\right)_o$$

Example: Combined open and closed section under torsion



$$\left(\frac{d\theta}{dz}\right)_T = \left(\frac{d\theta}{dz}\right)_c = \left(\frac{d\theta}{dz}\right)_o$$

$$(GJ)_T\left(\frac{d\theta}{dz}\right)_T = (GJ)_c\left(\frac{d\theta}{dz}\right)_c + (GJ)_o\left(\frac{d\theta}{dz}\right)_o$$

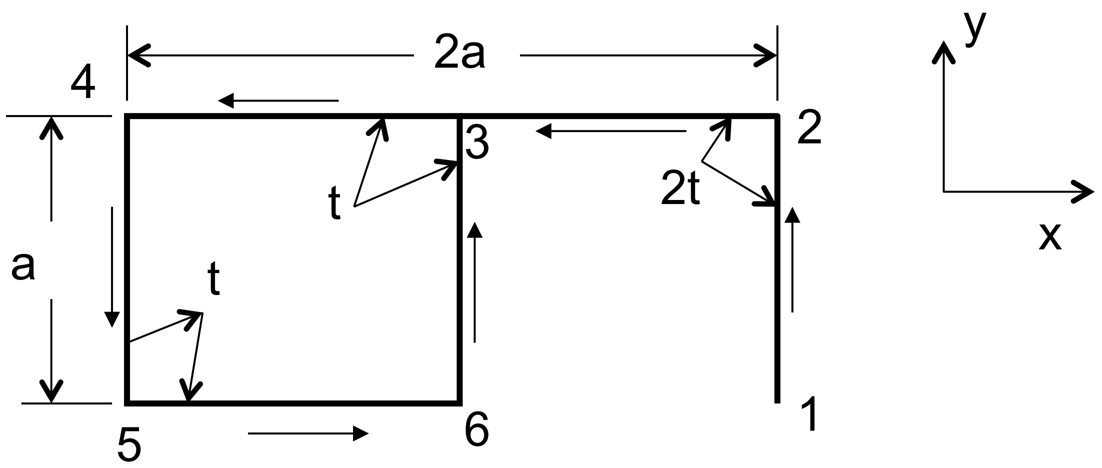
- combining: $(GJ)_T = (GJ)_c + (GJ)_o$

- and if the same material is used for the closed and open portions of the cross-section: $G_T = G_c = G_o = G$

- we obtain:

$$J_T = J_c + J_o$$

Example: Combined open and closed section under torsion



$$J_T = J_c + J_o$$

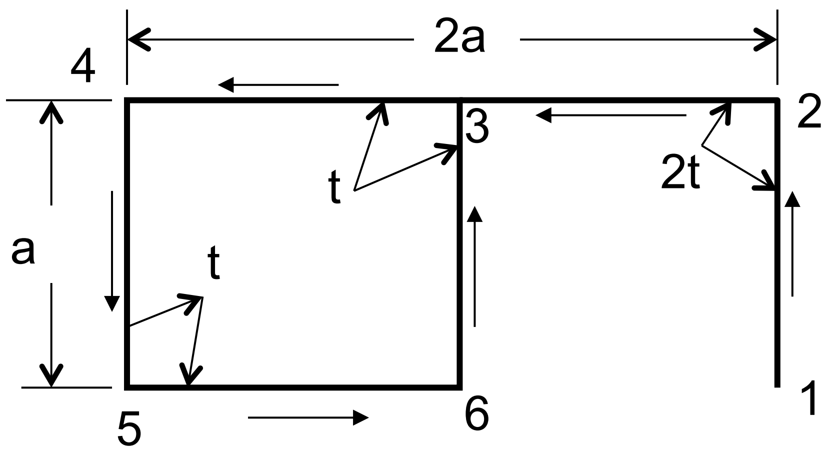
- for the open portion, using eq. (4.11): $J = \frac{1}{3}\sum_{i=1}^{N} s_i t_i^3$ we get

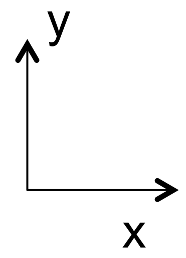
$$J_o = a\frac{(2t)^3}{3} + a\frac{(2t)^3}{3} = \frac{16}{3}at^3$$

- for the closed section we have from eq. (5.12): $\frac{d\theta}{dz} = \frac{1}{2A}\oint\frac{q_s}{tG}ds$

- for q_s=const=q and using (3.44), T=2Aq: $\frac{d\theta}{dz} = \frac{T_c}{4A^2}\oint\frac{ds}{Gt}$

Example: Combined open and closed section under torsion





$$J_T = J_c + J_o$$

$$J_o = \frac{16}{3} a t^3$$

$$\frac{d\theta}{dz} = \frac{T_c}{4A^2} \oint \frac{ds}{Gt}$$

- and using (3.31), $T = GJ \dfrac{d\theta}{dz}$ we get

$$\frac{d\theta}{dz} = \frac{GJ_c}{4A^2} \frac{d\theta}{dz} \oint \frac{ds}{Gt} \Rightarrow \boxed{J_c = \frac{4A^2}{\oint \frac{ds}{t}}}$$

