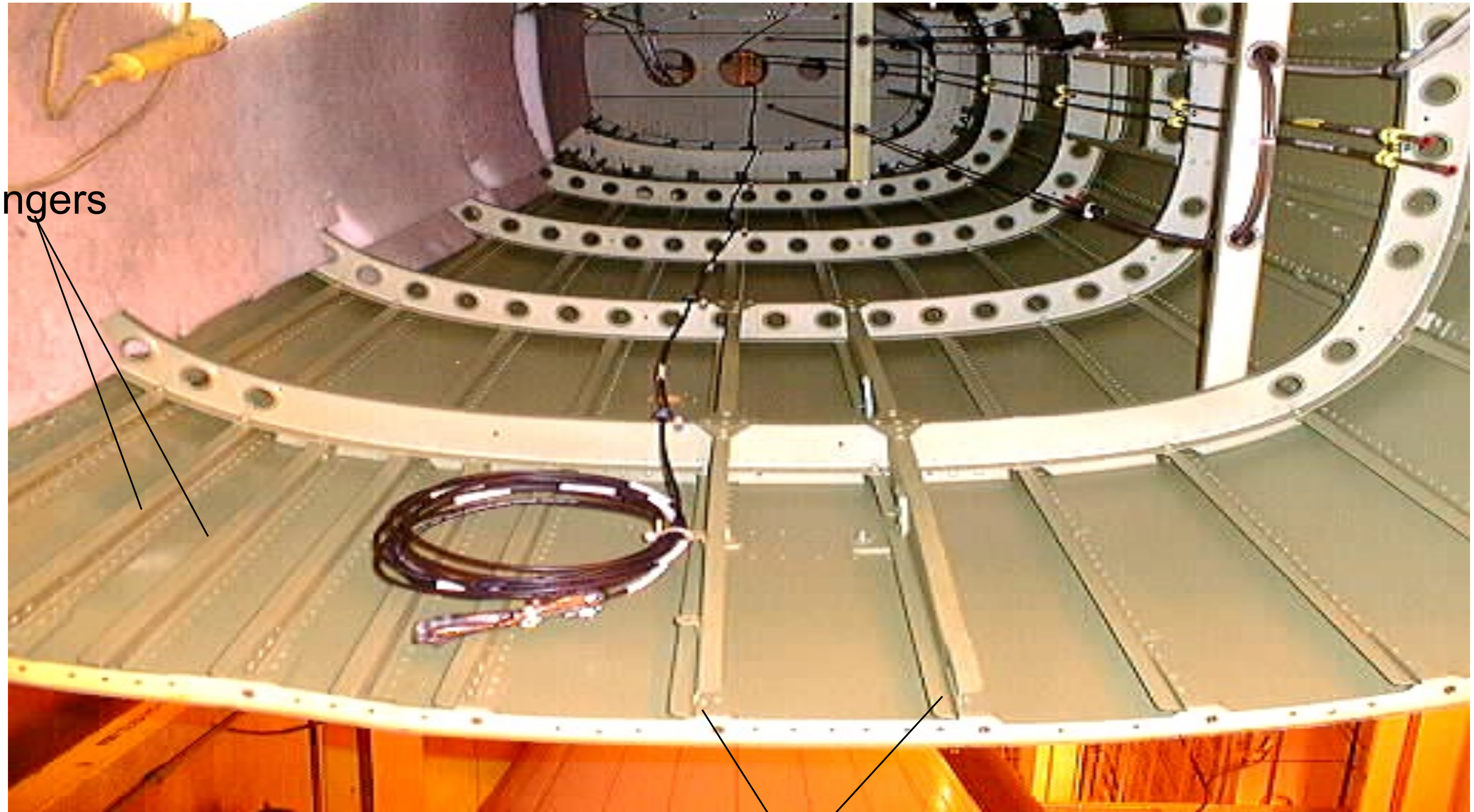
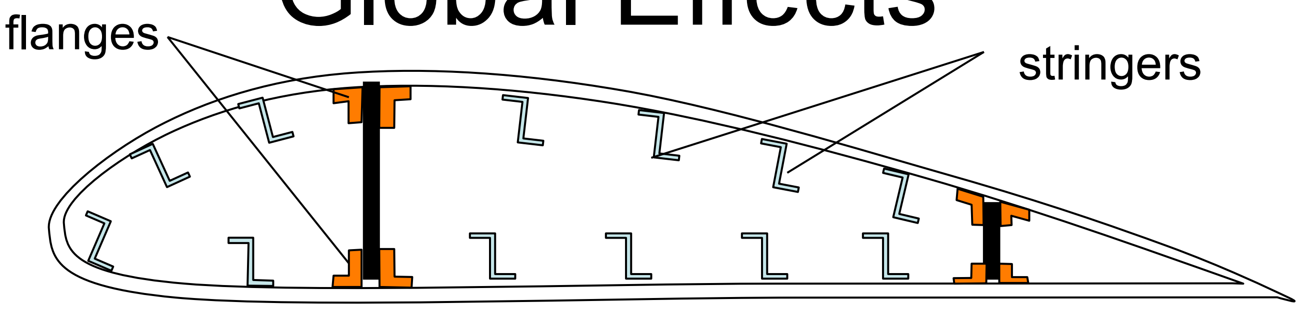
Megson: 20, 20.1, 20.2, 20.3, 20.3.1, 20.3.2, 20.3.3, 20.3.5

Structural Idealization

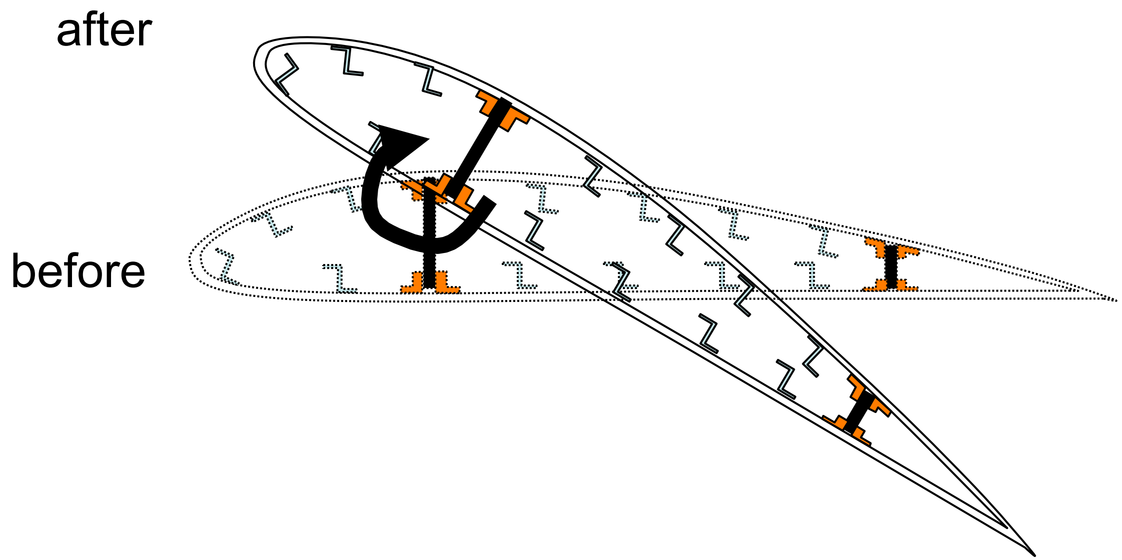


Structural Idealization – Local vs Global Effects



- In general, a wing (or a fuselage) bends up or down and twists
- the different components are used to resist the local loads:
  - skins take shear loads due to torsion and some bending loads (tension or compression) due to bending
  - stringers take tension or compression loads due to bending
  - spars take bending and shear loads

Structural Idealization – Local vs Global Effects



• for pure torsion, the rate of twist (and the resulting stresses) are inversely proportional to J: the greater the value of J the lower the stresses and angle of rotation

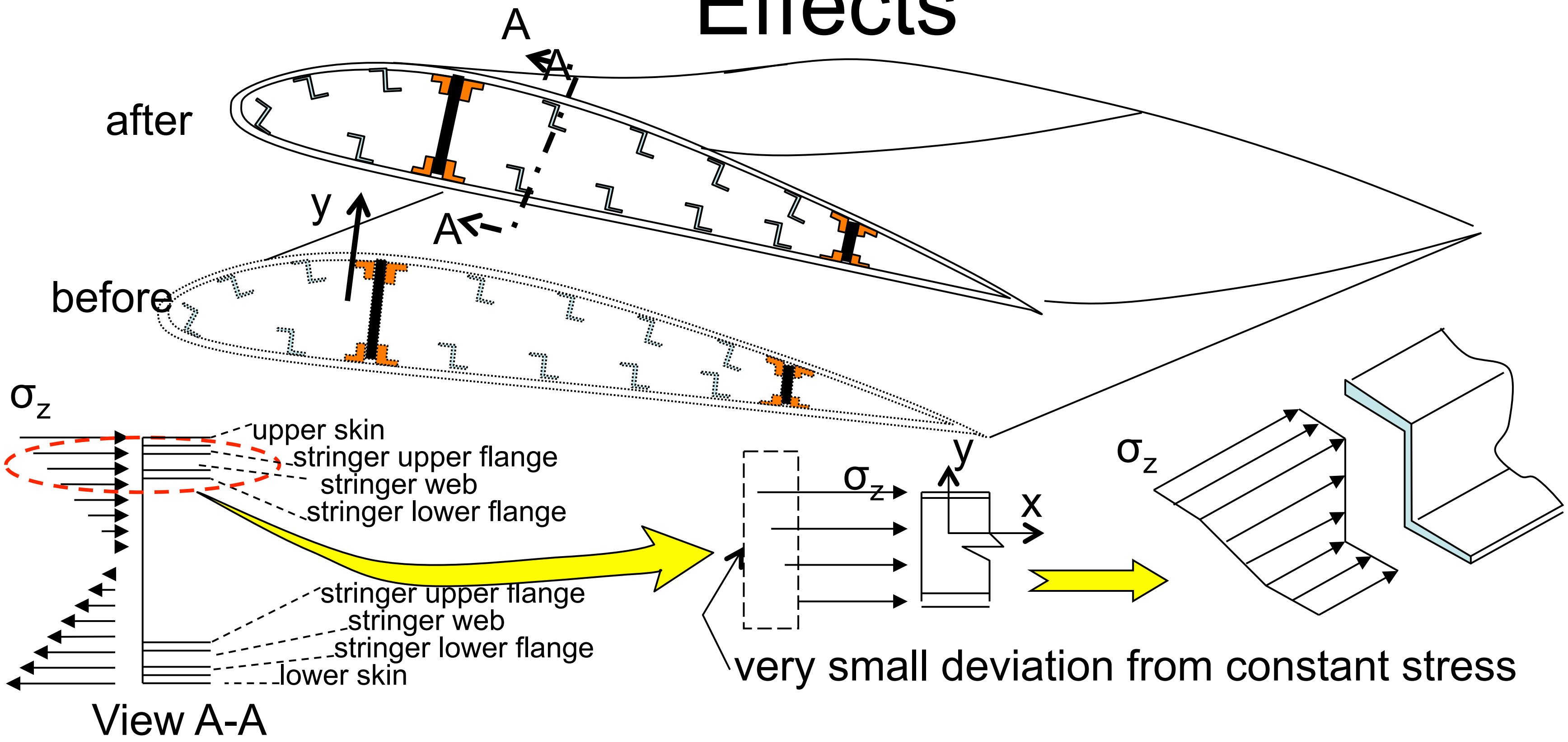
• from the previous lecture:

$$J_c = \frac{4A^2}{\oint \frac{ds}{t}}$$

valid for a closed section; A is the enclosed area of the airfoil. It can be seen that the contribution of the stringers to J is negligible (sum of $h_i t_i^3/3$) if they are open and very small if they are closed (because their enclosed area is very small compared to the enclosed area of the airfoil

• therefore, the shear stresses in the stringers, being inversely proportional to their individual J are negligible compared to the shear stresses in the skin

