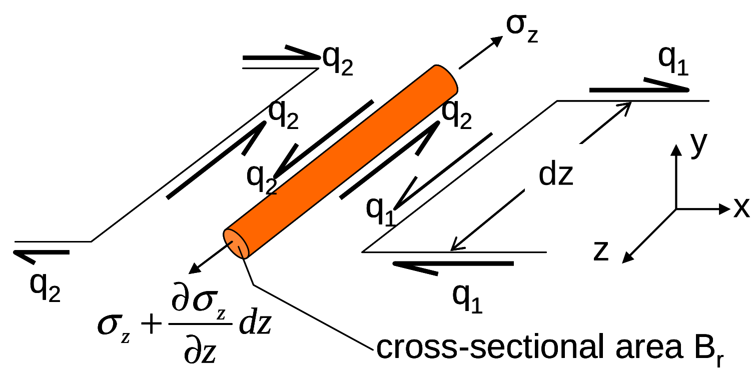
Calculating the shear flow between booms using the axial load



- we found before, eq. (7.10), that the shear flows across a boom are given by

$$q_2 - q_1 = -\frac{\partial \sigma_z}{\partial z} B_r \qquad (7.10)$$

where B_r is the area of the rth boom

- for a B_r that does not change with z, we can write

$$\frac{\partial \sigma_z}{\partial z} B_r = \frac{\partial (\sigma_z B_r)}{\partial z}$$

- but

$$\sigma_z B_r = P_r$$

$$\Longrightarrow \quad \frac{\partial \sigma_z}{\partial z} B_r = \frac{\partial P_r}{\partial z}$$

then, (7.10) becomes:

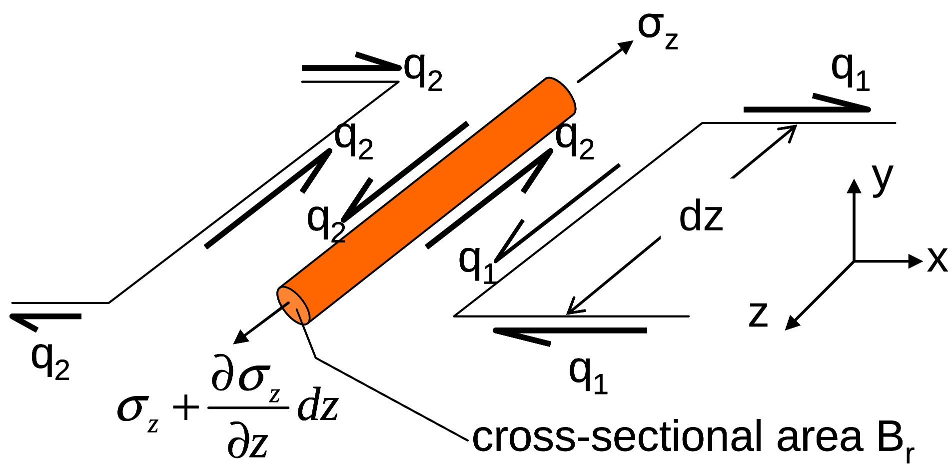
where P_r is the axial load in boom r; note that P_r is, in general, a function of z

$$q_2 - q_1 = -\frac{\partial P_r}{\partial z} \qquad (8.1)$$

note difference in sign from Megson!!!

