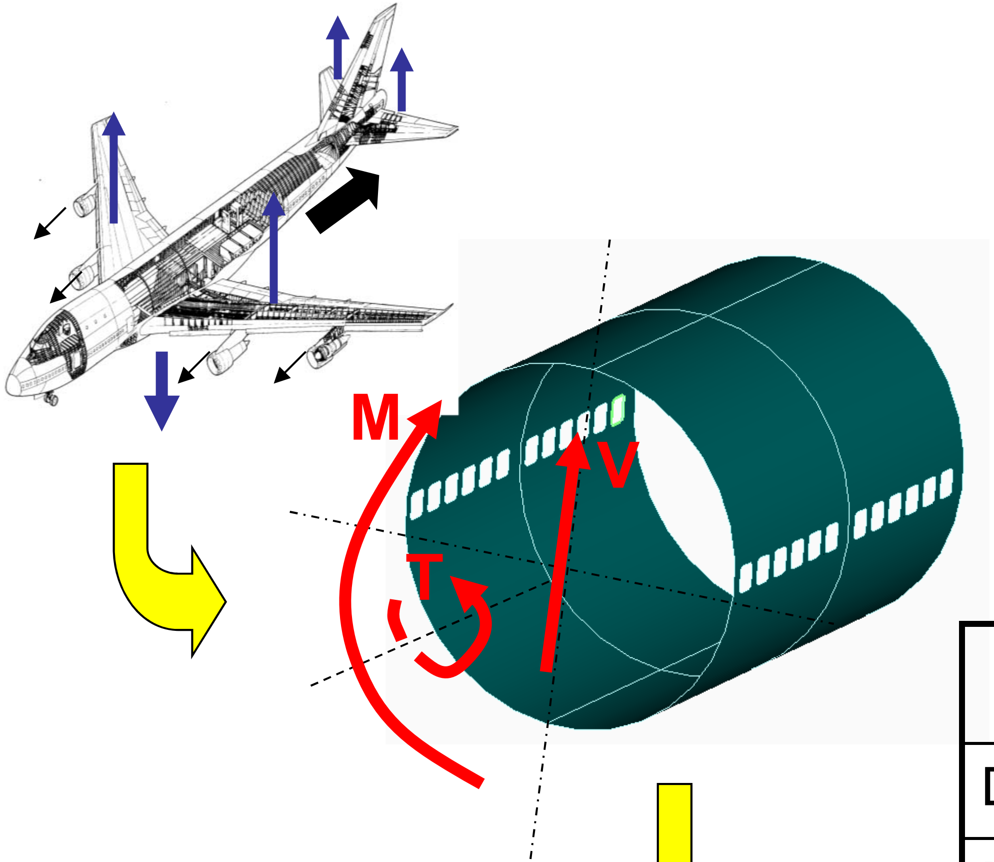
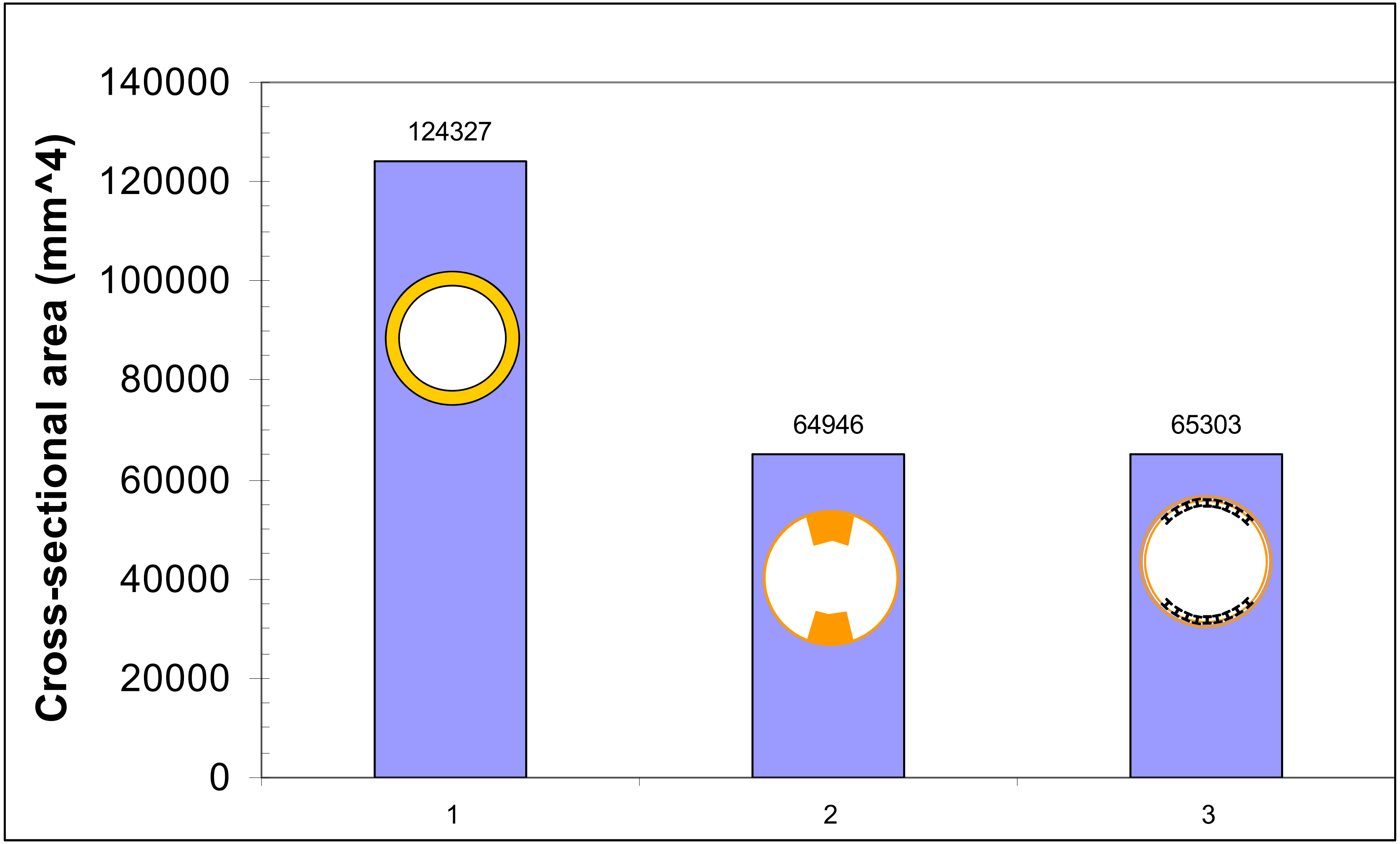
"Running" example – Fuselage cross-section



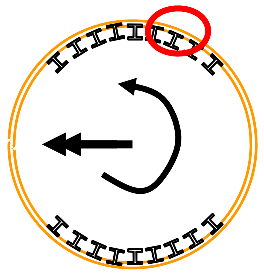
Property	Value
Diameter(m)	4.0
M (MNm)	60
V (kN)	660
T (kNm)	30

So far…

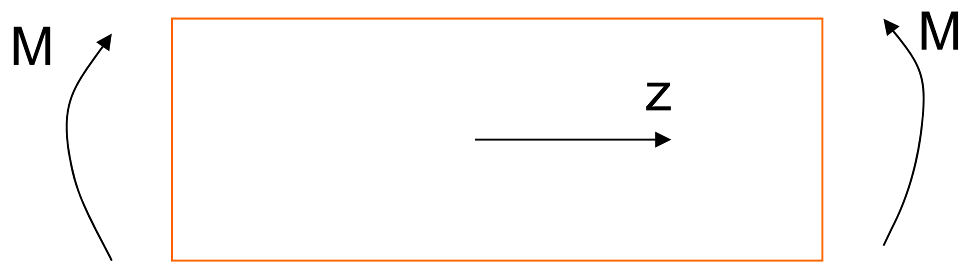


Cross-sectional area (mm^4)
140000
120000
100000
80000
60000
40000
20000
0
124327
64946
65303
1
2
3