Structural Idealization – Local vs Global Effects

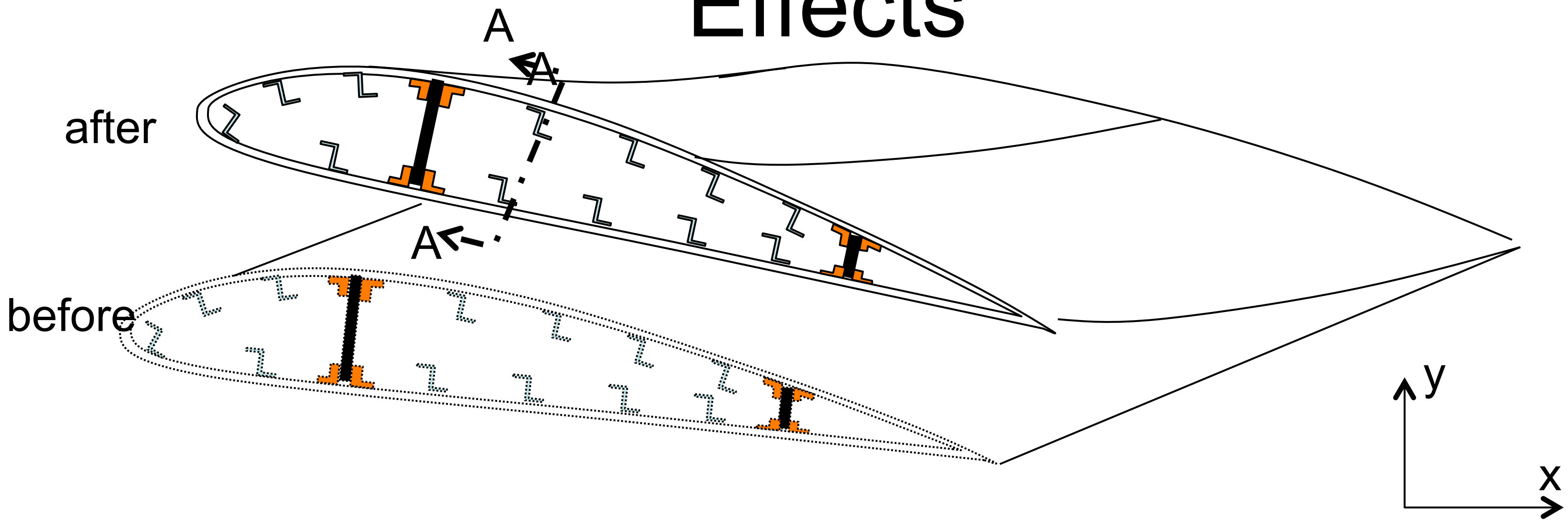


• for bending, whether it is caused by shear or moment, a bending stress distribution develops across the depth of the wing

• locally, over a single stringer, the stress varies very little from the top of the stringer to the bottom of the stringer

• therefore, to a first approx’n, the stringer stress can be assumed constant with

Structural Idealization – Local vs Global Effects

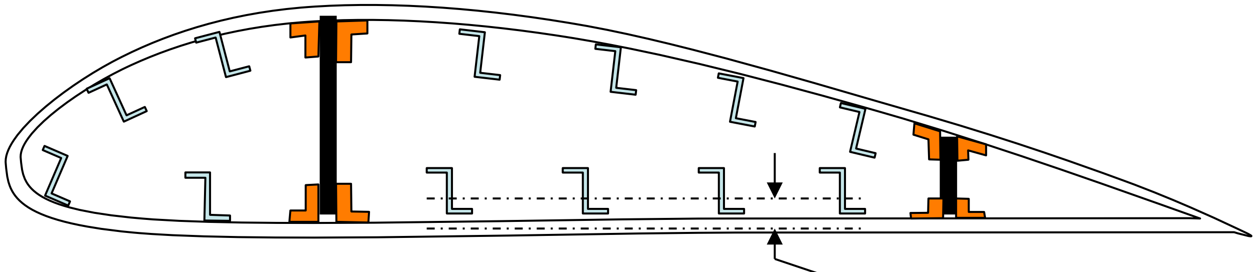


• similarly, the skins themselves have an even smaller variation of direct stress across their thickness and can also be assumed to be under normal stress σ_{zskin} that is constant with y

Structural Idealization – Local vs Global Effects

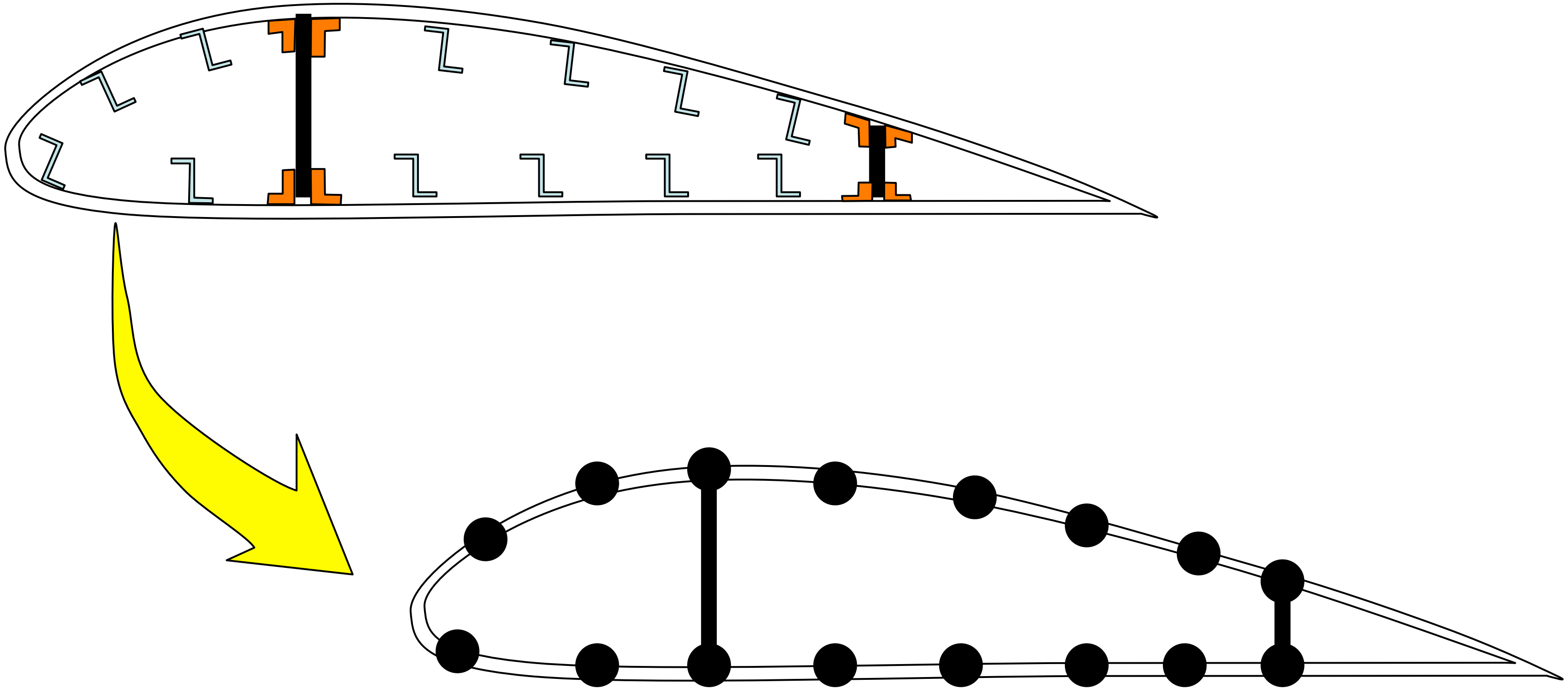
- to summarize:
  - skins carry shear stress from torsion or shear and (some) normal stress from bending; the normal stress is assumed to not vary through the thickness of the skin
  - stringers carry normal stress from bending; the normal stress is constant for each stringer

Structural Idealization – Simplified model



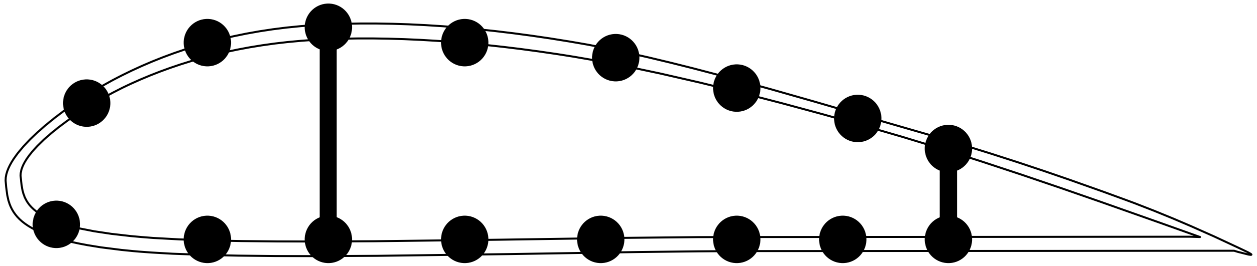
- because the stringer (and flange) dimensions are very small relative to the overall dimensions of the wing cross-section, the neutral axis of the stringers is very close to the neutral axis of the adjacent skin
- we can therefore assume that the two coincide
- then…

Structural Idealization – Simplified model



• stringers and flanges are replaced by lumps of area called “booms” (or “flanges”) which carry only normal stresses

Structural Idealization – Simplified model



- there are now two possibilities:
  - skin is allowed to carry normal stresses
  - skin is not allowed to carry normal stresses

most common in practice

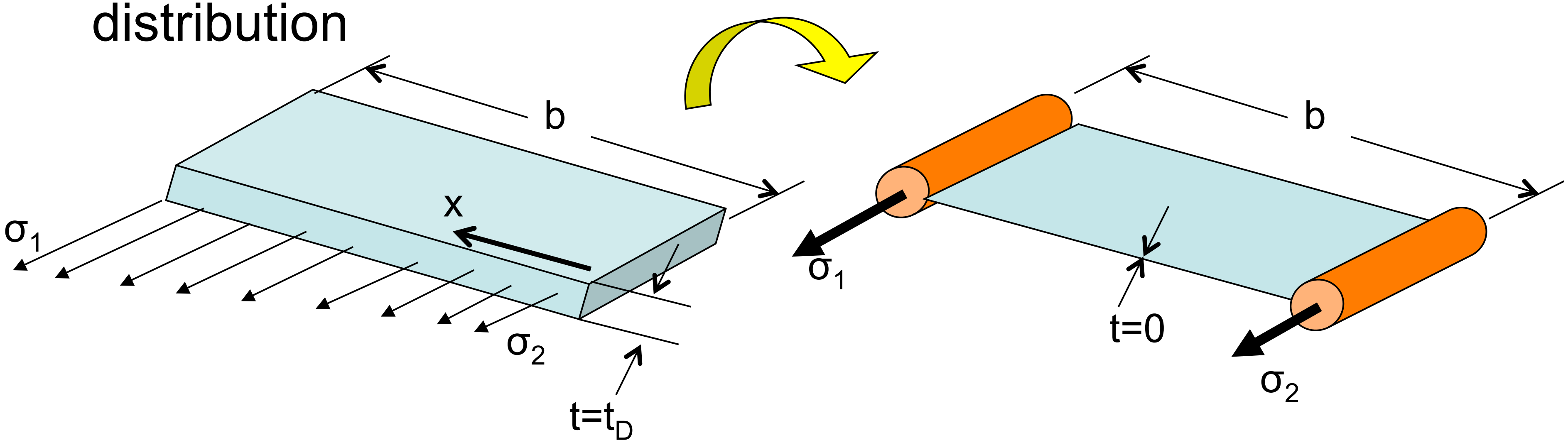
- in the first case, the skin thickness is unchanged and normal stresses are carried by booms and skin according to their respective stiffnesses

(for direct stresses only!)