Calculating the shear flow between booms using the axial load

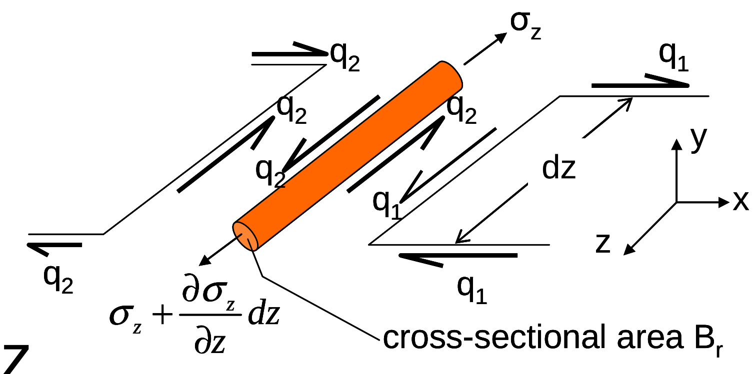
$$q_2 - q_1 = -\frac{\partial P_r}{\partial z} \qquad (8.1)$$



- note that if P_r does not depend on z, $\partial P_r/\partial z=0$ and $q_2=q_1$ this would mean that we have a constant shear flow, which is the case of pure torsion (for which $P_r=0$) or, if the section is open, $q_1=0$ and hence $q_2=0$, which is the case of (P_r=const), i.e., pure tension or compression

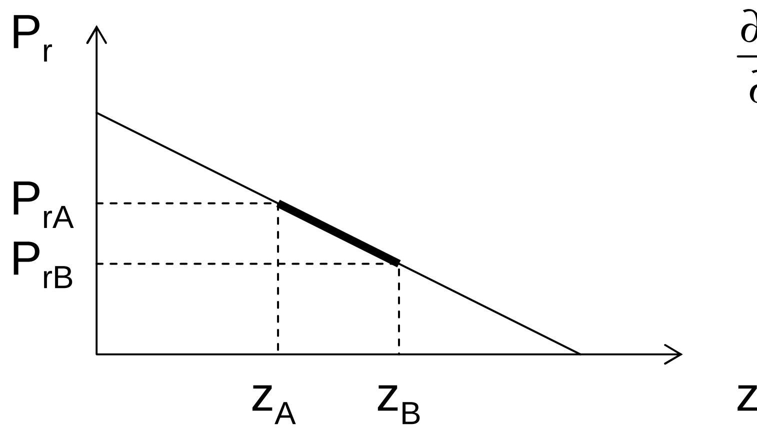
Calculating the shear flow between booms using the axial load

$$q_2 - q_1 = -\frac{\partial P_r}{\partial z} \qquad (8.1)$$



- the trick is to determine $\partial(P_r)/\partial z$

- assume that P_r varies linearly with z
(if the variation is non-linear, the answer is approximate)



$$\frac{\partial P_r}{\partial z} = -\frac{(P_{rA} - P_{rB})}{(z_B - z_A)} = \frac{(P_{rB} - P_{rA})}{(z_B - z_A)} \qquad (8.2)$$

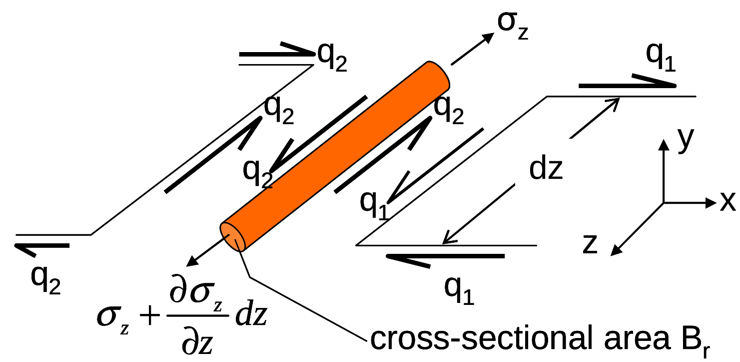
- let z_B-z_A= 1 unit of length

- then, numerically, if ΔP_r=P_{rB}-P_{rA}

$$\frac{\partial P_r}{\partial z} = -\Delta P_r \qquad (8.3)$$

units are off!