Design for torsion showed…



front view

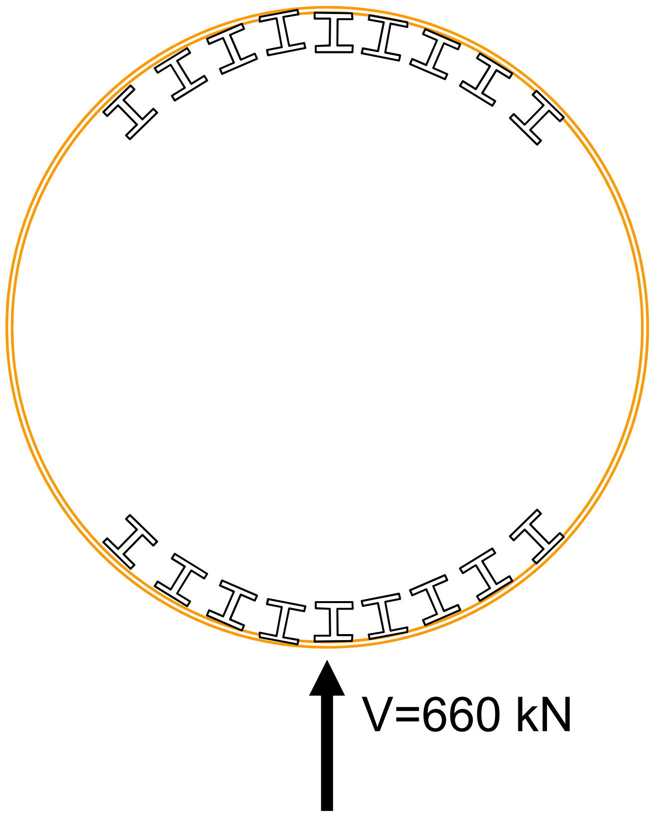


edge view

- the presence of constant bending moment M did not change the constant shear flow caused by the torque T
- we found in lecture 6 that the skin thickness required to accommodate the torque T was 0.004mm which, being smaller than the currently used value of 0.5 mm has no affect on the design

Design for shear load V

- current design:

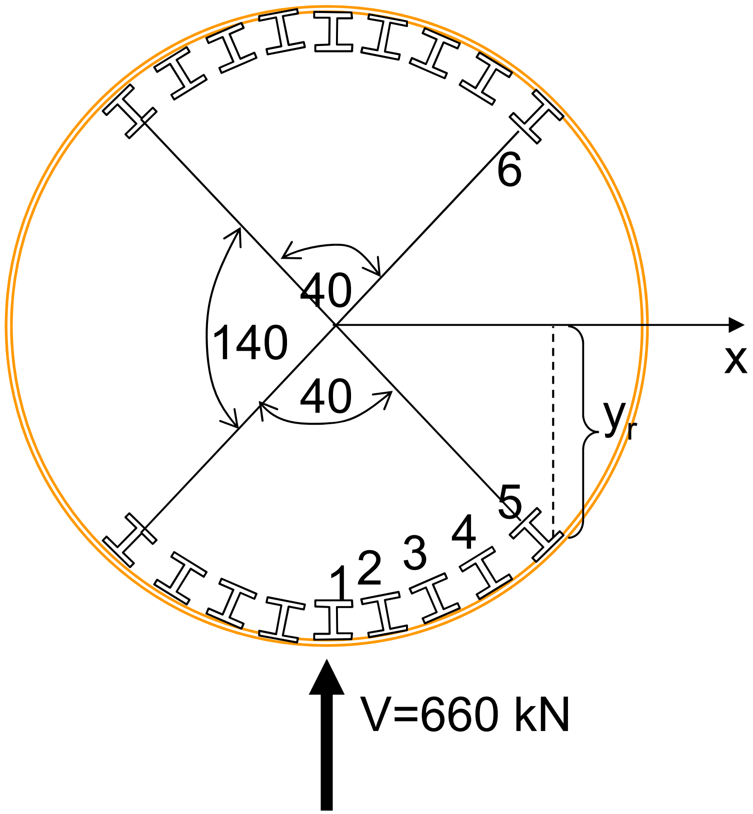


- Determine the skin thickness to avoid failure under V

- Note that any bending moment caused by V should also be taken into account by checking that the I stiffeners can cope with both M and V(L-x) with L and x TBD

- For now, V(L-x) is not accounted for

Design for shear load V



- assume the skin carries no normal loads
- treat the stiffeners as booms and, conse-rvatively, do not include any skin in the boom areas
- stiffeners (or booms) are 5 degrees apart

why?

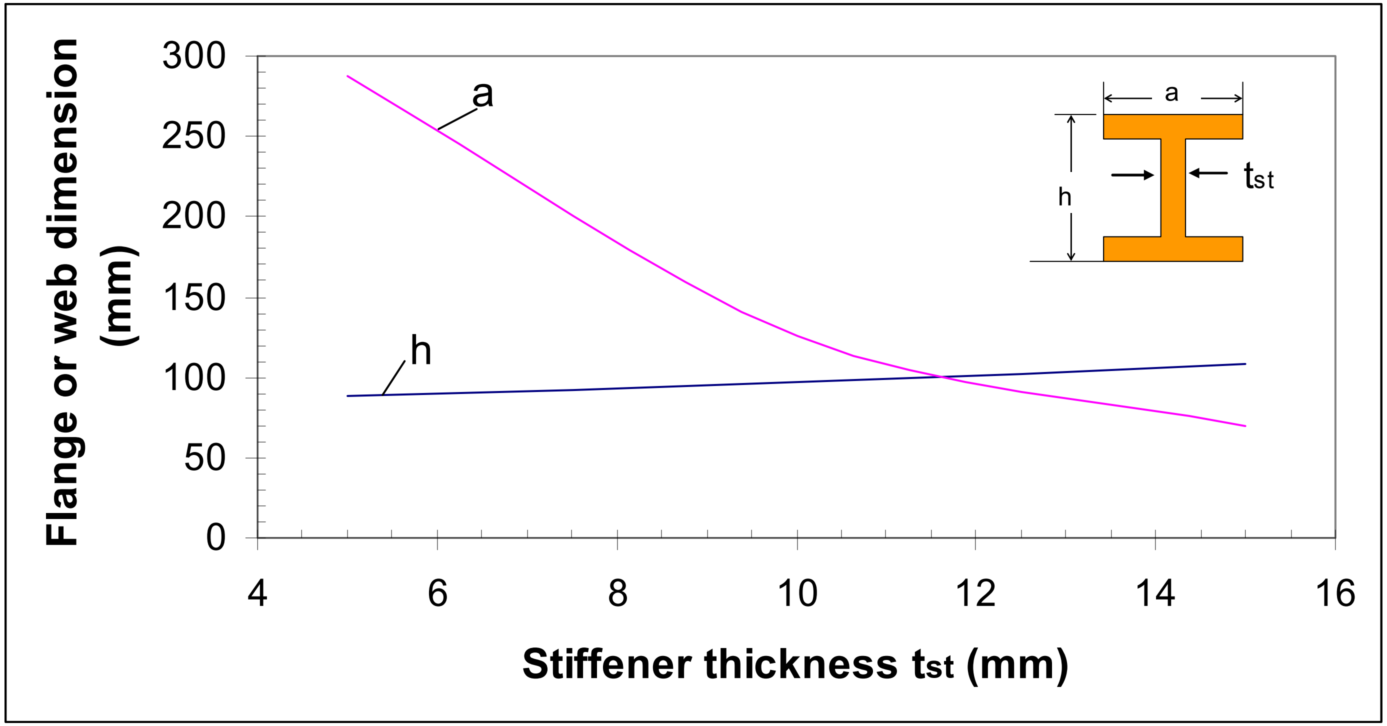
- start from boom 1 (where q=0) and proceed counter-clockwise
- note $q_{12} \neq 0$ (one boom is already passed)
- use

$$\boxed{I_{xx} = \sum_{r=1}^{18} A_r y_r^2}$$

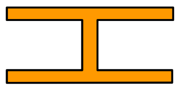
$$q_s = -\frac{V}{I_{xx}} \sum_r A_r y_r$$

Stiffened fuselage: Stiffener design

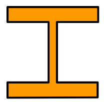
- for **nine** stiffeners at top and bottom, a and h as a function of t_{st} are given by



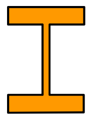
- approx to scale (except thickness)



t_{st}=5mm



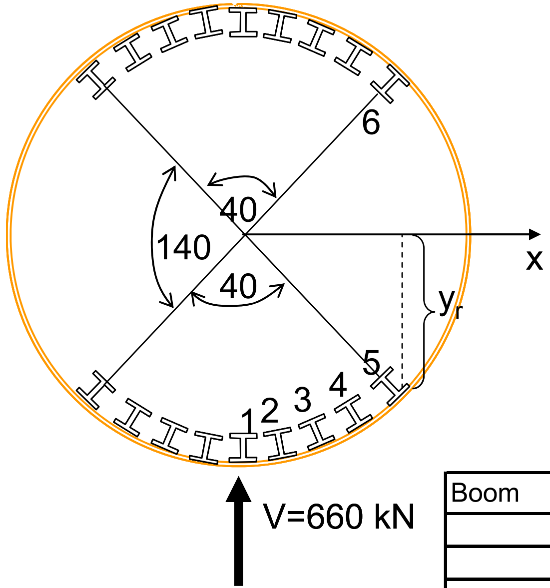
t_{st}=10mm



t_{st}=15mm

much better designs!!