- in the second case, the skin thickness is zero, the boom areas are adjusted to include the area of the skin and all normal stresses are carried by the booms

Determination of boom area - Skins

• skin of thickness t_D carrying (linear) normal stress distribution

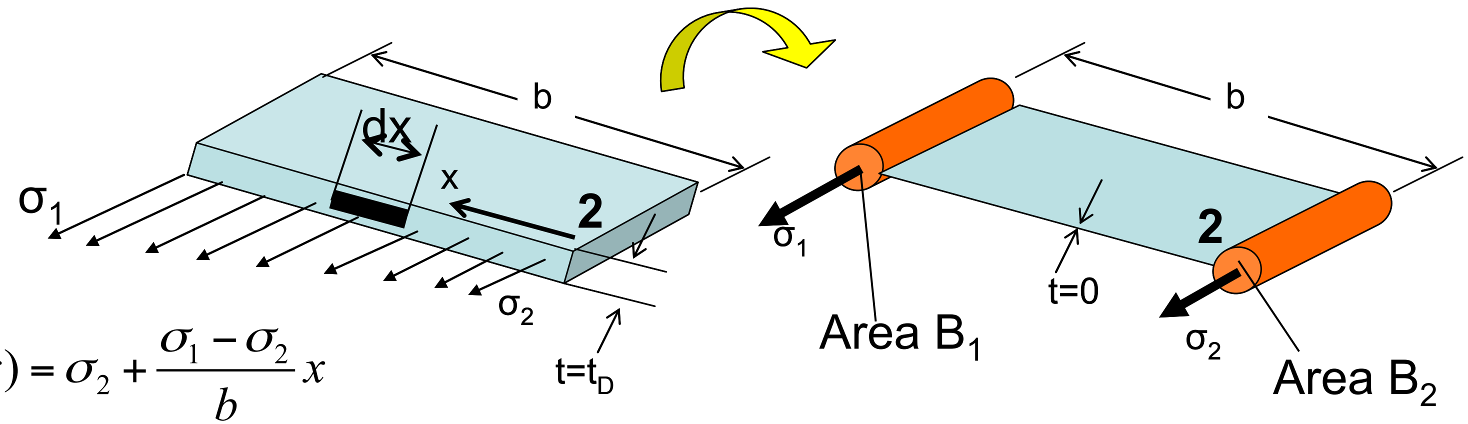


• for design, we want the extreme stresses to be reproduced in our idealization (hence σ_1 and σ_2 on the booms)

• a linear distribution of stress with extreme values σ_1 and σ_2 is given (for x defined as shown) by:

$$\sigma(x) = \sigma_2 + \frac{\sigma_1 - \sigma_2}{b} x \qquad (7.1)$$

Determination of boom area - Skins



$$\sigma(x) = \sigma_2 + \frac{\sigma_1 - \sigma_2}{b} x$$

- determine now the boom areas such that the idealized structure produces the same moment

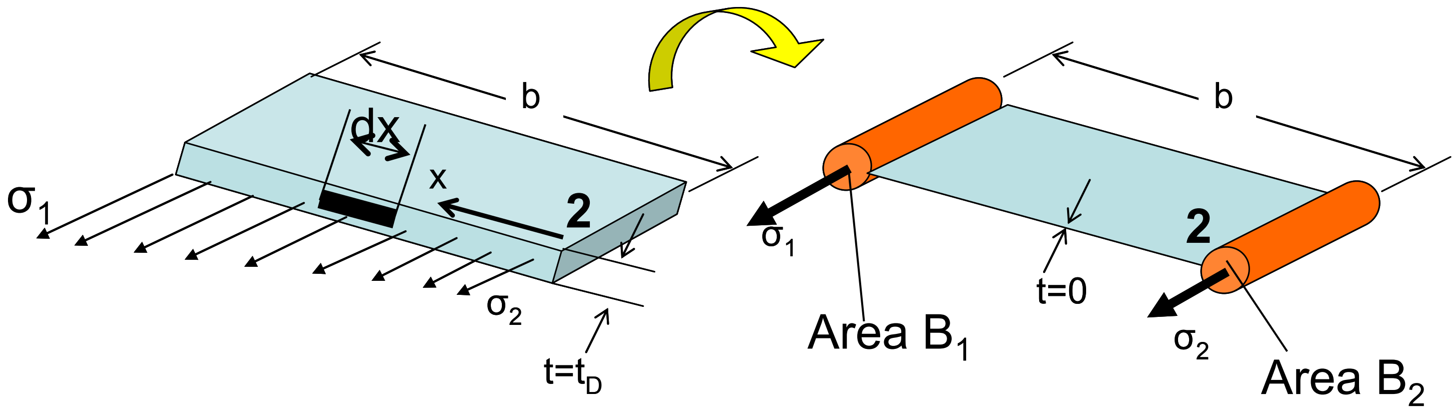
- taking moments about point 2 (in the real structure):

$$M_2 = \int_0^b \sigma(x) t_D x dx = \int_0^b \left(\sigma_2 + \frac{\sigma_1 - \sigma_2}{b} x \right) t_D x dx = \sigma_2 t_D \frac{b^2}{2} + (\sigma_1 - \sigma_2) t_D \frac{b^2}{3} \qquad (7.2)$$

- this should be equal to the moment caused about point 2 in the idealized structure:

$$M_2 = \sigma_1 B_1 b \qquad (7.3)$$

Determination of boom area - Skins



- equating the moments M_2 and solving for the area B_1:

$$\boxed{B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)} \tag{7.4}$$

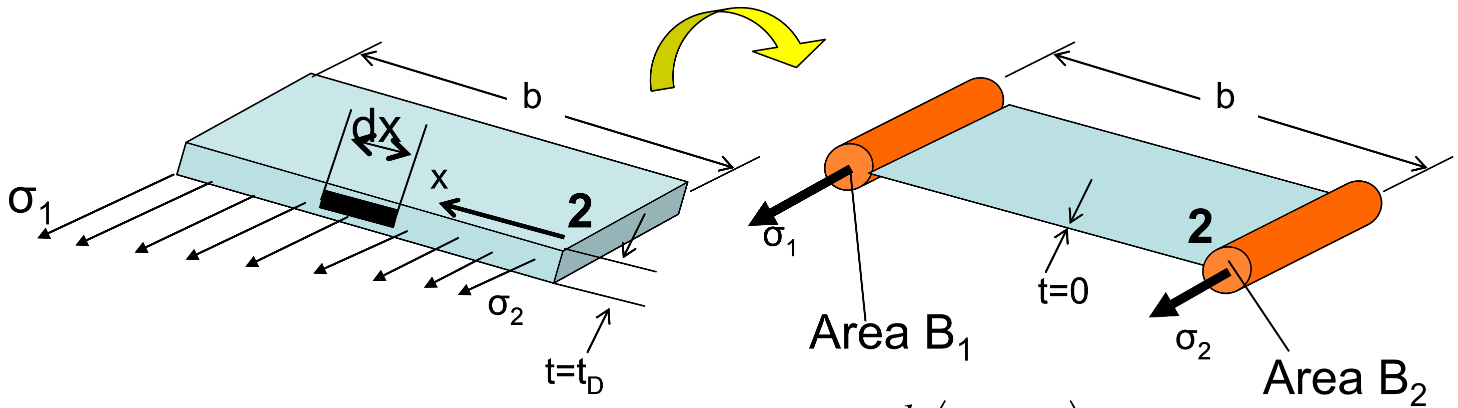
- require now also that the total force created in the two cases is the same:

$$\int_0^b \sigma(x) t_D dx = B_1\sigma_1 + B_2\sigma_2 \Rightarrow \int_0^b \left(\sigma_2 + \frac{\sigma_1 - \sigma_2}{b}x\right) t_D dx = B_1\sigma_1 + B_2\sigma_2 \tag{7.5}$$

- carrying out the integration and substituting for B_1 from (7.4):

$$\boxed{B_2 = \frac{t_D b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)} \tag{7.6}$$

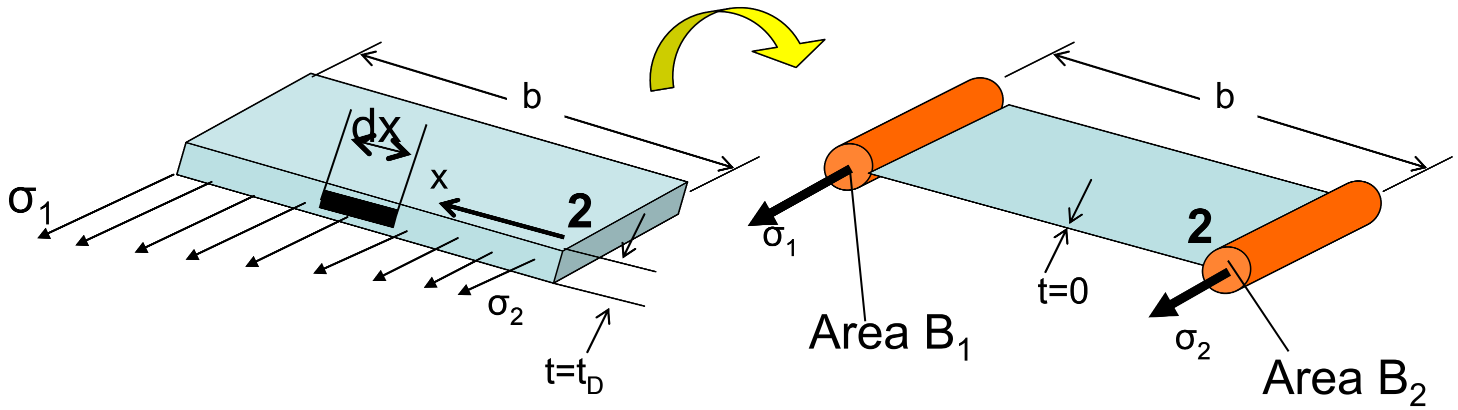
Determination of boom area - Skins



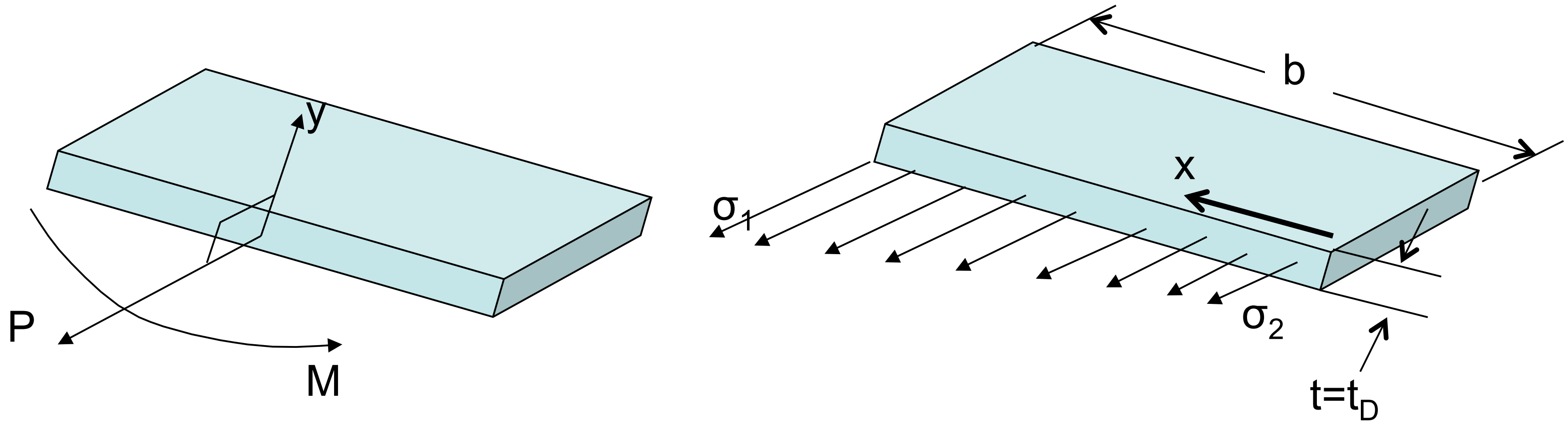
$$B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right) \qquad B_2 = \frac{t_D b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)$$

- interestingly, the model says that B_1, which has higher stress ($\sigma_1 > \sigma_2$) is smaller than B_2
- for the model to work, one needs to know the ratio σ_1 / σ_2
  - if the skin is under pure tension or compression, $\sigma_1 = \sigma_2$
  - if the skin is under pure bending (about an axis perpendicular to x) then $\sigma_1 = -\sigma_2$
  - what if the skin is under both bending and axial loads as above?