Calculating the shear flow between booms using the axial load



$$q_2 - q_1 = -\frac{\partial P_r}{\partial z} \qquad (8.1)$$

$$\frac{\partial P_r}{\partial z} = -\Delta P_r \qquad (8.3)$$

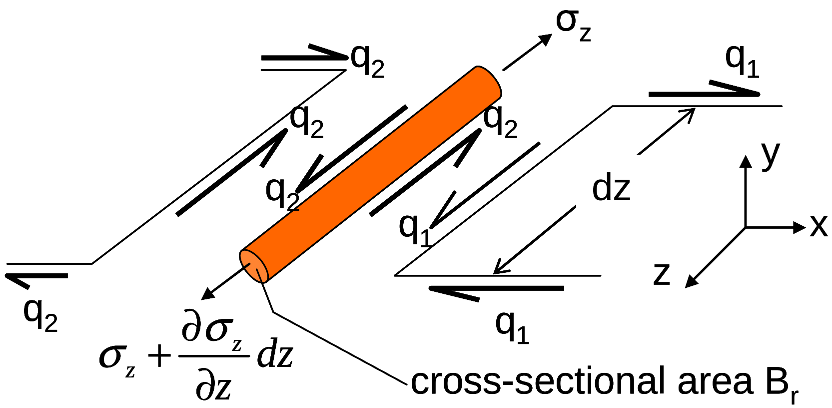
- combining:

$$q_2 - q_1 = \Delta P_r \qquad (8.4)$$

where ΔP_r is the change in axial load in boom r over a length of boom = 1 unit of length

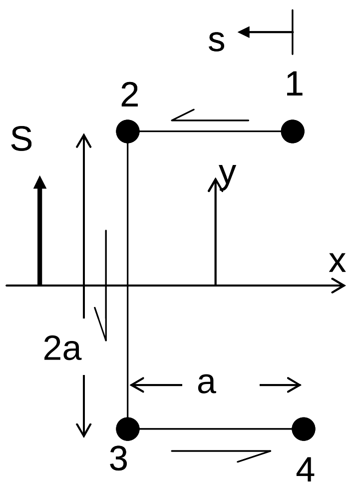
Calculating the shear flow between booms using the axial load

$$q_2 - q_1 = \Delta P_r \qquad (8.4)$$



• so if the change in axial load (per unit length) is known, q_2-q_1 is known

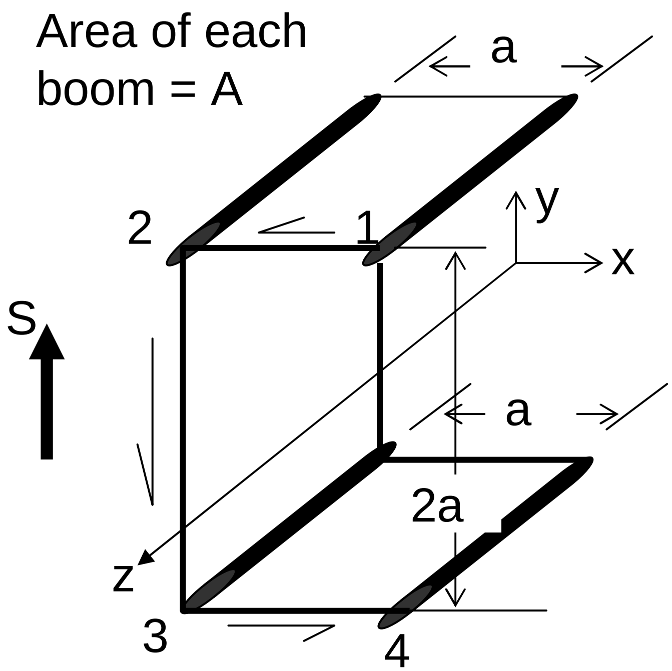
• as an example, consider the case solved in previous lecture:



C- section beam under shear S with four booms at the corners, each of area A; flange length=a, web height=2a. The solution we found:

$$\begin{cases} q_{12} = -\dfrac{S}{4a} \\ q_{23} = -\dfrac{S}{2a} \\ q_{34} = -\dfrac{S}{4a} \end{cases}$$