Design for shear load V



- for t_{str}=10mm, a=125mm, h=97.8mm

  and A_{str}=3278mm^2

- create table:

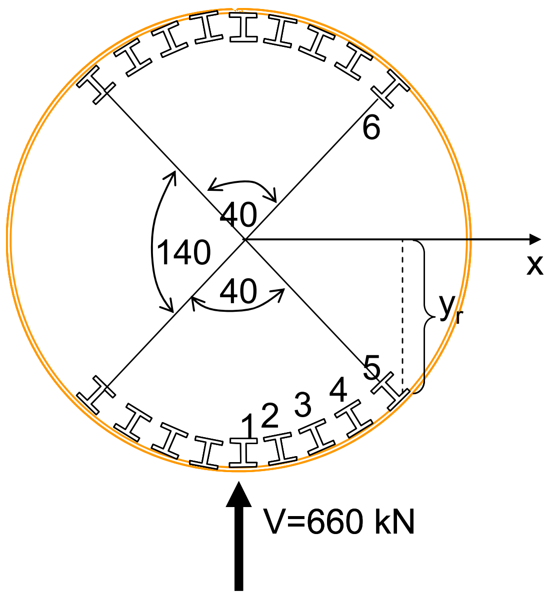
$$q_s = -\frac{V}{I_{xx}} \sum_r A_r y_r$$

Boom	Area (mm^2)	angle r	y_r (mm)	A y_r	A y_r^2	q	(N/mm)
1	3278	0	-2000	-6556000	13112000000	12	19.28343
2	3278	0.087266	-1992.389396	-6531052.441	13012399629	23	38.49348
3	3278	0.174533	-1969.615506	-6456399.629	12716624822	34	57.48394
4	3278	0.261799	-1931.851653	-6332609.717	12233662547	45	76.1103
5	3278	0.349066	-1879.385242	-6160624.822	11578187369	56	94.2308
6	3278		1879.385242	6160624.822	11578187369		

$I_{xx} = \sum^{18} A_r y_r^2$ 2.24387E+11 mm^4

- note that remaining shear flows are obtained by symmetry

Design for shear load V



- Now:

$$\tau = \frac{q}{t} \Rightarrow t = \frac{q}{\tau}$$

- and for failure:

$$t = \frac{q}{\tau_y}$$

with τ_y=275.1MPa from lecture 6

- therefore:

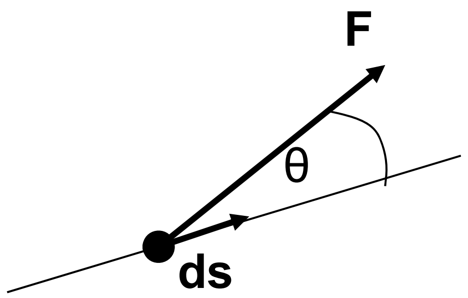
q	(N/mm)	t (mm)
12	19.28	0.07
23	38.49	0.14
34	57.48	0.21
45	76.11	0.28
56	94.23	0.34

largest thickness value is still lower than our assumed skin thickness of 0.5 mm so our current design is still OK

4.1, 4.2.2, 4.2.3,
5.1, 5.2, 5.3

Energy Methods – Castigliano's theorems

• Work:

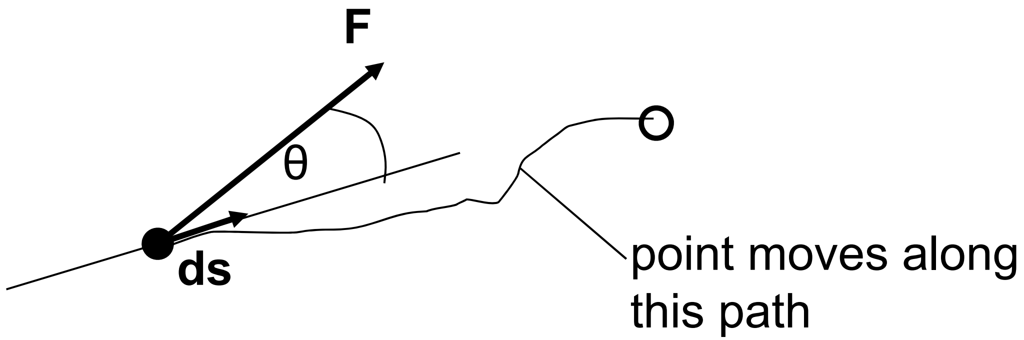


• a force acting on a point produces work

• if that point moves by a distance ds (in any direction) the work done is

$$\mathbf{F.\ ds} = F \cos\theta \ ds \qquad (12.1)$$

Work done vs Potential Energy



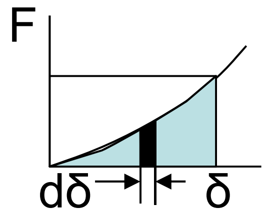
• in general, F varies as the point moves along an (arbitrary) path so the total work along the path is ∫**F.ds**

• define now the potential energy U stored in the system when a force F is acting and causes displacement δ as the work done by F:

$$U = \int F.ds = \int_0^{\delta} F\left(d\delta\right) \qquad (12.2)$$

(δ in the direction of F)

• for the simple case of a spring,

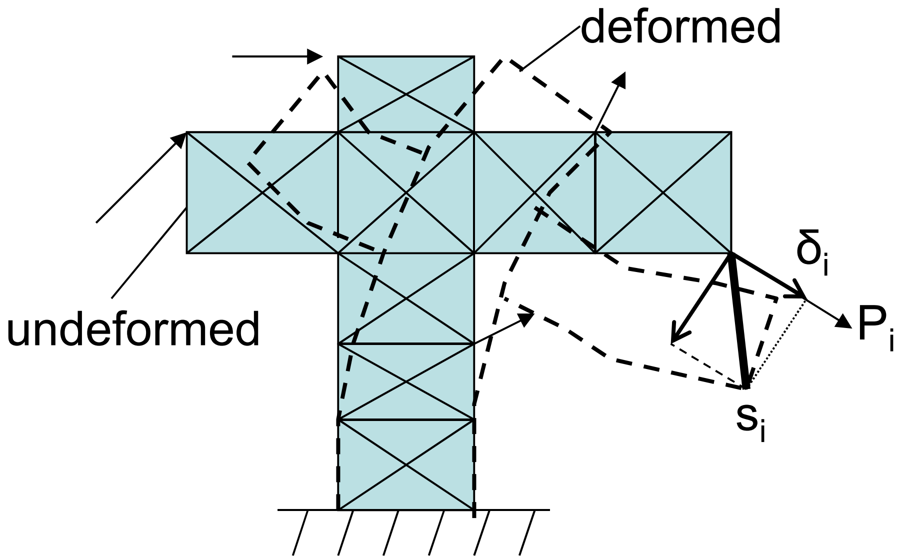


U is the area under the F versus δ curve

Work done vs Potential Energy

• the work W done by force F is stored in the system as potential energy U

• a general system loaded by an arbitrary set of loads $\mathbf{P_i}$ will reach an equilibrium state with $\mathbf{s_i}$ the corresponding displacements; the total potential energy is the sum of all contributions:



$$U = \sum_i \int^{s_i} P_i \bullet ds_i = \sum_i \int^{\delta_i} P_i d\delta_i \qquad (12.3)$$

Note that because of the constraints exerted by the entire structure to point i, the overall displacement s_i is not necessarily parallel to P_i

Complementary Work and Complementary Energy

- so far, forces P_i were applied to the system and displacements s_i resulted when the system reached equilibrium
- if instead, displacements s_i are applied, the system will reach equilibrium and the forces corresponding to the applied displacements will be P_i; this means that as s_i is applied, the corresponding force changes until equilibrium is reached and the force becomes P_i
- the work done by s_i (as s_i is exerted) is the complementary work $W_c = \int \mathbf{s} . d\mathbf{F}$
- and the energy stored in the system as a result is the complementary energy C

C is a bad choice for symbol (=compliance) but Megson uses it

Complementary Work and Complementary Energy

- then, the complementary energy C is given by (notice analogy with potential energy)

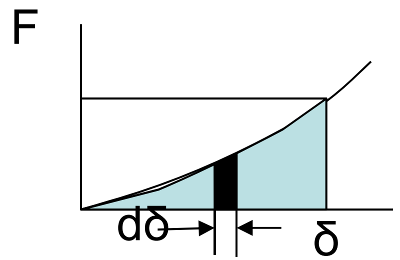
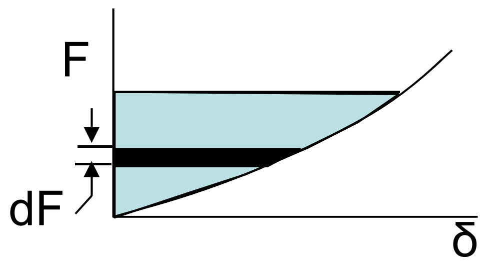
$$C = \int s \bullet dF = \int_0^{F(\delta)} \delta dF \tag{12.4}$$