Determination of boom area - Skins

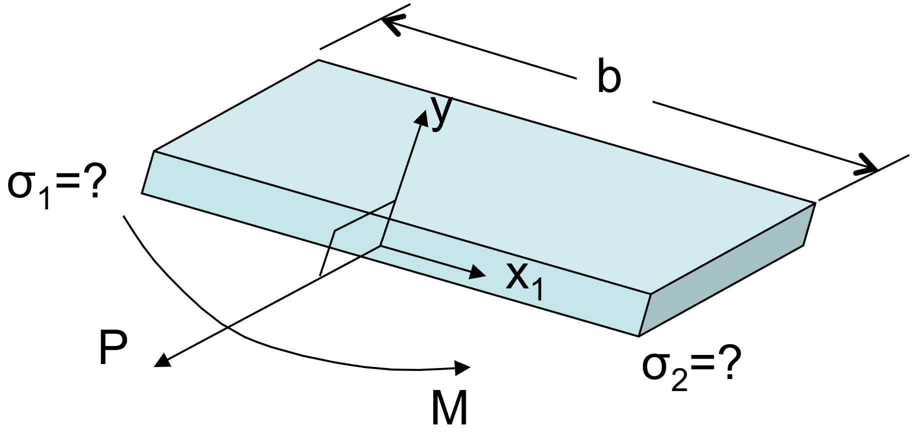


- for the case for which the skin can be modeled as a beam:



P, M known

Determination of boom area - Skins



- the applied stress on the skin is given by

$$\sigma = \frac{P}{A} + \frac{Mx_1}{I_{yy}} = \frac{P}{t_D b} + \frac{Mx_1}{\frac{t_D b^3}{12}} \qquad (7.7)$$

- then $\sigma_1=\sigma(x_1=-b/2)$ and $\sigma_2=\sigma(x_1=+b/2)$; therefore:

$$\sigma_1 = \frac{P}{t_D b} - \frac{Mb}{2\frac{t_D b^3}{12}} = \frac{P}{t_D b} - \frac{6M}{t_D b^2} \qquad (7.8)$$

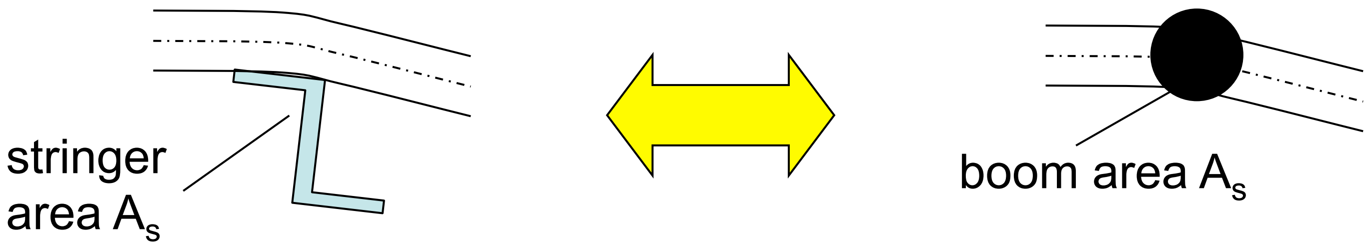
$$\sigma_2 = \frac{P}{t_D b} + \frac{Mb}{2\frac{t_D b^3}{12}} = \frac{P}{t_D b} + \frac{6M}{t_D b^2} \qquad (7.9)$$

σ_1 and σ_2 change as P and M change; therefore, B_1 and B_2 change as loading changes; so the solution for B_1 and B_2 depends on loading!

Determination of boom area – Stringers and flanges

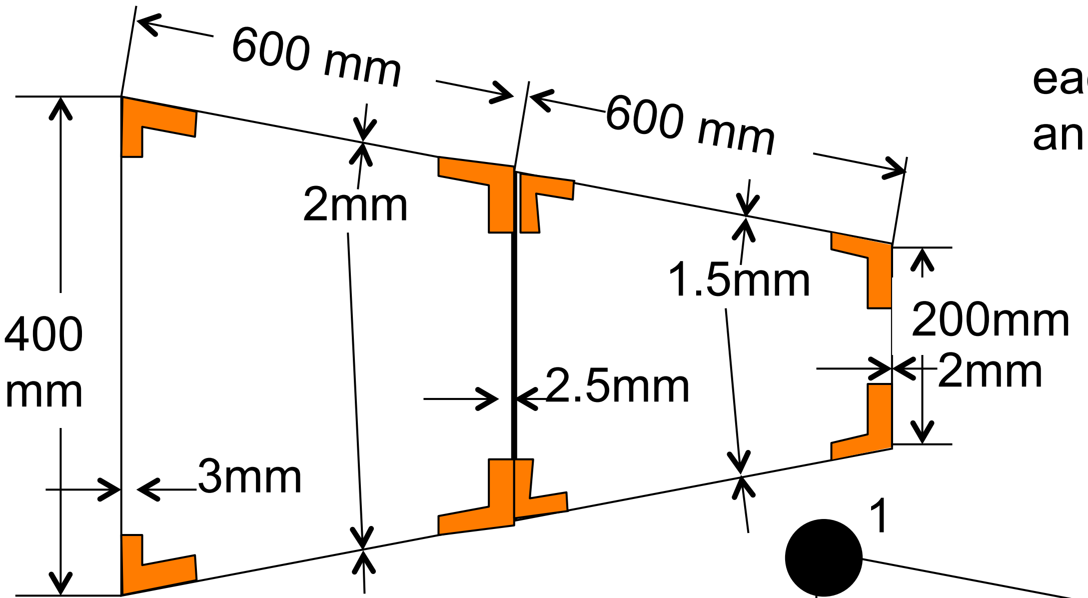
• as already mentioned (a) the difference between the neutral axis of the stringers and that of the skin is neglected (b) any variation of stress along the stringer cross-section is neglected (normal stress on a stringer is constant)

• then, each stringer (or flange) can be represented by a boom of equal area at the mid-skin at that location

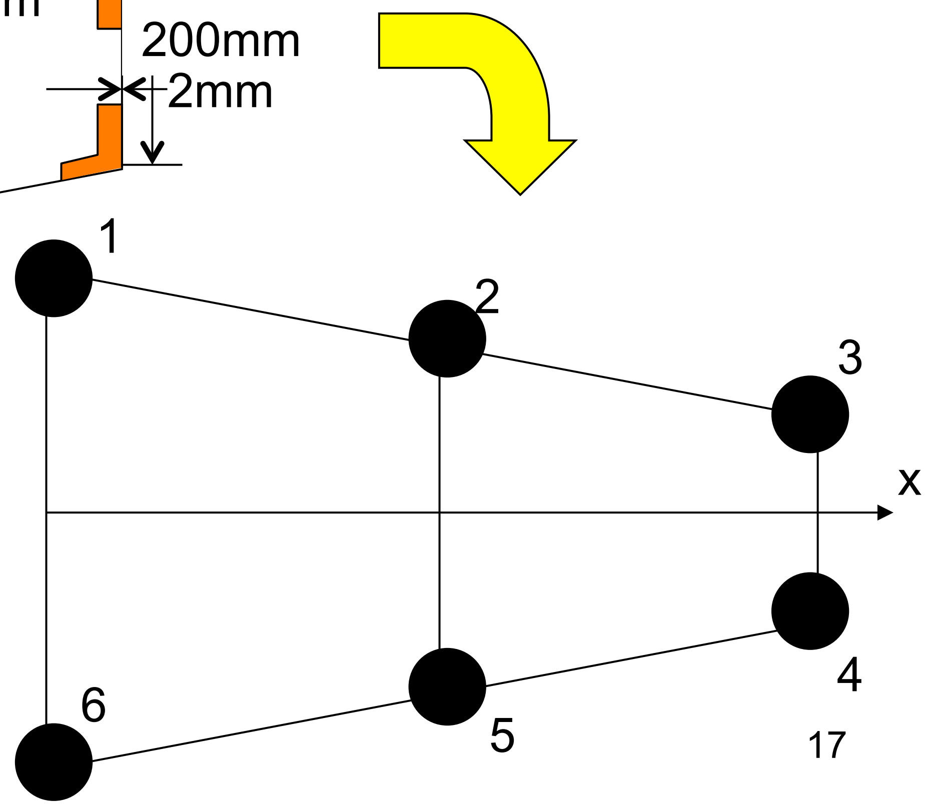


•exercise caution if you intend to move the stringer to another boom location (preserve symmetry, equiv loads..)

Example: boom areas for wingbox in pure bending

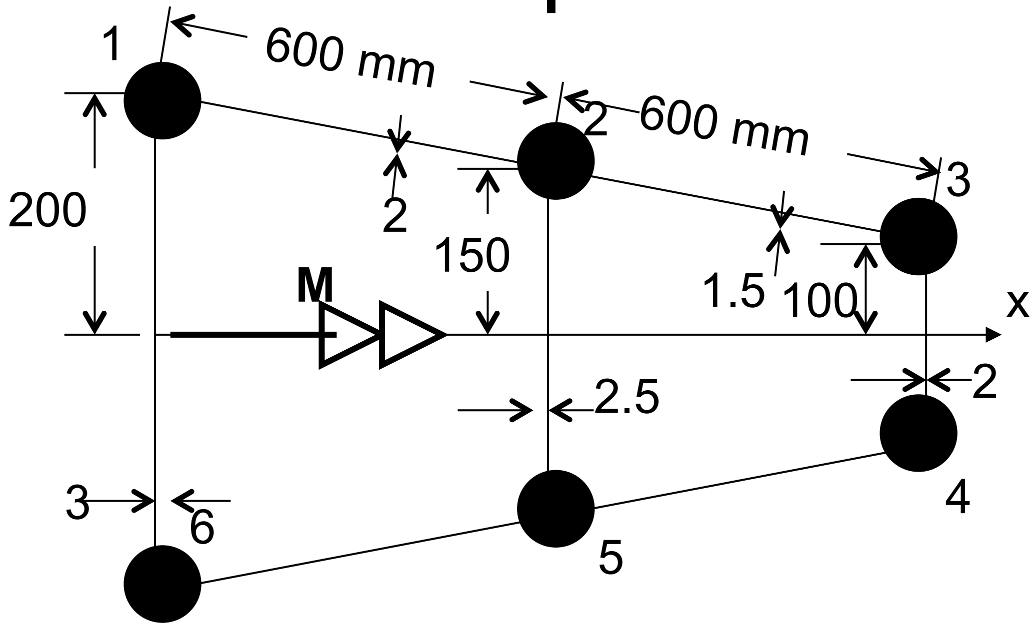


each (orange) flange has an area of 300 mm^2



Determine the areas of the 6 booms when the applied load is a moment about the x axis