Calculating the shear flow between booms using the axial load



$$q_2 - q_1 = \Delta P_r \qquad (8.4)$$

• the change in load can be related to the change in axial (direct) stress

• concentrating on boom 1 (r=1):

$$\Delta P_1 = A \Delta \sigma_z$$

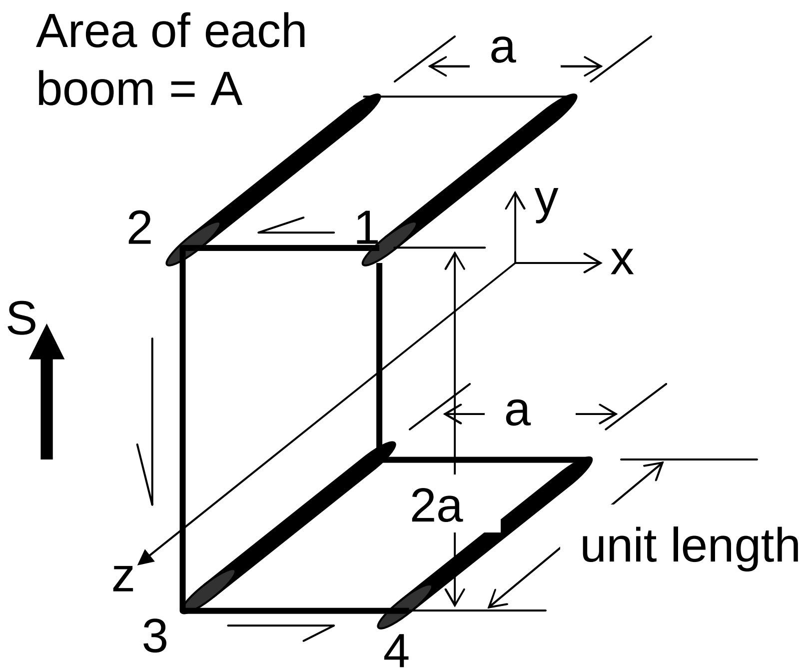
• using the bending equation:

$$\sigma_z = \frac{M_x y}{I_{xx}}$$

• we can then write

$$\Delta P_1 = A \frac{y}{I_{xx}} \Delta M_x = A \frac{a}{4Aa^2} \Delta M_x = \frac{\Delta M_x}{4a}$$

Calculating the shear flow between booms using the axial load



$$q_2 - q_1 = \Delta P_r \qquad (8.4)$$

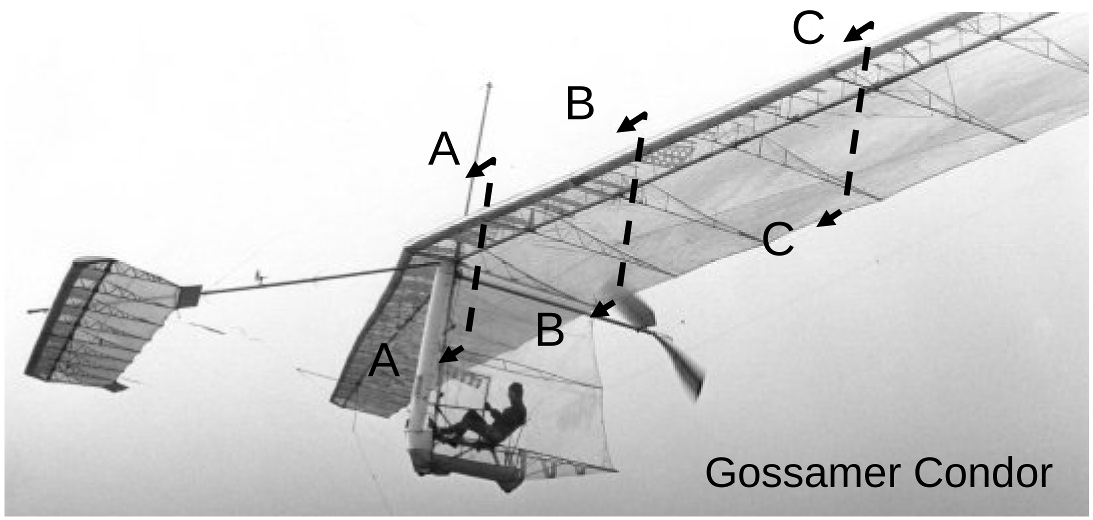
$$\Delta P_1 = A\frac{y}{I_{xx}}\Delta M_x = A\frac{a}{4Aa^2}\Delta M_x = \frac{\Delta M_x}{4a}$$