- and for a general system with many displacements s_i applied at i points:

$$C = \sum_i \int_0^{P_i} s_i \bullet dP_i = \sum_i \int_0^{P_i} \delta_i dP_i \tag{12.5}$$

Again, pay attention to the fact that s_i is in some arbitrary direction but δ_i is the component of s_i **parallel** to P_i

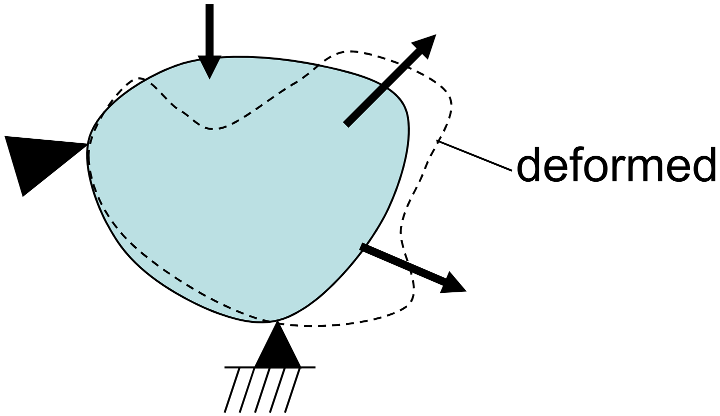
Potential energy U versus complementary energy C

Potential energy U	Complementary energy C
• constant applied forces P_i causing displacements s_i • displacements are variable • limits of integration on displacements	• constant applied displacements s_i causing forces P_i • forces are variable • limits of integration on forces
	

When is U=C??

Principle of virtual work

Given a structure in equilibrium, if a system of forces acts on it to disturb it (slightly) from equilibrium to reach a new equilibrium state, then the work done by the system of forces equals the energy stored in the structure



energy is stored in the structure as it deforms; the amount of energy stored = work done by applied forces; if the forces were removed the structure would return to its original condition and the amount of energy released would equal the work of deformation

it might appear obvious that "energy can neither be created or destroyed" but the above expressed in mathematical form is a very powerful tool in solving structural problems (e.g. the finite element method is based on it)

Principle of virtual work for a particle

- To keep it simple, consider the case of a particle instead of a body. In this case, there is no stored energy (the particle has no way of storing energy)
- The principle of virtual work then states that

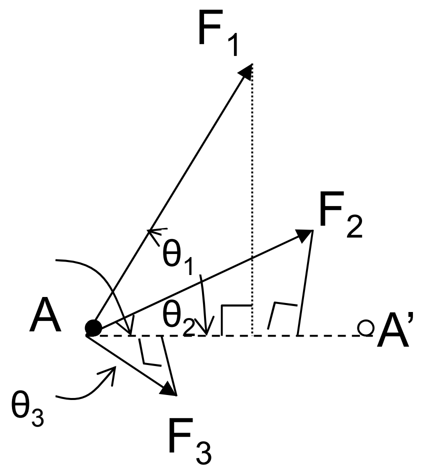
if a particle is disturbed from equilibrium by a system of forces and it reaches a new equilibrium state, the work done during the "disturbance" is zero

- note that the work is zero because the internal energy for a particle is zero

- the disturbances or changes from one equilibrium state to another have to be small **to keep the forces from changing**

Principle of virtual work for a particle

- proof of principle of virtual work for a particle
- suppose a system of forces is acting on a particle

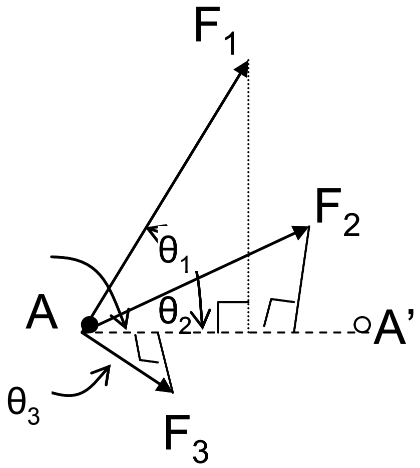


AA'=Δv

- and suppose the particle moves a small (virtual) distance Δv from A to A'
- the work done during this excursion is, from (12.3) (exchanging work for potential energy in this case):

CK1

Principle of virtual work for a particle



AA'=Δv

$$U = \sum_i \int^{s_i} P_i \bullet ds_i = \sum_i \int^{\delta_i} P_i d\delta_i \qquad (12.3)$$

$$W = \sum_{i=1}^{N} F_{i\Delta v}\delta_{i\Delta v} \qquad (12.6)$$

where $F_{i\Delta v}$ is the component of F_i along Δv and

$\delta_{i\Delta v}$ is the component of displacement along Δv

- but since the particle is moved from A to A', the component of displacement is **for all forces** Δv:

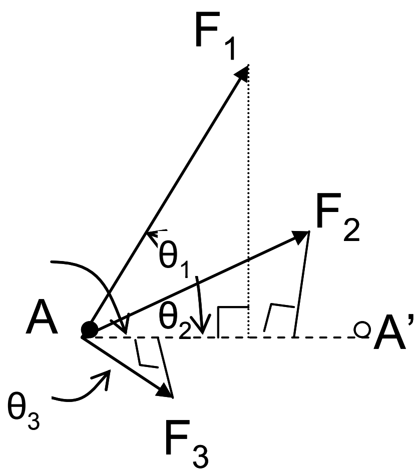
$$\delta_{i\Delta v} = \Delta v$$

Slide 18

CK1 Deltav is a small (infinitesimal) deflection ds (analogous to dx, or dy, or dz in a cartesian coordinate system). So each force causing an infinitesimal deflection will cause the same ds or Deltav

Christos Kassapoglou; 22-3-2011

Principle of virtual work for a particle



AA'=Δv

$$W = \sum_{i=1}^{N} F_{i\Delta v}\delta_{i\Delta v} \qquad (12.6)$$

$$\delta_{i\Delta v} = \Delta v$$

- now the component of force F_i along Δv is given by

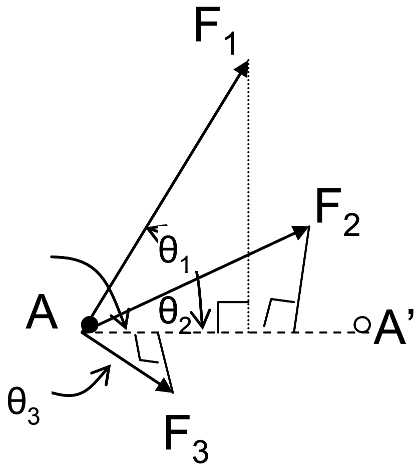
$$F_{i\Delta v} = F_i \cos\theta_i$$

where θ_i is the angle between the line of action of F_i and AA'(=Δv)

- substituting in eq. (12.6):

$$W = \sum_{i=1}^{N} F_i \cos\theta_i (\Delta v) = \Delta v \sum_{i=1}^{N} F_i \cos\theta_i \qquad (12.7)$$

Principle of virtual work for a particle



AA'=Δv

$$W = \sum_{i=1}^{N} F_i \cos\theta_i (\Delta v) = \Delta v \sum_{i=1}^{N} F_i \cos\theta_i \qquad (12.7)$$

- but $\Sigma F_i \cos\theta_i$ is the sum of the components of all the forces along Δv; So if R is the resultant of all the forces:

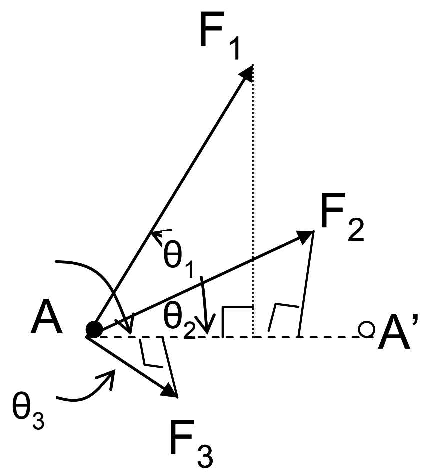
$$\sum_{i=1}^{N} F_i \cos\theta_i = R\cos\theta_R$$

where θ_R is the angle between the resultant of all forces with Δv

- combining:

$$W = \Delta v R \cos\theta_R \qquad (12.8)$$

Principle of virtual work for a particle



AA'=Δv

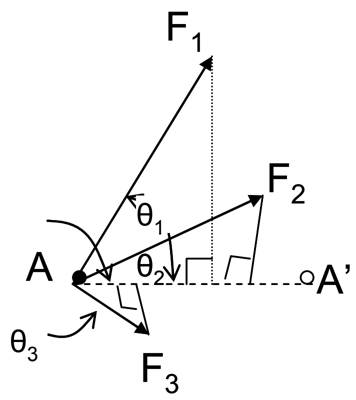
$$W = \Delta v R \cos\theta_R \qquad (12.8)$$

• suppose now the particle is in equilibrium

• then, the total force R acting on it must be zero (otherwise the particle flies off!)