Example: boom areas for wingbox in pure bending



- Consider first, the contribution of the skins to the boom areas
- Since this is pure bending under moment M, the normal stress anywhere around the wingbox is given by

$$\sigma = \frac{My}{I_{xx}}$$

- therefore,

$$\sigma_1 = \frac{M(200)}{I_{xx}}$$

$$\sigma_2 = \frac{M(150)}{I_{xx}}$$

$$\sigma_3 = \frac{M(100)}{I_{xx}}$$

$$\sigma_6 = -\frac{M(200)}{I_{xx}}$$

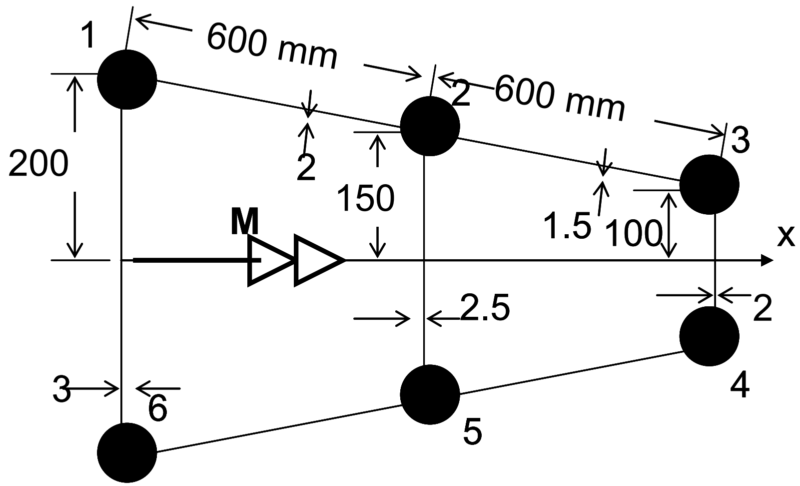
from which:

$$\frac{\sigma_1}{\sigma_2} = \frac{200}{150}$$

$$\frac{\sigma_2}{\sigma_3} = \frac{150}{100}$$

$$\frac{\sigma_1}{\sigma_6} = \frac{\sigma_2}{\sigma_5} = \frac{\sigma_3}{\sigma_4} = -1$$

Example: boom areas for wingbox in pure bending



$$\frac{\sigma_1}{\sigma_2} = \frac{200}{150}$$

$$\frac{\sigma_2}{\sigma_3} = \frac{150}{100}$$

$$\frac{\sigma_1}{\sigma_6} = \frac{\sigma_2}{\sigma_5} = \frac{\sigma_3}{\sigma_4} = -1$$

$$B_1 = \frac{t_D b}{6}\left(2 + \frac{\sigma_2}{\sigma_1}\right)$$

$$B_2 = \frac{t_D b}{6}\left(2 + \frac{\sigma_1}{\sigma_2}\right)$$

• go to each boom now and apply eq (7.4) or (7.6) and use the subscript "s" to denote the skin contribution

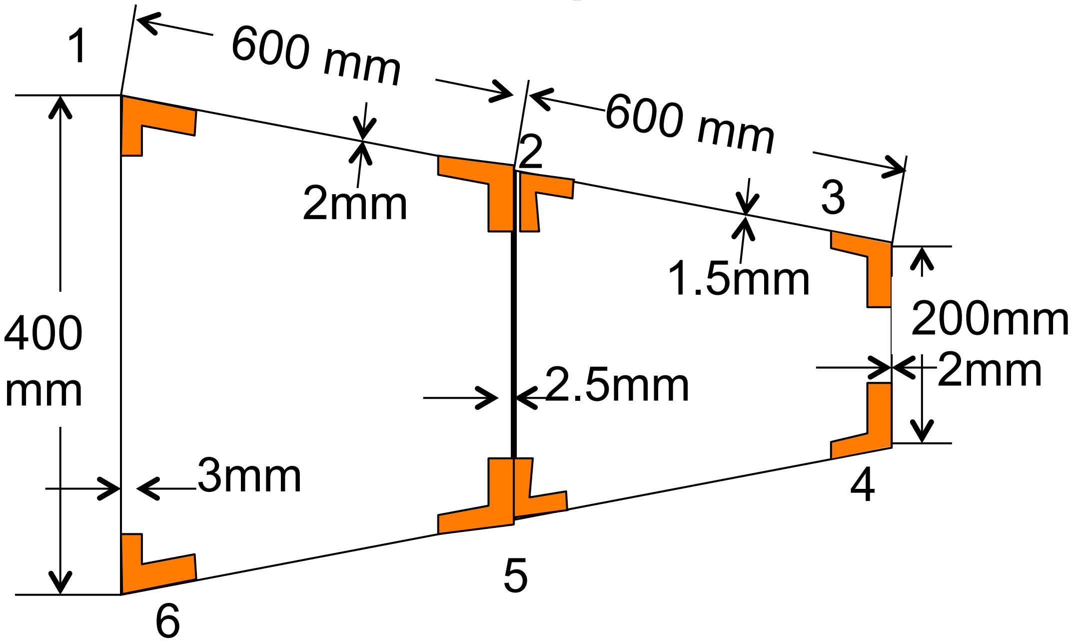
$$B_{1s} = \frac{2(600)}{6}\left(2 + \frac{150}{200}\right) + \frac{3(400)}{6}(2-1) = 750$$

$$B_{2s} = \frac{1.5(600)}{6}\left(2 + \frac{100}{150}\right) + \frac{2(600)}{6}\left(2 + \frac{200}{150}\right) + \frac{2.5(300)}{6}(2-1) = 1191.7$$

(units are mm^2)

$$B_{3s} = \frac{1.5(600)}{6}\left(2 + \frac{150}{100}\right) + \frac{2(200)}{6}(2-1) = 591.7$$

Example: boom areas for wingbox in pure bending



- now consider the contribution of the flanges to the boom areas
- each flange has area 300 mm^2

B_{1f}=300

B_{2f}=300+300=600

B_{3f}=300

- combining:

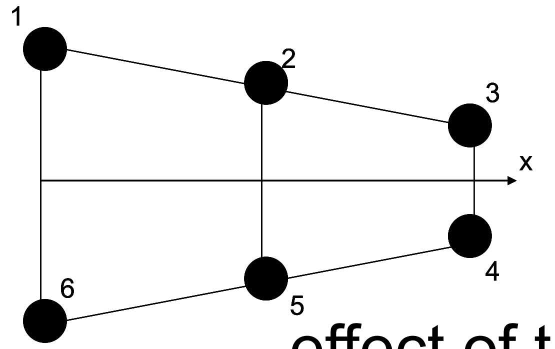
$$B_1 = B_{1s} + B_{1f} = 750 + 300 = 1050 mm^2$$

$$B_2 = B_{2s} + B_{2f} = 1191.7 + 600 = 1791.7 mm^2$$

$$B_3 = B_{3s} + B_{3f} = 591.7 + 300 = 891.7 mm^2$$

- B_4, B_5, and B_6 are, by symmetry, the same as B_3, B_2 and B_1 respectively

Effect of booms in stress calculations



• suppose that, for a given loading, the boom areas have been calculated

• to proceed with the analysis, the effect of the booms on different quantities such as moments of inertia must be determined

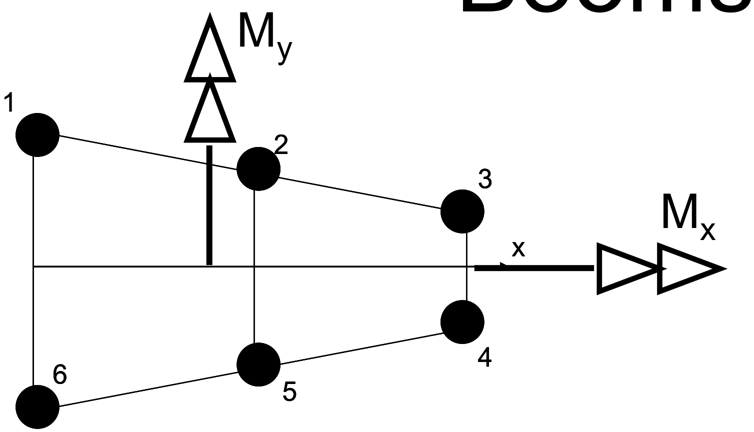
• for example, determining the neutral axis location, requires that eq (2.0) is satisfied

$$\int_A \sigma_z dA = 0$$

(pure bending, no net axial force)

• note that the area A is the area that carries normal stresses and, thus, the calculated neutral axis location is for the boom area (i.e. do not include in the calculation of the neutral axis, I_{xx}, etc., the skin if it is already included in the booms)

Booms in bending



• we are interested in the normal stresses

• the bending equation (2.5) from before,

$$\sigma_z = \frac{I_{xx}M_y - I_{xy}M_x}{I_{xx}I_{yy} - I_{xy}^2}x + \frac{I_{yy}M_x - I_{xy}M_y}{I_{xx}I_{yy} - I_{xy}^2}y \qquad (2.5)$$

• but all quantities refer exclusively to the portion of the cross-section that carries normal stresses

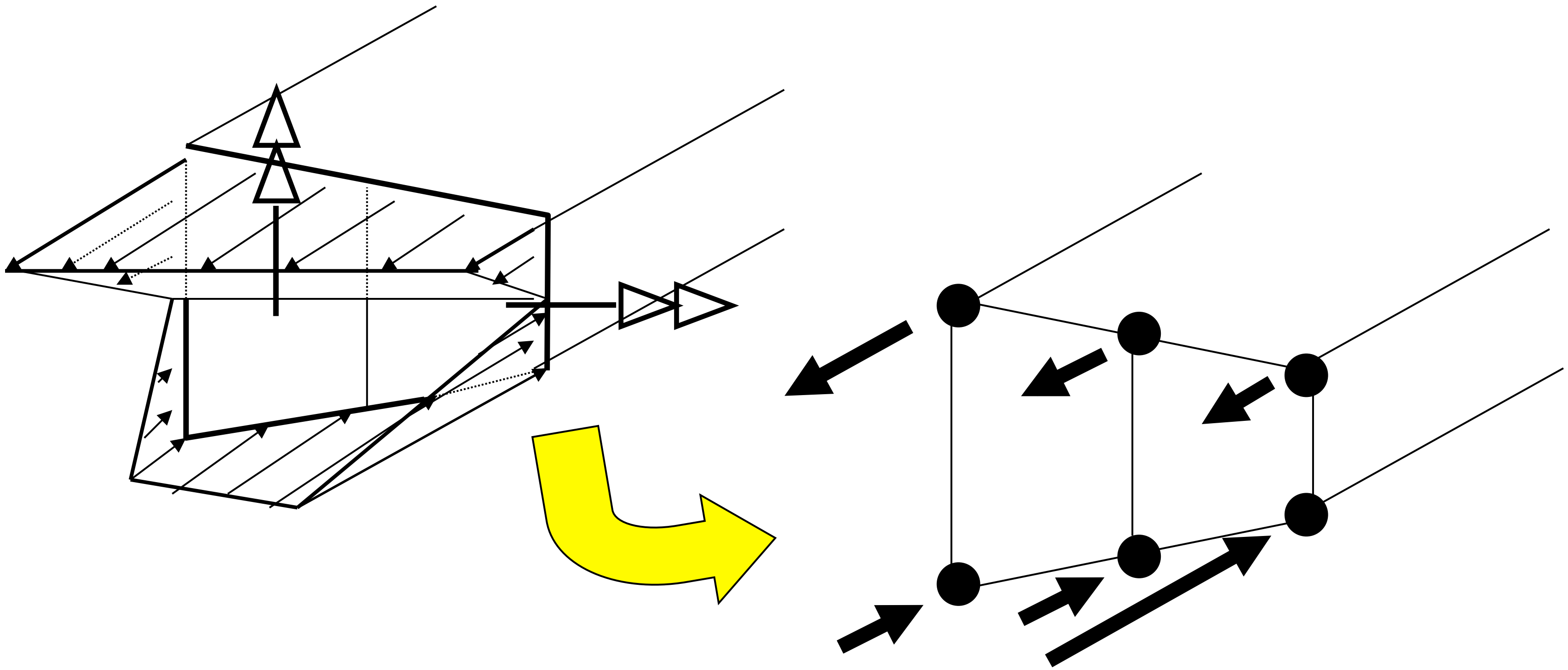
• this means that the **neutral axis** is calculated using **only the areas that carry normal stresses**

• the **moments of inertia** use **only areas that carry normal stresses** and

• the x,y coordinates where stresses are evaluated refer to a coordinate system at the centroid of the areas that carry normal stresses