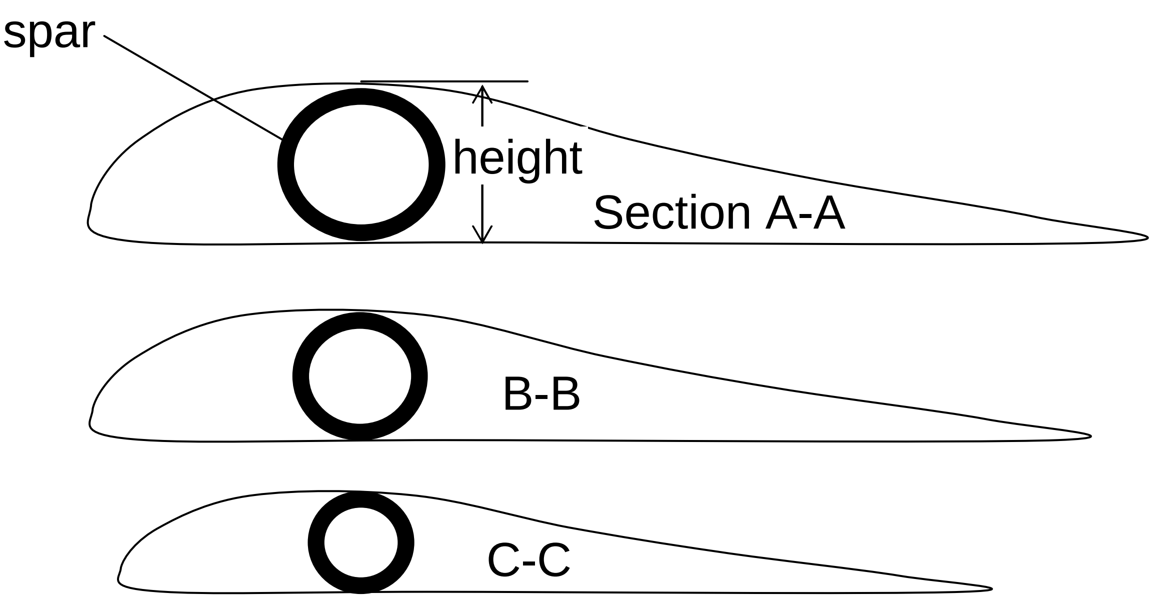
- the change in moment per unit length ΔM_x is obtained by considering the moment change caused by the applied load S, over the unit length

- the change in moment ΔM_x equals the force S times the moment arm (=1 since unit length) => ΔM_x =(S)x(1)=S. But since the moment is decreasing for increasing z, ΔM_x =-S

- substituting in (8.4): $q_2 - q_1 = \frac{-S}{4a}$ which is the same as we had before!