• if R=0 then W=0 !! (Q.E.D.)

Principle of virtual work for a particle



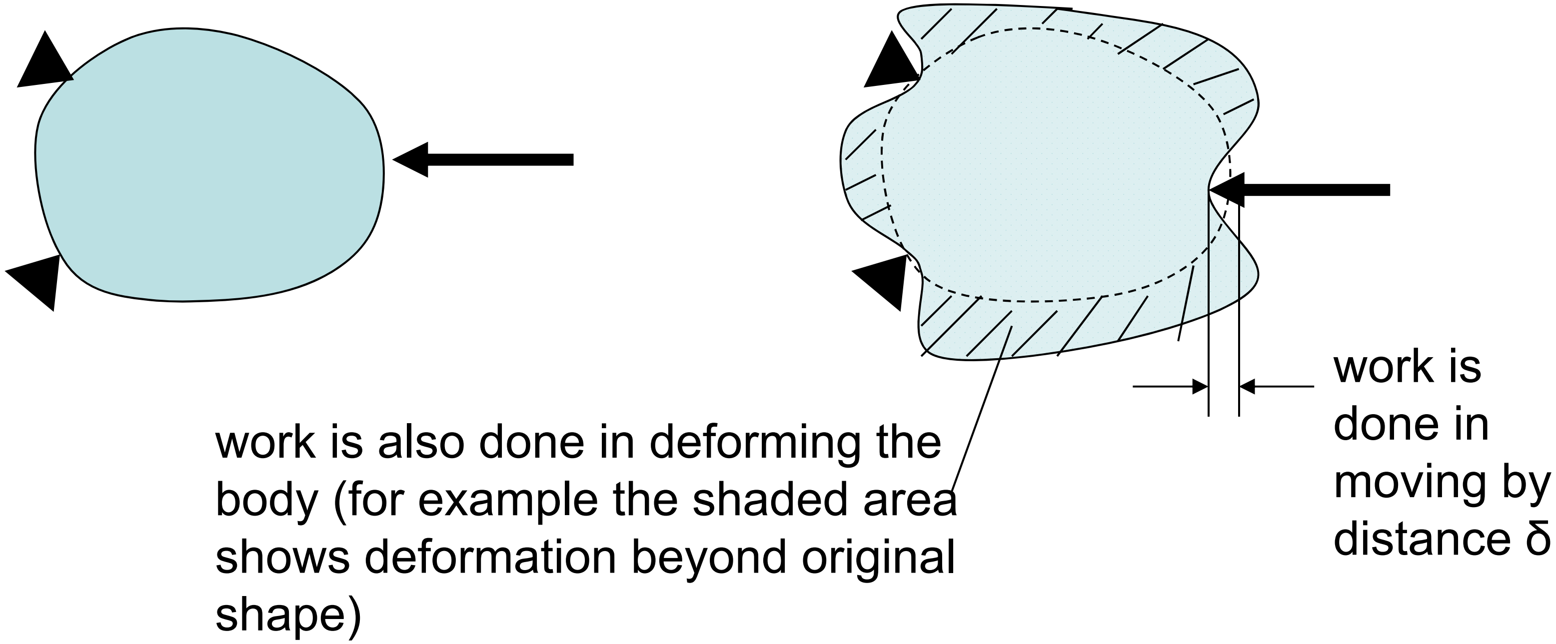
• we have demonstrated that if a particle is disturbed slightly to a new equilibrium position, the (virtual) work done is zero

• once again, note that we want the disturbance to be small so the orientations and magnitudes of the forces do not change in order to maintain equilibrium

Principle of minimum complementary energy

- consider a body (consisting of particles) that is in equilibrium under a system of forces P_i
- corresponding to each P_i the local displacement is Δ_i (in the direction of P_i)
- suppose now we apply virtual forces δP_i along the directions Δ_i
- the total virtual work done in the system will be the sum of the energy stored in the system under the loads δP_i plus the work done in locally moving the points on which each δP_i acts

Principle of minimum complementary energy



• important note: when a force is acting on a deformable body, work is done because the point of action moves (= force x distance) plus energy is stored in the body as it strains

Principle of minimum complementary energy

- therefore, for our case of a body in equilibrium under forces P_i and deflections Δ_i, and virtual forces δP_i, the total virtual work is

$$Work \quad done = W = \sum_{i=1}^{N} \Delta_i \left(\delta P_i \right)$$

Work done (Force x displacement) by the externally applied forces (12.9)

- and the (complementary) energy stored by deformation is, from eq. (12.4)

$$C = \int s \bullet dF = \int_0^{F(\delta)} \delta dF \tag{12.4}$$

- but, according to the principle of virtual work, the two are equal

$$W = C \Rightarrow -C + W = 0 \tag{12.10}$$

Principle of minimum complementary energy

$$W = C \Rightarrow -C + W = 0 \qquad (12.10)$$

- recall, however, that the work and energy we are talking about arose from **small disturbances caused by virtual forces acting on the body**; therefore, these are incremental changes in the work and energy of the body

- if C is the energy stored in the body when the virtual forces act, then –C is the work done to the body in straining it by the same forces

- in an analogous fashion, if W is the work done by the external forces, -W is the energy that is consumed by the body when the forces act

Principle of minimum complementary energy

• at this point it appears we are playing with words and, in some sense, we are; this is an issue of perspective

– book-keep the total work of the system: W-C

– book-keep the total energy of the system: C-W

• choosing the latter, we have shown that incremental forces δP_i make the quantity C-W zero

• but the quantity C-W is the incremental change in the total energy of the system caused by the incremental forces δP_i

• so denoting the total internal energy by C_i and the total work by $-W_e$ we can write the incremental change in total energy δ(total energy)

Principle of minimum complementary energy

incremental total energy change W-C=0

incremental total energy change= δ(total energy)= $\delta(C_i+W_e)=0$

where $\delta W_e=-W$ for incremental changes

- therefore,

$$\delta\left(C_i + W_e\right) = 0 \tag{12.11}$$

where

$C_i = \int_0^{F(\delta)} \delta(F)dF$ is the internal complementary energy (in terms of forces) (12.12)

$W_e = -\sum_{i=1}^{N} \Delta_i P_i$ is the work done by the body (negative of work done by external forces) note that $\delta W_e=-\Sigma\Delta_i\, \delta P_i \Rightarrow W_e=-\Sigma\Delta_i P_i$ (12.13)

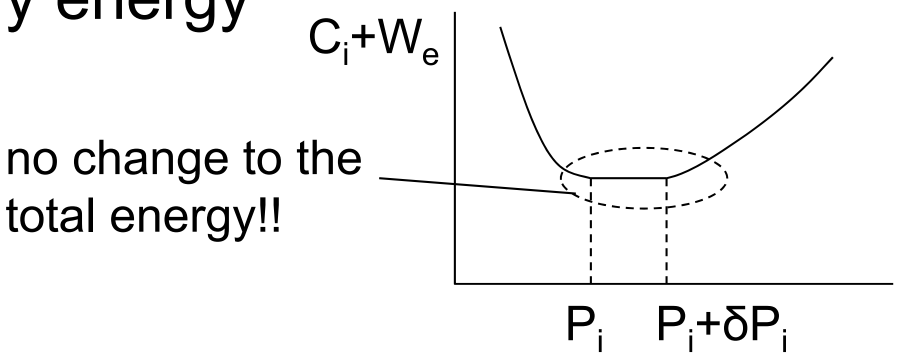
- note that the increment of a quantity δ(..) is also called a variation of the quantity (…) and it is analogous (but not the same) as the differential d(…); eq (12.11) can be derived rigorously by using calculus of variations

Principle of minimum complementary energy
(what is the big deal?)