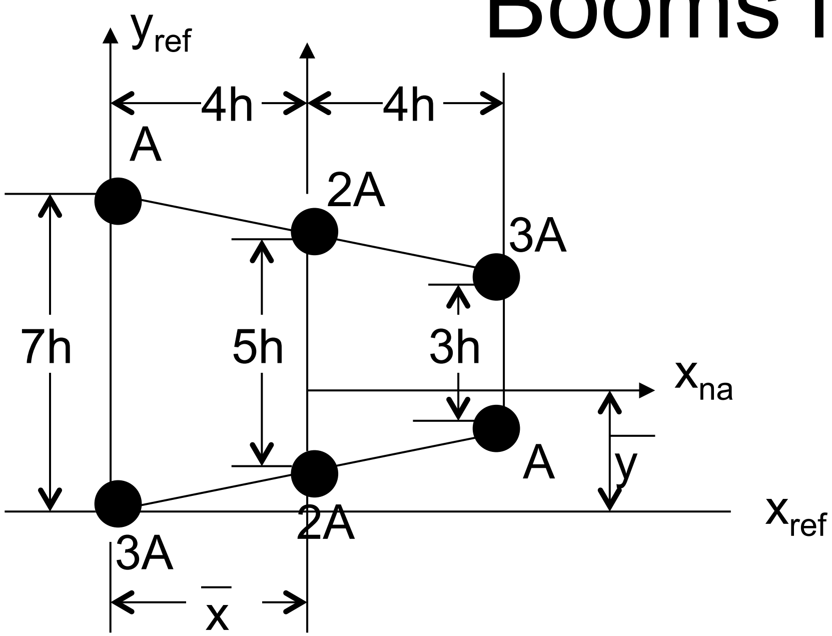
Booms in bending

• in the extreme case where all the skin has been divided into boom areas, the normal stresses are only the stresses in the booms and there are no normal stresses in-between



distributed stresses become simple point stresses on the booms

Booms in bending



$$\bar{y} = \frac{A7h + 2A6h + 3A5h + A2h + 2Ah}{12A} = \frac{19}{6}h$$

$$\bar{x} = \frac{2A4h + 2A4h + 3A8h + A8h}{12A} = 4h$$

moments of inertia include only Steiner terms:

$$I_{xx} = \sum y_i^2 A_i$$

$$I_{yy} = \sum x_i^2 A_i$$

$$I_{xy} = \sum x_i y_i A_i$$

- therefore:

$$I_{xx} = A\left(7 - \frac{19}{6}\right)^2 h^2 + 2A\left(6 - \frac{19}{6}\right)^2 h^2 + 3A\left(5 - \frac{19}{6}\right)^2 h^2 + A\left(\frac{19}{6} - 2\right)^2 h^2 + \ldots$$

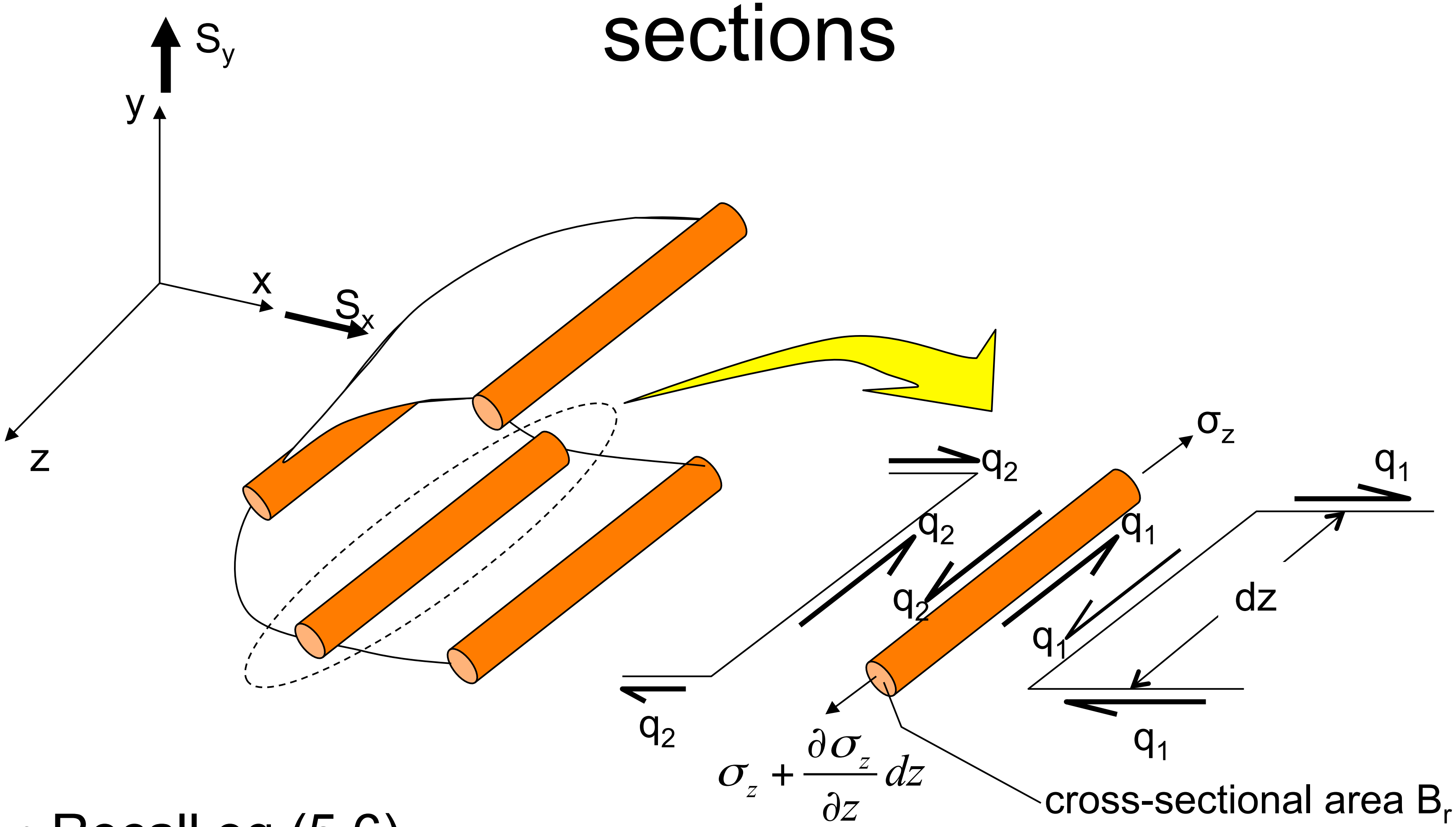
$$I_{yy} = A(4h)^2 + 3A(4h)^2 + A(4h)^2 + 3A(4h)^2 = 128Ah^2$$

$$I_{xy} = -A\left(7 - \frac{19}{6}\right)4h^2 + 3A\left(5 - \frac{19}{6}\right)4h^2 - A\left(\frac{19}{6} - 2\right)4h^2 + 3A\frac{19}{6}4h^2 = 40Ah^2$$

then proceed with eq (2.5)

- note that even if the original shape is symmetric, if the resulting boom configuration is not symmetric, $I_{xy} \neq 0$

Booms under shear loads – open sections

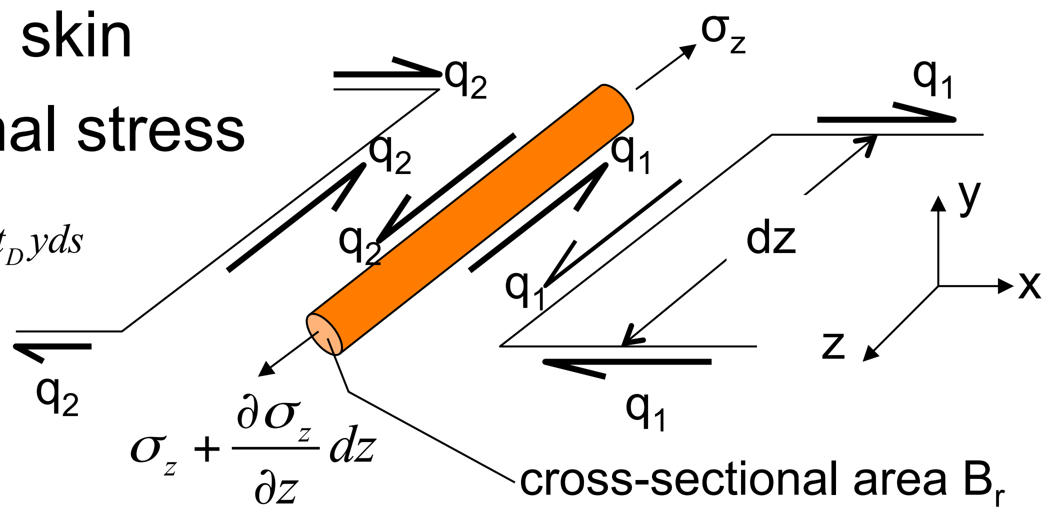


- Recall eq (5.6)

$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s txds - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s tyds \qquad (5.6)$$

Booms under shear loads – open sections

- this eq refers only to the skin portion that carries normal stress



$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s t_D x ds - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s t_D y ds$$

hence the use of t_D which equals the skin thickness t if skin is fully effective, and zero if skin carries shear only

- this equation does NOT account for the effect of booms
- consider z-dir equilibrium of the boom:

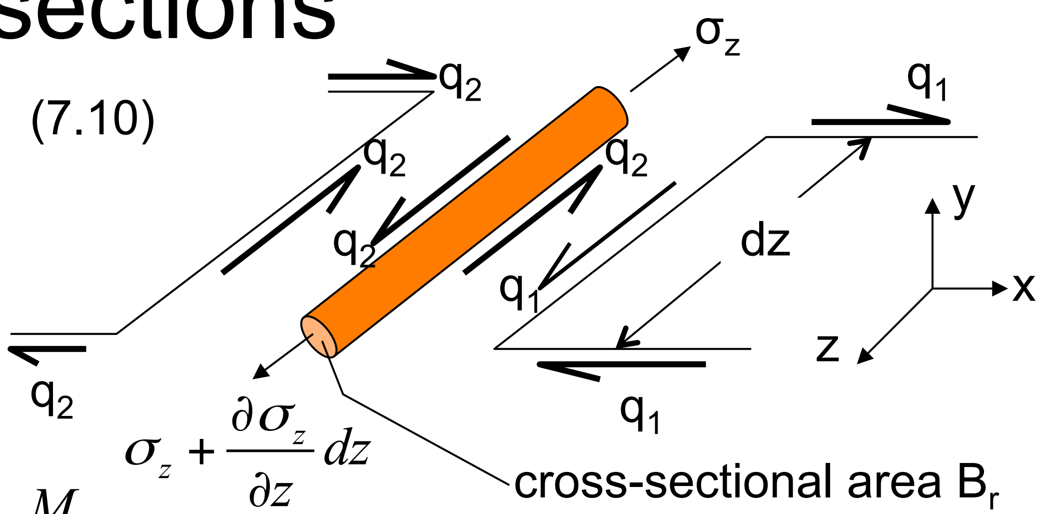
$$\left(\sigma_z + \frac{\partial \sigma_z}{\partial z} dz\right)B_r + q_2 dz - q_1 dz - \sigma_z B_r = 0$$

- which leads to

$$q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z} B_r \qquad (7.10)$$

Booms under shear loads – open sections

$$q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z} B_r \qquad (7.10)$$



• from bending theory, eq (2.5) gives:

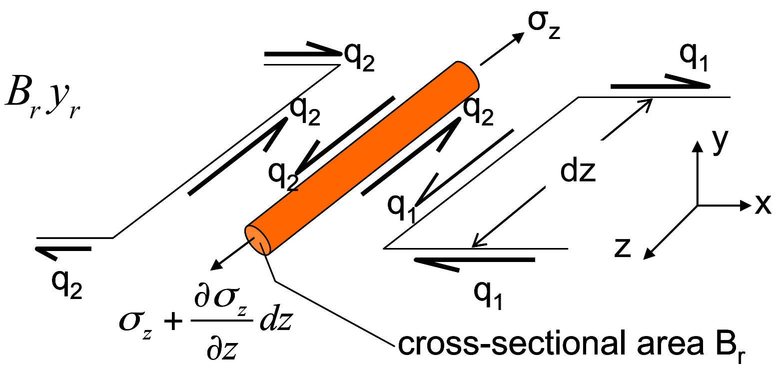
$$\sigma_z = \frac{I_{xx} M_y - I_{xy} M_x}{I_{xx} I_{yy} - I_{xy}^2} x + \frac{I_{yy} M_x - I_{xy} M_y}{I_{xx} I_{yy} - I_{xy}^2} y$$