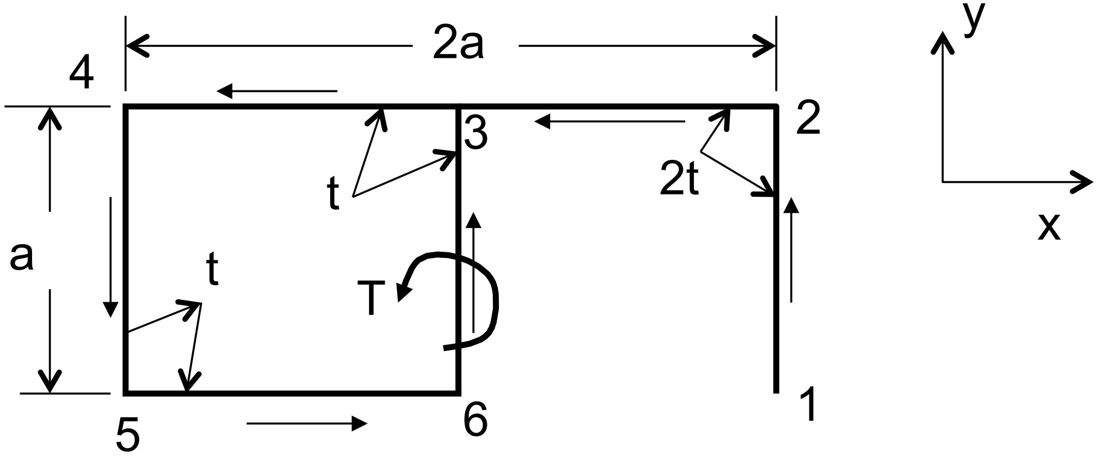
- and substituting values:

$$J_c = \frac{4\left(a^2\right)^2 t}{4a} = a^3 t$$

- therefore, $J_T = a^3 t + \dfrac{16}{3} a t^3$

note that the contribution from the open portion, J_o is negligible compared to the contribution from the closed portion

Example: Combined open and closed section under torsion



$$J_T = a^3 t + \frac{16}{3} a t^3$$

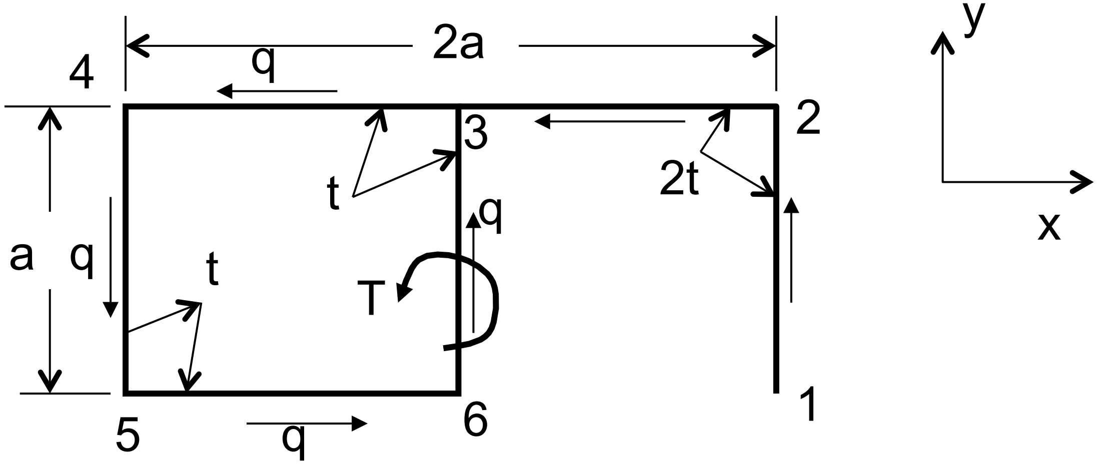
$$T = GJ \frac{d\theta}{dz}$$

• solving now for the rate of twist: $\frac{d\theta}{dz} = \frac{T}{GJ_T} = \frac{T}{G\left(a^3 t + \frac{16}{3} a t^3\right)}$

• the shear flow q in the closed portion is given by combining the rate of twist just found with

$$\left.\begin{aligned} T_c &= GJ_c \frac{d\theta}{dz} \\ T_c &= 2Aq = 2a^2 q \end{aligned}\right\} \underbrace{2a^2 q}_{T_c} = G\underbrace{\left(a^3 t\right)}_{J_c} \underbrace{\frac{T}{G\left(a^3 t + \frac{16}{3} a t^3\right)}}_{d\theta/dz} \Rightarrow q = \frac{Tat}{2\left(a^3 t + \frac{16}{3} a t^3\right)}$$