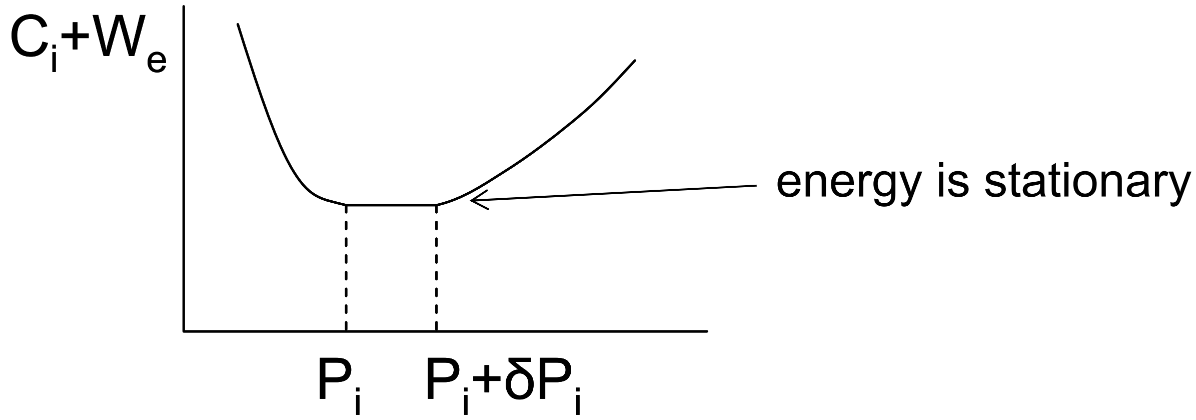
- so far, we have shown that if a system is in equilibrium, an incremental change in its internal energy is equal to an incremental change in work done by virtual forces

- or, the total complementary energy of the system (internal complementary + work done) has zero change (variation) in response to incremental changes in the applied forces

- if we make a plot of total complementary energy as a function of load P_i, as $P_i \rightarrow P_i + \delta P_i$ there is **no change** to the total complementary energy



Principle of minimum complementary energy



- therefore:

If a body is in equilibrium under a set of applied forces, the internal state of stress is such that the total complementary energy is stationary

usually, stationary implies a minimum

Principle of minimum complementary energy - Implications

• this is a very powerful theorem to use in obtaining (approximate) solutions to structural problems

• given a structure under applied loads we can select approximate expressions for the stresses caused by the applied loads; these expressions are in terms of unknown constants

• if we satisfy stress equilibrium, we can then determine the unknown constants by minimizing the total complementary energy

• i.e., according to the principle of minimum complementary energy, of all acceptable stress expressions (acceptable means they satisfy the equilibrium equations and force boundary conditions), the best solution is the one that minimizes the complementary energy

Principle of minimum potential energy

• an exactly analogous theorem can be derived for the total potential energy

• instead of varying loads or stresses, here we vary displacements

• then

also known as strain-displacement eqns

If the displacements in a body satisfy compatibility and the displacement boundary conditions then if the body is in equilibrium the total potential energy is minimized

Not in Megson

Castigliano's Second Theorem

• two ways to derive it

• first, use the principle of virtual work; for a body in equilibrium, if the load P_i is changed by a small amount, ΔP_i with corresponding deflection along its line of action equal to Δ_i, the work done equals the change ΔC in internal complementary energy C_i (see also eqs. 12.9,12.10)

$$\left(\Delta P_i\right)\Delta_i = \Delta C \qquad (12.14)$$

• solving for Δ_i

$$\Delta_i = \frac{\Delta C}{\Delta P_i} \qquad (12.15)$$

• in the limit, for infinitesimal changes in P_i and C:

$$\boxed{\Delta_i = \frac{\partial C_i}{\partial P_i}} \qquad (12.16)$$

note the "unfortunate" use of subscripts here: i in C_i is a subscript denoting internal energy while i in Δ_i and P_i are dummy indices ranging from 1 to N the number of applied loads

Castigliano's Second Theorem

- second way (for the mathematically inclined): use principle of minimum complementary energy
- from eq. (12.11),

$$\delta\left(C_i + W_e\right) = 0 \tag{12.11}$$

- if we use eq (12.13) to express the work W_e

$$W_e = -\sum_{i=1}^{N} \Delta_i P_i \tag{12.13}$$

- we get

$$\delta\left(C_i - \sum_{i=1}^{N} \Delta_i P_i\right) = 0 \tag{12.17}$$

Castigliano's Second Theorem

$$\delta\left(C_i - \sum_{i=1}^{N} \Delta_i P_i\right) = 0 \qquad (12.17)$$

• recall now that C_i is expressed in terms of the applied loads P_i; so a small change δC_i can be expressed using the first terms of a Taylor series expansion,

$$\delta C_i = \frac{\partial C_i}{\partial P_1}\delta P_1 + \frac{\partial C_i}{\partial P_2}\delta P_2 + \ldots \frac{\partial C_i}{\partial P_N}\delta P_N \qquad (12.18)$$

• and

$$\delta \sum_{i=1}^{N} \Delta_i P_i = \Delta_1 \delta P_1 + \Delta_2 \delta P_2 + \ldots \Delta_N \delta P_N \qquad (12.19)$$

• placing (12.18) and (12.19) into (12.17) and collecting terms:

$$\left(\frac{\partial C_i}{\partial P_1} - \Delta_1\right)\delta P_1 + \left(\frac{\partial C_i}{\partial P_2} - \Delta_2\right)\delta P_2 + \ldots \left(\frac{\partial C_i}{\partial P_N} - \Delta_N\right)\delta P_N = 0 \qquad (12.20)$$

Castigliano's SecondTheorem

$$\left(\frac{\partial C_i}{\partial P_1} - \Delta_1\right)\delta P_1 + \left(\frac{\partial C_i}{\partial P_2} - \Delta_2\right)\delta P_2 + \ldots\left(\frac{\partial C_i}{\partial P_N} - \Delta_N\right)\delta P_N = 0 \qquad (12.20)$$

• now the incremental changes in the loads δP_1, δP_2, etc. are arbitrary (they can be anything)

• this means they cannot (only) assume values that will make the entire left hand side of (12.20) equal to zero; because this would be a few fixed sets of values and the equation would only hold for those particular sets of δP_1, δP_2, … and not for **any** set

• then, the only way (12.20) can be satisfied with δP_1, δP_2, … arbitrary is if:

$$\left(\frac{\partial C_i}{\partial P_1} - \Delta_1\right) = 0, \quad \left(\frac{\partial C_i}{\partial P_2} - \Delta_2\right) = 0, \quad \ldots\left(\frac{\partial C_i}{\partial P_N} - \Delta_N\right) = 0 \qquad (12.21)$$

Castigliano's Second Theorem

$$\left(\frac{\partial C_i}{\partial P_1} - \Delta_1\right) = 0, \quad \left(\frac{\partial C_i}{\partial P_2} - \Delta_2\right) = 0, \quad \ldots \left(\frac{\partial C_i}{\partial P_N} - \Delta_N\right) = 0 \qquad (12.21)$$

- eq (12.21) can be written in general:

$$\Delta_i = \frac{\partial C_i}{\partial P_i} \qquad (12.16)$$

- which is identical to eq (12.16) we found before, QED.

Castigliano's Second Theorem – Discussion

$$\Delta_i = \frac{\partial C_i}{\partial P_i} \qquad (12.16)$$

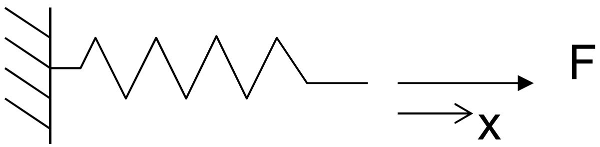
- the theorem states that if the internal energy of a structure is **expressed in terms of applied loads**, (that's why it is called complementary as opposed to potential energy when it would be expressed in terms of displacements) then the displacement at any location **parallel to a locally applied load** equals the partial derivative of the internal energy with respect to that load

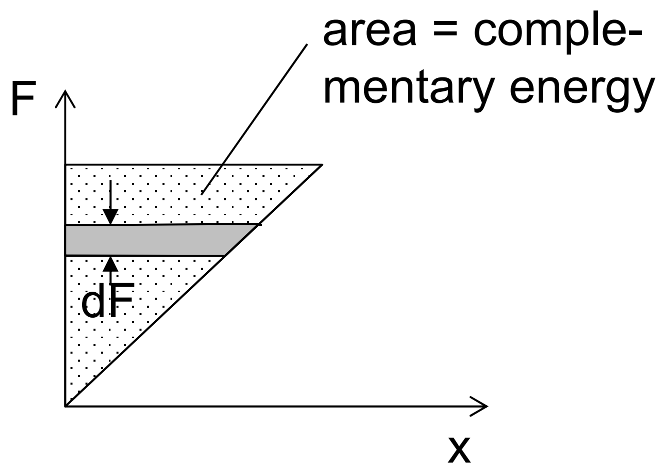
- note that partial derivatives are involved because C_i is in general a function of more than one loads P_1, P_2, …

Castigliano's Second Theorem – Examples

• we must first find the complementary energy of some frequently encountered systems