• substituting in (7.10) and noting that x, y are the coordinates x_r, y_r of the rth boom:

$$q_2 - q_1 = -\frac{I_{xx}\frac{\partial M_y}{\partial z} - I_{xy}\frac{\partial M_x}{\partial z}}{I_{xx} I_{yy} - I_{xy}^2} B_r x_r - \frac{I_{yy}\frac{\partial M_x}{\partial z} - I_{xy}\frac{\partial M_y}{\partial z}}{I_{xx} I_{yy} - I_{xy}^2} B_r y_r$$

Booms under shear loads – open sections

$$q_2 - q_1 = -\frac{I_{xx}\dfrac{\partial M_y}{\partial z} - I_{xy}\dfrac{\partial M_x}{\partial z}}{I_{xx}I_{yy} - I_{xy}^{\;2}} B_r x_r - \frac{I_{yy}\dfrac{\partial M_x}{\partial z} - I_{xy}\dfrac{\partial M_y}{\partial z}}{I_{xx}I_{yy} - I_{xy}^{\;2}} B_r y_r$$



- but from (2.10)

$$S_y = \frac{\partial M_x}{\partial z} \qquad S_x = \frac{\partial M_y}{\partial z} \qquad (2.10)$$

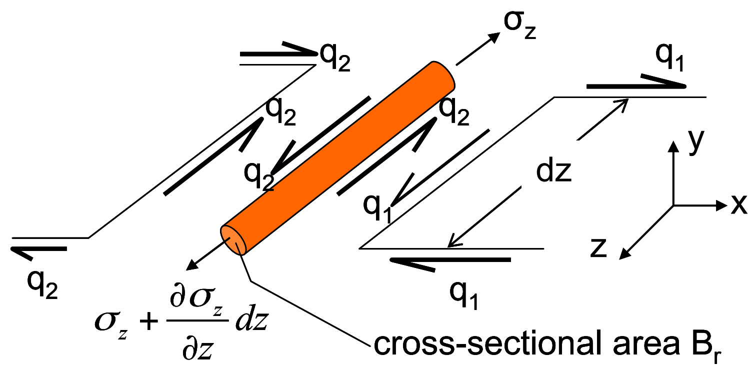
- substituting,

$$q_2 - q_1 = -\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx}I_{yy} - I_{xy}^{\;2}} B_r x_r - \frac{S_y I_{yy} - S_x I_{xy}}{I_{xx}I_{yy} - I_{xy}^{\;2}} B_r y_r \qquad (7.11)$$

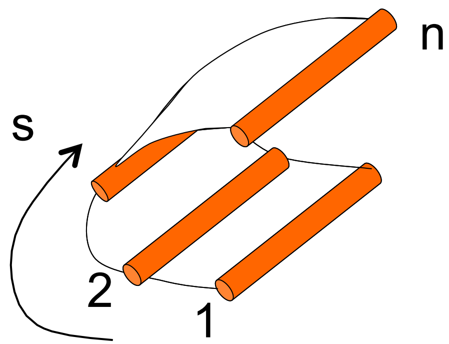
this equation gives the change in shear flow across a boom which carries an axial load $\sigma_z B_r$

Booms under shear loads – open sections

$$q_2 - q_1 = -\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} B_r x_r - \frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} B_r y_r$$



- suppose now we have n booms



- the shear flow after the nth boom will be given by (a) the standard shear flow equation when there are no booms PLUS (b) the contribution of all the booms up to that point:

$$\boxed{q_s = -\frac{I_{xx} S_x - I_{xy} S_y}{I_{xx} I_{yy} - I_{xy}^2}\left[\int_0^s t_D x\,ds + \sum_{r=1}^{n} B_r x_r\right] - \frac{I_{yy} S_y - I_{xy} S_x}{I_{xx} I_{yy} - I_{xy}^2}\left[\int_0^s t_D y\,ds + \sum_{r=1}^{n} B_r y_r\right]} \qquad (7.12)$$

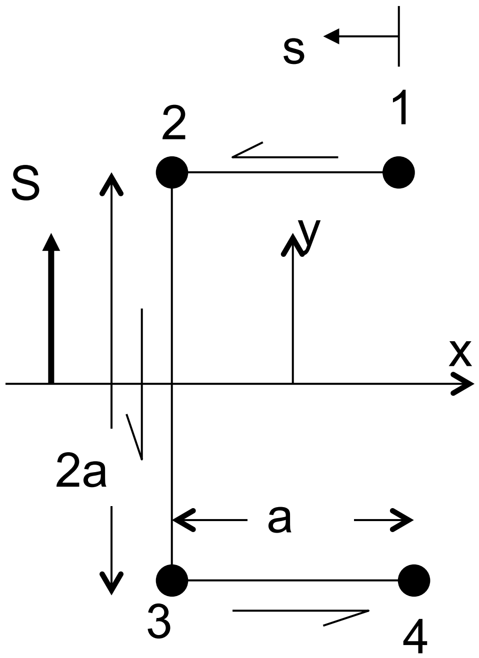
Booms under shear loads – open sections

$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^{\;2}}\left[\int_0^s t_D x\,ds + \sum_{r=1}^{n} B_r x_r\right] - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^{\;2}}\left[\int_0^s t_D y\,ds + \sum_{r=1}^{n} B_r y_r\right] \qquad (7.12)$$

- simplification:
  - suppose the skin carries only shear stresses (=> t_D=0)
  - suppose also that the booms (and not necessarily the skin) have at least one axis of symmetry (=>I_{xy}=0)
  - then:

$$q_s = -\frac{S_x}{I_{yy}}\sum_{r=1}^{n} B_r x_r - \frac{S_y}{I_{xx}}\sum_{r=1}^{n} B_r y_r \qquad (7.13)$$

Booms under shear loads – open sections Example



Area of each boom=A

Thickness = t everywhere

Skin carries only shear

Determine the shear flows

• since skin carries only shear and the boom pattern has at least one axis of symmetry, eq. (7.13) is valid

• then, for only S_y applied, (7.13) becomes:

$$q_s = -\frac{S_y}{I_{xx}} \sum_{r=1}^{n} B_r y_r \quad (7.13a)$$

• now B_r=A and y_1=y_2=a, y_3=y_4=-a

• also, $I_{xx} = 4Aa^2$

$$q_{12} = -\frac{S}{4Aa^2} Aa = -\frac{S}{4a}$$

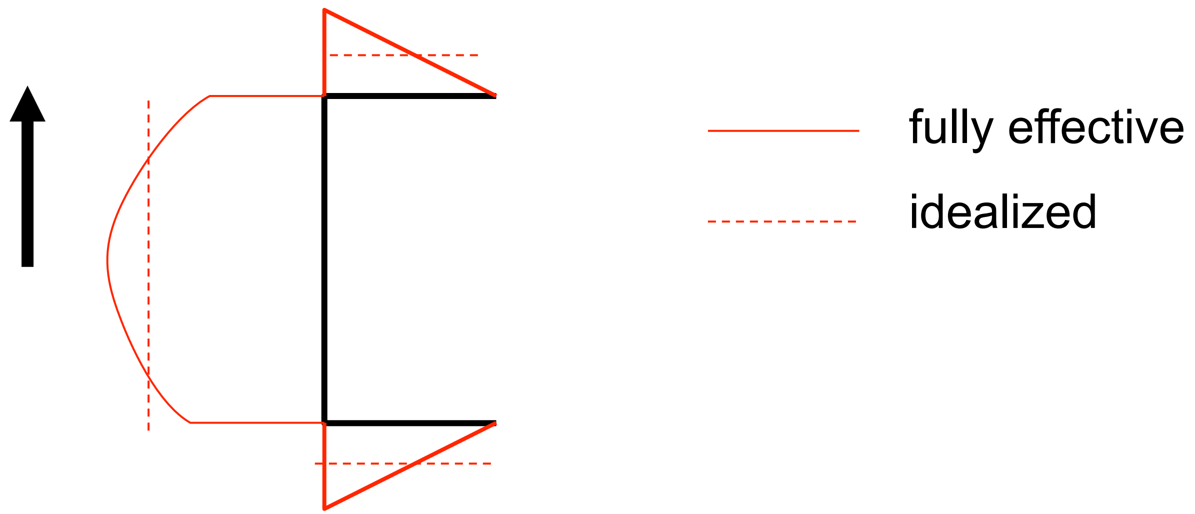
$$q_{23} = -\frac{S}{4a} - \frac{S}{4a} = -\frac{S}{2a}$$

$$q_{34} = -\frac{S}{2a} - \frac{S}{4Aa^2} A(-a) = -\frac{S}{4a}$$

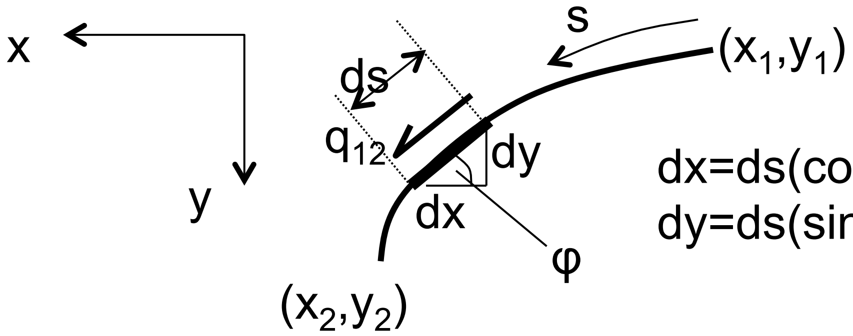
Booms under shear loads – open sections – some useful conclusions

• when we idealize the skins to carry only shear loads, the shear flows between booms are constant as in the previous example

• these constant shear flows are the **average** shear flows that we would get if we had fully effective skins (carrying bending loads); in the previous example, q_{12} and q_{34} would be linear in s while q_{23} would be quadratic



Booms under shear loads – open sections – some useful conclusions



dx=ds(cosφ)
dy=ds(sinφ)

q_{12} = const = q

horiz force component=qds(cosφ)

vert force component=qds(sinφ)

- if we assume an idealized structure, calculation of the total force in any direction is very simple

- total force in x direction is

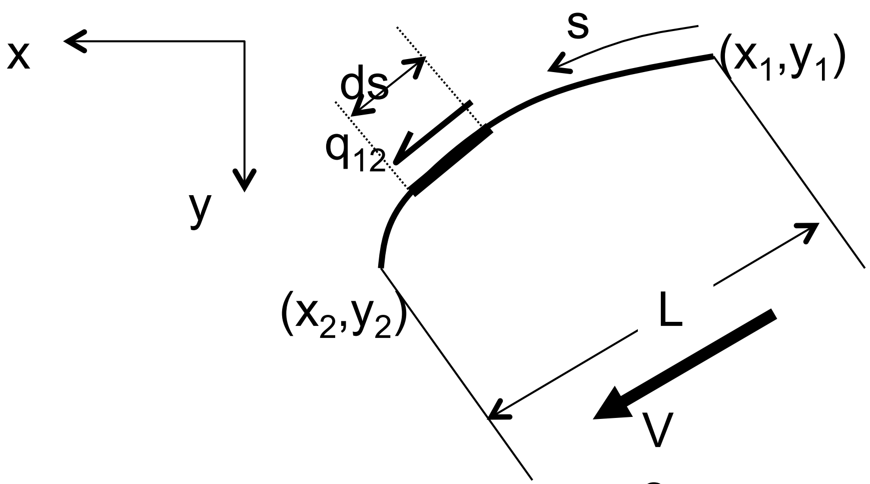
$$S_x = \int_1^2 q\,ds(\cos\phi) = q\int_1^2 dx = q(x_2 - x_1)$$

- total force in y direction is

$$S_y = \int_1^2 q\,ds(\sin\phi) = q\int_1^2 dy = q(y_2 - y_1)$$

total force in any direction between two points equals the shear flow times the distance between the points **parallel to that direction**

Booms under shear loads – open sections – some useful conclusions

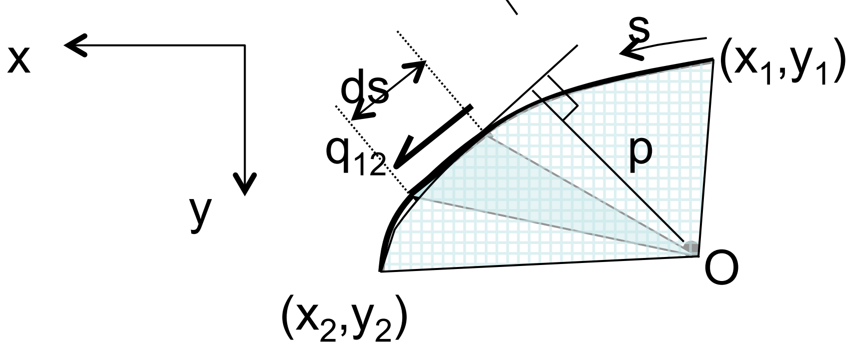


- the resultant force on this skin is given by

$$V=\sqrt{S_x^{\ 2}+S_y^{\ 2}}=q\underbrace{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}_{L}$$

$$V=qL \qquad (7.14)$$

acting parallel to a line connecting the end points!!