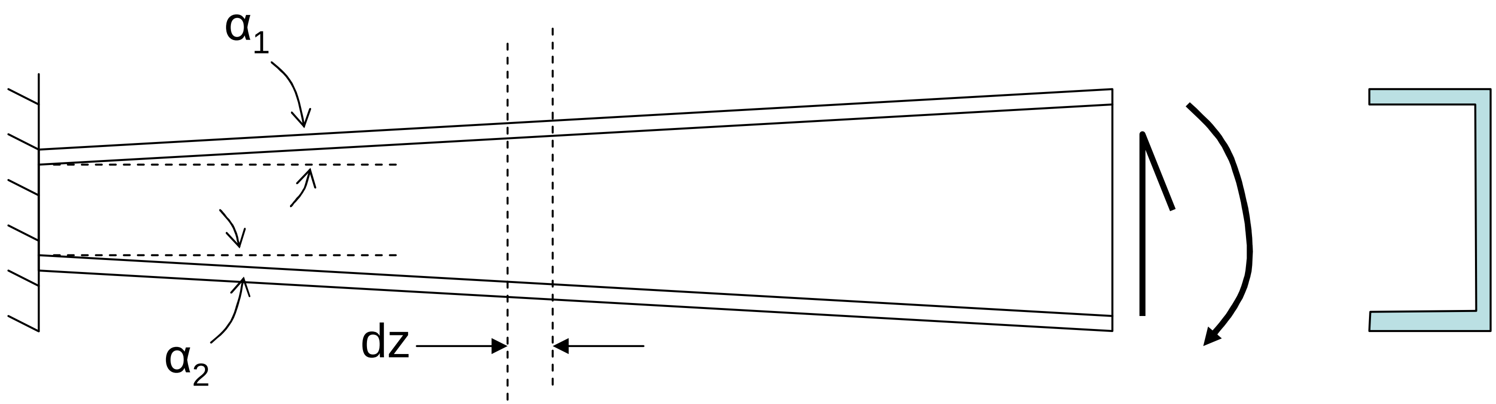
Taper in structures



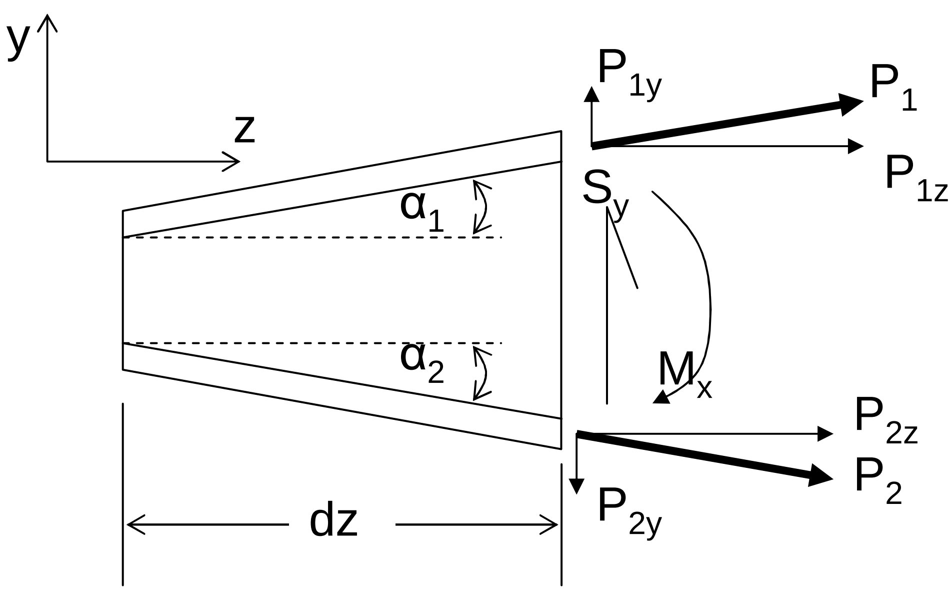


For aerodynamic reasons, the spar height must decrease moving outboard; how to analyze?