Linear Spring





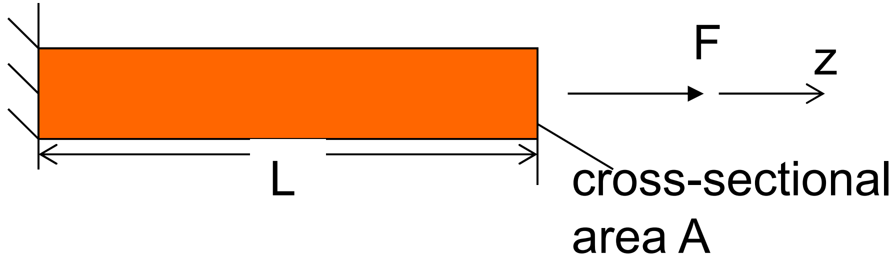
linear spring with spring constant k;
force F causes displacement x=> F = kx

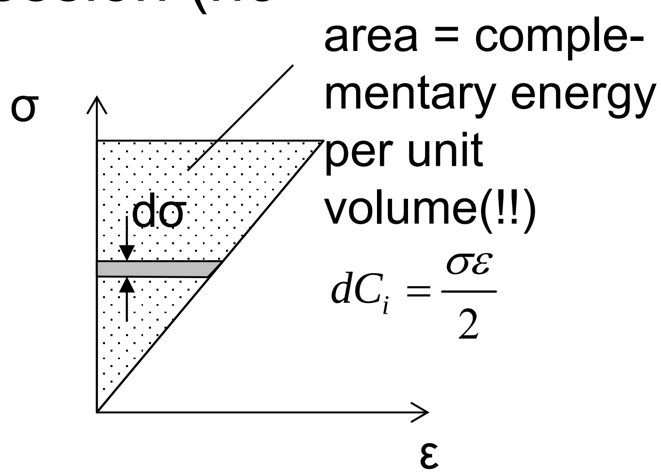
$$C_i = \int_0^F x dF \qquad \text{but} \qquad x = \frac{F}{k} \qquad \text{substituting:}$$

$$\boxed{C_i = \int_0^F \frac{F}{k} dF = \frac{F^2}{2k}} \qquad (12.22)$$

Castigliano's Second Theorem – Examples

Bar (or beam) in tension or compression (no buckling)





$$dC_i = \frac{\sigma\varepsilon}{2}$$

$$C_i = \int_{vol} dC_i = \iiint \frac{\sigma\varepsilon}{2} dxdydz = \int_0^L \frac{A\sigma\varepsilon}{2} dz$$

but $\sigma = E\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E}$

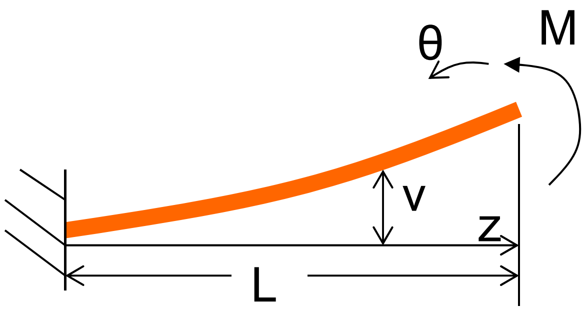
$$\sigma = \frac{F}{A}$$

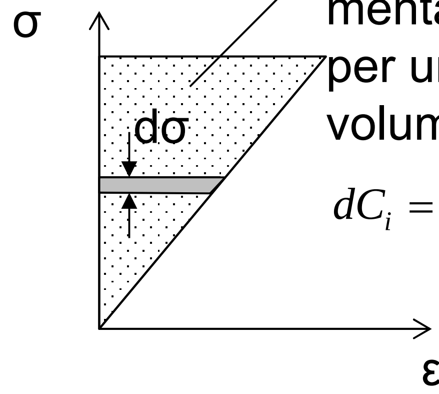
$$C_i = \int_0^L \frac{F^2}{2EA} dz$$

- and since F, E, A, are independent of z: $\boxed{C_i = \frac{F^2 L}{2EA}}$ (12.23)

Castigliano's Second Theorem – Examples

Beam under bending moment M





$$dC_i = \frac{\sigma\varepsilon}{2}$$

we know $\sigma = E\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E}$ so $dC_i = \frac{\sigma^2}{2E}$

from bending theory, lecture 2, $\sigma_z = -\frac{My}{I}$

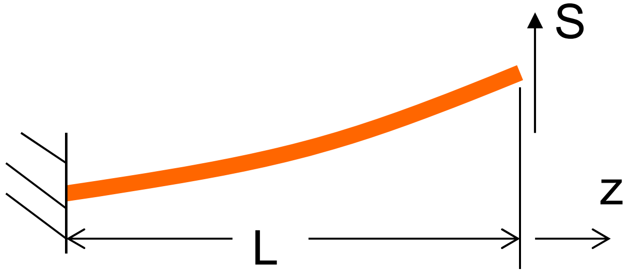
combining the two: $dC_i = \frac{M^2 y^2}{2EI^2}$

then, $C_i = \int_{vol} dC_i = \iiint \frac{M^2 y^2}{2EI^2} dxdydz$ but, by definition, $\iint y^2 dxdy = I$

therefore, $$\boxed{C_i = \int_0^L \frac{M^2}{2EI} dz} \qquad (12.24)$$

Castigliano's Second Theorem – Examples

Beam under shear load S



the shear force S causes two stresses at any beam cross-section: normal and shear; both contribute to the energy C_i but the shear contribution is negligible compared to the bending contribution

therefore, we can use the expression we derived before for the applied moment by properly evaluating the moment caused by S; at any station:

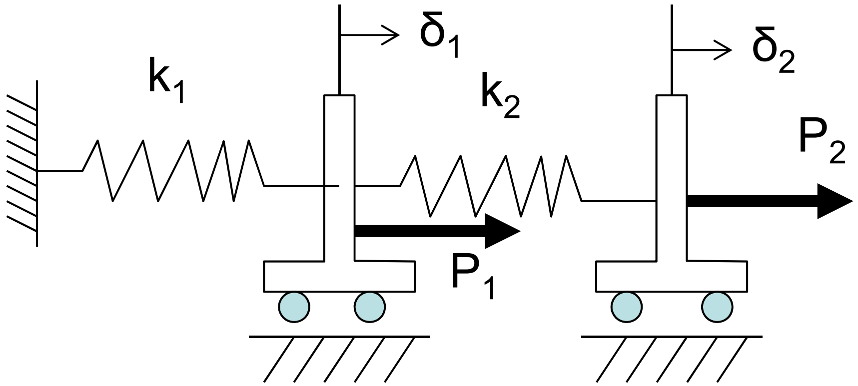
$$M = S(L-z)$$

substituting in (12.24): $C_i = \int_0^L \frac{M^2}{2EI} dz$ (12.24)

$$\boxed{C_i =} \int_0^L \frac{S^2 (L-z)^2}{2EI} dz = \int_0^L \frac{S^2}{2EI}\left(L^2 - 2Lz + z^2\right) dz = \frac{S^2}{2EI}\left[L^2 z - 2L\frac{z^2}{2} + \frac{z^3}{3}\right]_0^L = \boxed{\frac{S^2 L^3}{6EI}} \quad (12.25)$$

Castigliano's Second Theorem – Application 1

- determine the displacements of the two springs shown



$$C_i = \int_0^F \frac{F}{k}\,dF = \frac{F^2}{2k} \qquad (12.22)$$

- from eq (12.22), the energy in spring 1 is

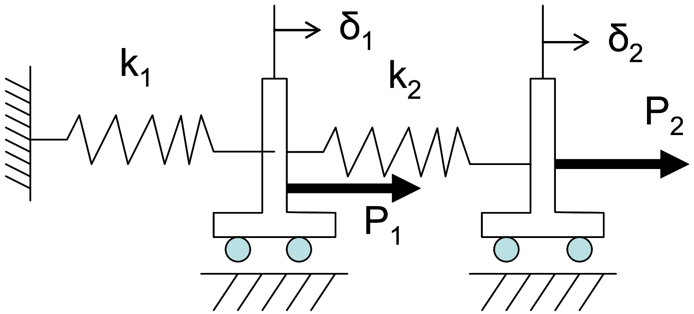
$$C_1 = \frac{F_1^2}{2k_1} \qquad (12.26)$$