- the resultant moment about any point O is given by

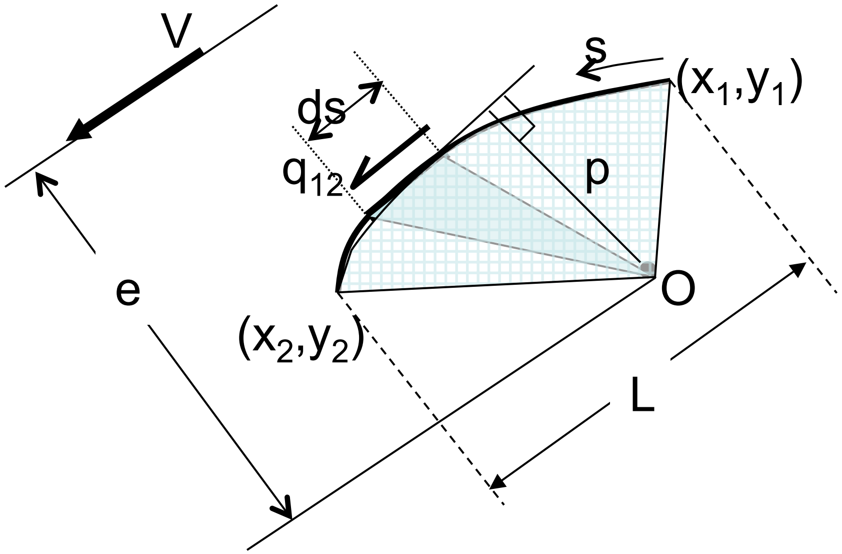
$$M=\int_1^2 qpds=q\int_1^2 pds$$

- but pds is twice the area of the shaded triangle

- then, going from 1 to 2, the integral is the area A enclosed by the skin and two lines connecting O to the skin ends:

$$\boxed{M=2Aq} \text{ !!!!} \qquad (7.15)$$

Booms under shear loads – open sections – some useful conclusions



$$V = qL \qquad (7.14)$$

$$M = 2Aq \qquad (7.15)$$

note that this is identical to eq. (3.44) from torsion theory:

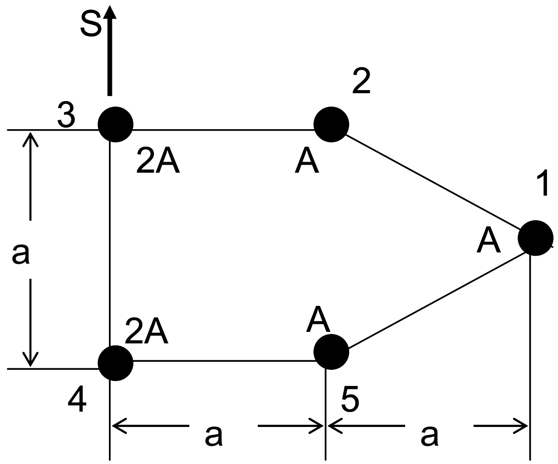
$$T = 2Aq \qquad (3.44)$$

- If V is the applied shear force causing shear flow $q(=q_{12})$, then, the distance e of the line of action of V from any point O can be determined

$$Ve = M = 2Aq$$

- and using (7.14) to substitute for V $\quad e = \dfrac{2A}{L} \qquad (7.16)$

Booms under shear loads – closed sections



In a manner analogous to the open sections, the shear flows can be obtained by combining the closed section without booms, eq. (5.8) with the equation giving the effect of a boom, eq (7.11)

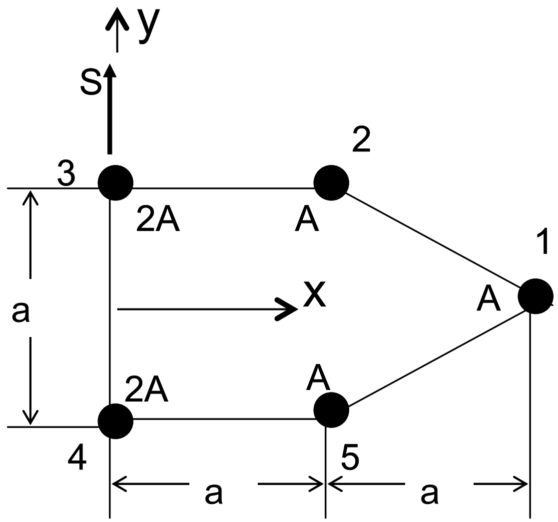
$$q_s - q_{s0} = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s txds - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\int_0^s tyds \qquad (5.8)$$

$$q_2 - q_1 = -\frac{S_xI_{xx} - S_yI_{xy}}{I_{xx}I_{yy} - I_{xy}^2}B_rx_r - \frac{S_yI_{yy} - S_xI_{xy}}{I_{xx}I_{yy} - I_{xy}^2}B_ry_r \qquad (7.11)$$

$$\boxed{q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_Dxds + \sum_{r=1}^{n} B_rx_r\right] - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_Dyds + \sum_{r=1}^{n} B_ry_r\right] + q_{so}} \qquad (7.17)$$

note similarity with (7.12)

Booms under shear loads – closed sections- Example



$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D x ds + \sum_{r=1}^{n} B_r x_r\right] - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D y ds + \sum_{r=1}^{n} B_r y_r\right] + q_{so} \qquad (7.17)$$

there is one axis of symmetry => $I_{xy}=0$

$S_x=0$, $S_y=S$

skin only carries shear => $t_D=0$

- eq. (7.17) then simplifies to

$$q_s = -\frac{S_y}{I_{xx}}\sum_{r=1}^{n} B_r y_r + q_{so} \qquad (7.17a)$$

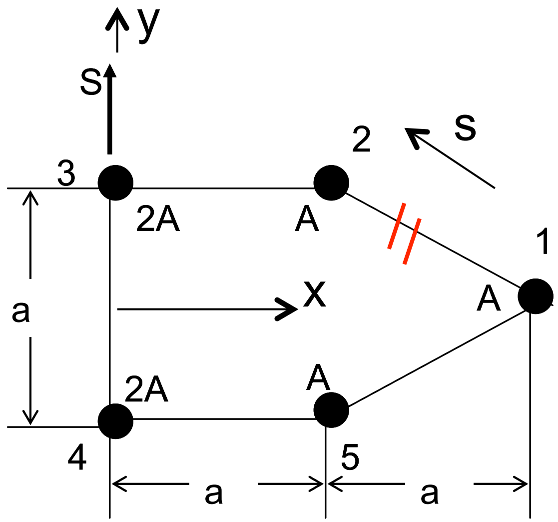
- with

$$I_{xx} = 2A\left(\frac{a}{2}\right)^2 + 2(2A)\left(\frac{a}{2}\right)^2 = \frac{3}{2}Aa^2$$

$B_1=B_2=B_5=A$ $y_1=0$; $y_2=-y_5=a/2$

$B_3=B_4=2A$ $y_3=-y_4=a/2$

Booms under shear loads – closed sections-Example



$$q_s = \underbrace{-\frac{S_y}{I_{xx}}\sum_{r=1}^{n} B_r y_r}_{q_{bs}} + q_{so}$$

$$I_{xx} = 2A\left(\frac{a}{2}\right)^2 + 2(2A)\left(\frac{a}{2}\right)^2 = \frac{3}{2}Aa^2$$

$B_1=B_2=B_5=A$ $\quad$ $y_1=0;\ y_2=-y_5=a/2$

$B_3=B_4=2A$ $\quad$ $y_3=-y_4=a/2$

Following standard procedures, cut, arbitrarily, between 1 and 2 and determine the shear flows for the open cross-section. Then:

$$q_{b12} = 0$$

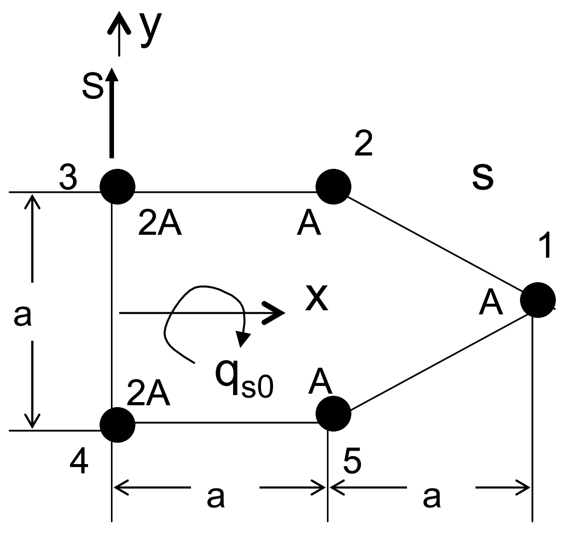
$$q_{b23} = -\frac{S}{\frac{3}{2}Aa^2}A\frac{a}{2} = -\frac{S}{3a}$$

$$q_{b34} = -\frac{S}{3a} - \frac{S}{\frac{3}{2}Aa^2}2A\frac{a}{2} = -\frac{S}{a}$$

$$q_{b45} = q_{23} = -\frac{S}{3a} \quad (symmetry)$$

$$q_{b51} = q_{12} = 0 \quad (symmetry)$$

Booms under shear loads – closed sections- Example



- Now close the cross-section and assume a constant shear flow q_{so} is applied (in the same direction as s)
- If we take moments about the point 4 we can use eq (5.10)

$$\oint_s p q_b ds + 2Aq_{s0} = 0 \qquad (5.10)$$

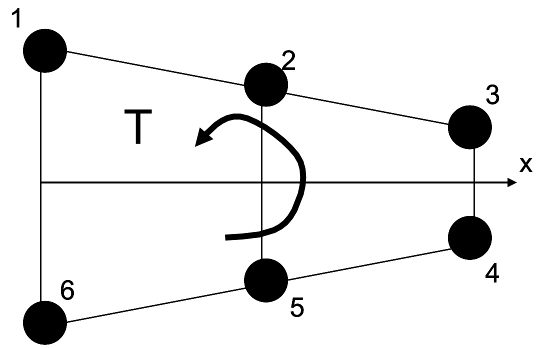
- since, as we found, $q_{b12}=q_{b51}=0$ and q_{34} and q_{45} do not contribute to the moments about 4, (5.10) becomes

$$q_{b23}a(a) + 2(a^2 + \frac{a^2}{2})q_{so} = 0$$

- using q_{b23} to solve for q_{so} gives: $q_{so} = -\dfrac{q_{b23}a^2}{2\frac{3}{2}a^2} = \dfrac{S}{9a}$

- adding q_{so} to q_{bij} gives the final answer

Booms under torsion loads – Closed or open section beams



• a pure torque T causes no normal stresses in the booms (unless the beam is constrained along its axis such as fixed-fixed)

• therefore, the booms have no effect on the shear flows in the skins and the solution from beams without booms are still valid