Example: Combined open and closed section under torsion



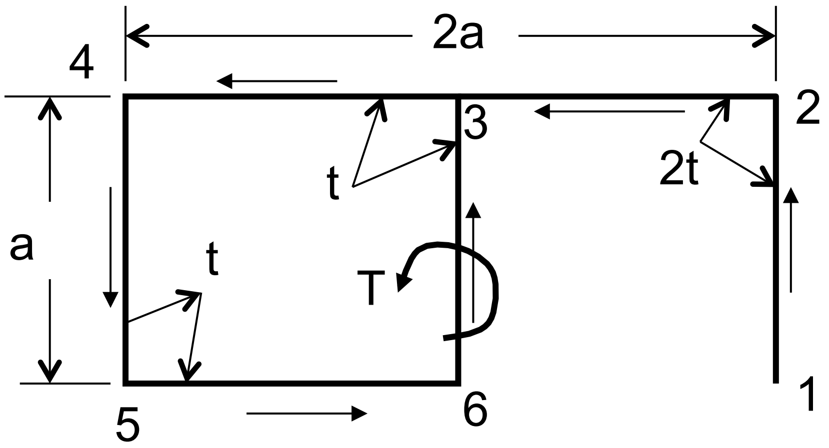
$$q = \frac{Tat}{2\left(a^3 t + \frac{16}{3} a t^3\right)}$$

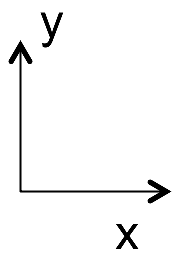
• the resulting shear stress τ in the closed portion can be found from τ=q/t:

$$\tau = \frac{Ta}{2\left(a^3 t + \frac{16}{3} a t^3\right)}$$

• finally, the max shear stress in the open portion is found from eq (4.4) $\tau_{yz} = 2Gx\frac{d\theta}{dz}$ which gives $\tau_{yz} = 2G\frac{2t}{2}\frac{d\theta}{dz} = 2Gt\frac{d\theta}{dz}$

Example: Combined open and closed section under torsion





$$\frac{d\theta}{dz} = \frac{T}{G\left(a^3 t + \frac{16}{3} a t^3\right)}$$

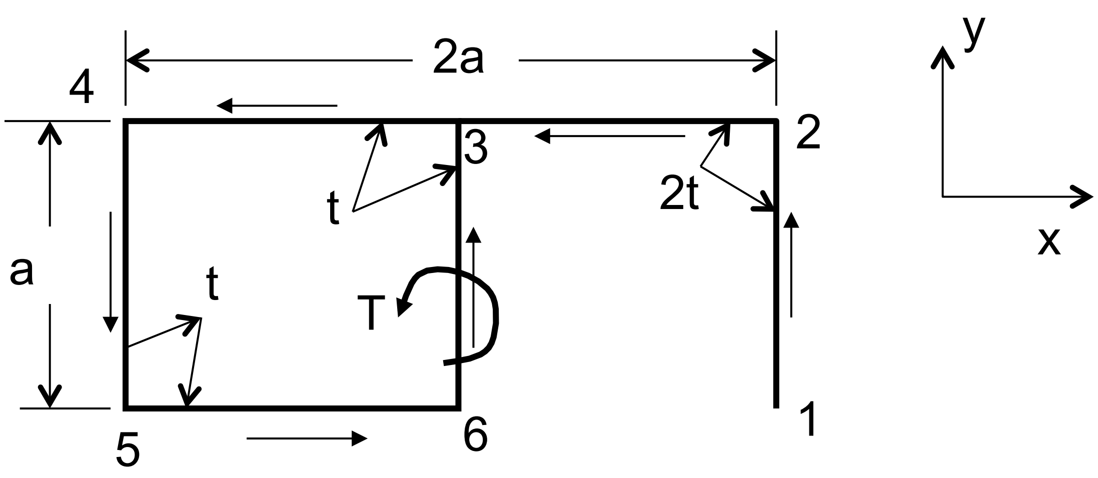
$$\tau_{yz} = 2G\frac{2t}{2}\frac{d\theta}{dz} = 2Gt\frac{d\theta}{dz}$$

- combining, the max shear stress in the open portion is

$$\tau_{yz} = 2G\frac{2t}{2}\frac{d\theta}{dz} = 2Gt\frac{T}{G\left(a^3 t + \frac{16}{3} a t^3\right)} = \frac{2T}{\left(a^3 + \frac{16}{3} a t^2\right)}$$

- note that if the thicknesses in 1-2 and 2-3 were not the same, one would have to also check the max shear stress in segment 2-3

Example: Combined open and closed section under torsion



- determine now the value of t so that there is no failure
- compare shear stress in closed and open portion
- find highest
- set that equal to τ_y and solve for t noting that t<<a

$$t = \frac{T}{2a^2\tau_y}$$