- similarly for spring 2,

$$C_2 = \frac{F_2^2}{2k_2} \qquad (12.27)$$

where F_1 and F_2 are the forces in the springs

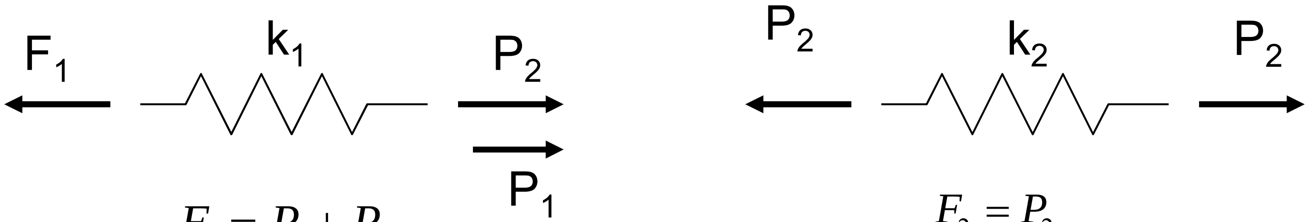
Castigliano's Second Theorem – Application 1



$$C_1 = \frac{F_1^2}{2k_1} \qquad (12.26)$$

$$C_2 = \frac{F_2^2}{2k_2} \qquad (12.27)$$

- need to determine F_1 and F_2
- free body diagram for each of the springs:



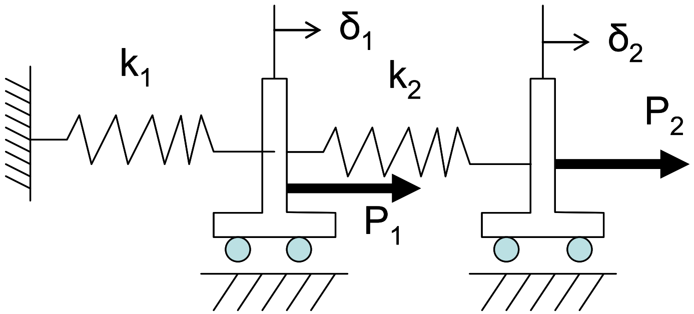
$$F_1 = P_1 + P_2$$

$$F_2 = P_2$$

- using (12.26) and (12.27), the total energy is given by:

$$C_1 + C_2 = \frac{\left(P_1 + P_2\right)^2}{2k_1} + \frac{P_2^2}{2k_2} \qquad (12.28)$$

Castigliano's Second Theorem – Application 1



$$C_1 + C_2 = \frac{\left(P_1 + P_2\right)^2}{2k_1} + \frac{P_2^{\,2}}{2k_2} \qquad (12.28)$$

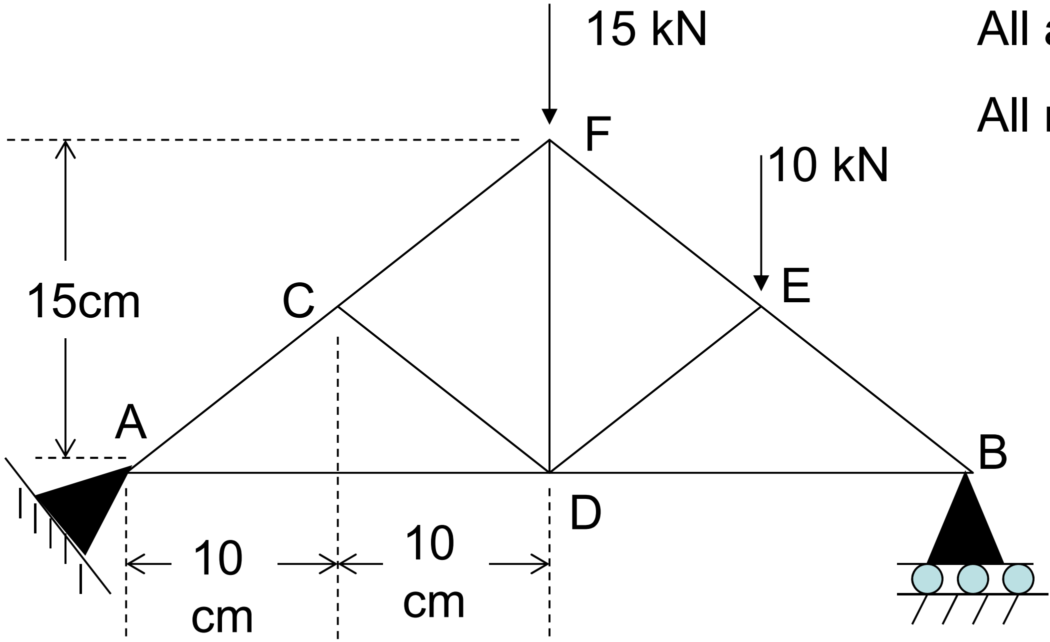
$$\Delta_i = \frac{\partial\left(C_1 + C_2\right)}{\partial P_i} \qquad (12.16)$$

• using Castigliano's 2nd theorem, eq (12.16), the deflections Δ_1 of spring 1 and Δ_2 for spring 2 are found to be:

$$\Delta_1 = \frac{P_1 + P_2}{k_1}$$

$$\Delta_2 = \frac{P_1 + P_2}{k_1} + \frac{P_2}{k_2}$$

Castigliano's Second Theorem – Application 2



All areas 25 mm^2

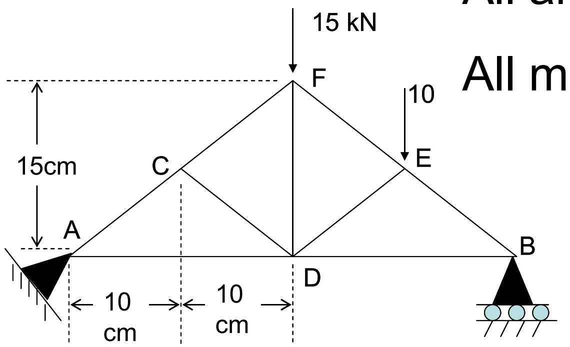
All members have stiffness E

(1) determine the deflection at point D under the given forces

(2) determine the horizontal force Q required at point B to have 0 horizontal deflection at that location

(3) with Q as found in (2), determine the new deflection at D

Castigliano’s Second Theorem – Application 2



All areas 25 mm^2

All members have stiffness E

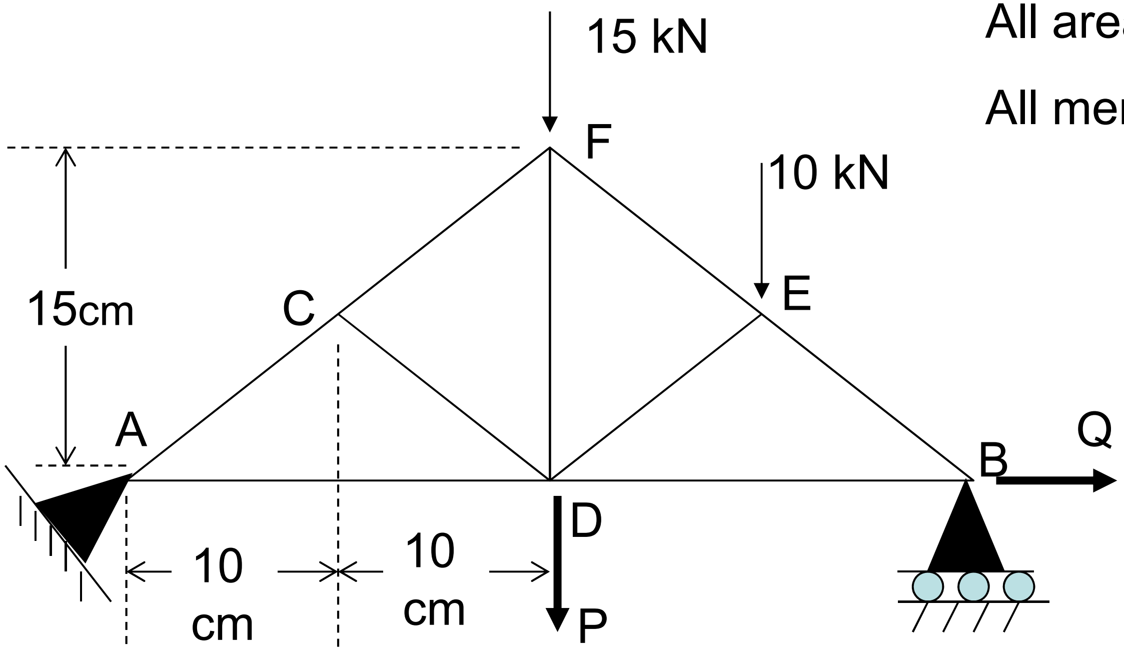
$$C_i = \frac{F^2 L}{2EA} \qquad (12.23)$$

• We want to use eq (12.23) which gives the complementary potential energy in a truss (or beam) member under tension or compression.

• since we are interested in deflections at points D (vertical) and B (horizontal) where no forces F are applied, we apply there **fictitious** forces in the direction of interest; we then compute the energy in terms of these forces, apply (12.23) and then set the fictitious forces back to zero

Castigliano's Second Theorem – Application 2

- P and Q are fictitious forces



All areas 25 mm^2

All members have stiffness E

- now determine the forces in all truss members in terms of P and Q