Tapered beam



- isolate an element of length dz and put it in equilibrium



$$P_1 = \frac{P_{1z}}{\cos\alpha_1}$$

$$P_{1y} = P_{1z}\tan\alpha_1$$

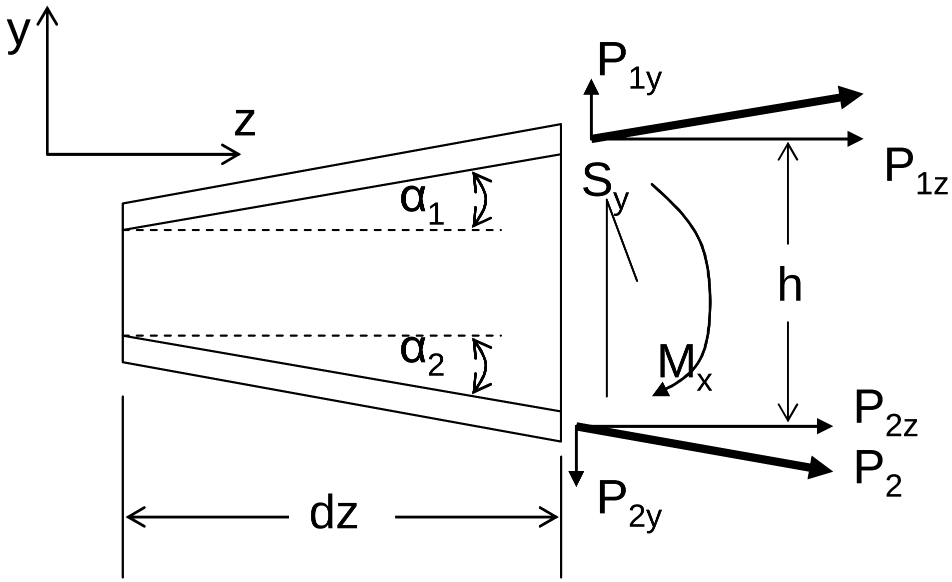
$$P_2 = \frac{P_{2z}}{\cos\alpha_2} \qquad (8.5)\text{-}(8.8)$$

$$P_{2y} = P_{2z}\tan\alpha_2$$

$$(neg \quad y \quad dir)$$

what are P_{1z} and P_{2z} ?

Tapered beam



$$P_1 = \frac{P_{1z}}{\cos\alpha_1}$$

$$P_{1y} = P_{1z}\tan\alpha_1$$

$$P_2 = \frac{P_{2z}}{\cos\alpha_2} \qquad (8.5)\text{-}(8.8)$$

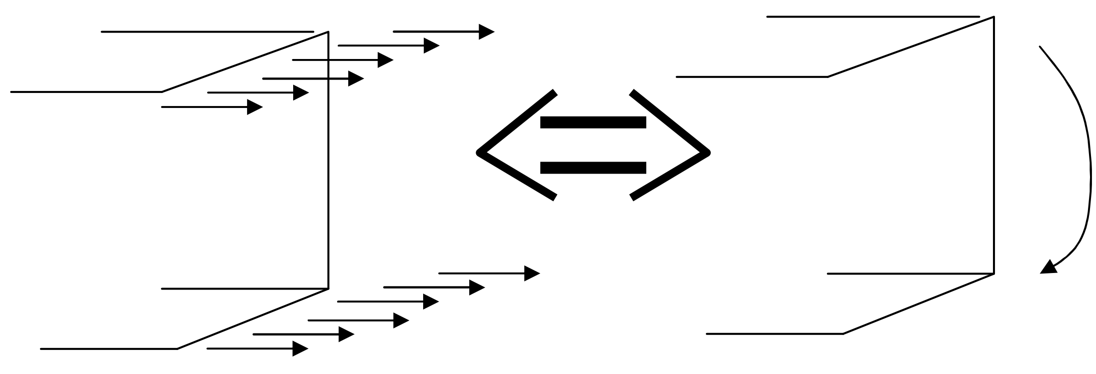
$$P_{2y} = P_{2z}\tan\alpha_2$$

$(neg \quad y \quad dir)$

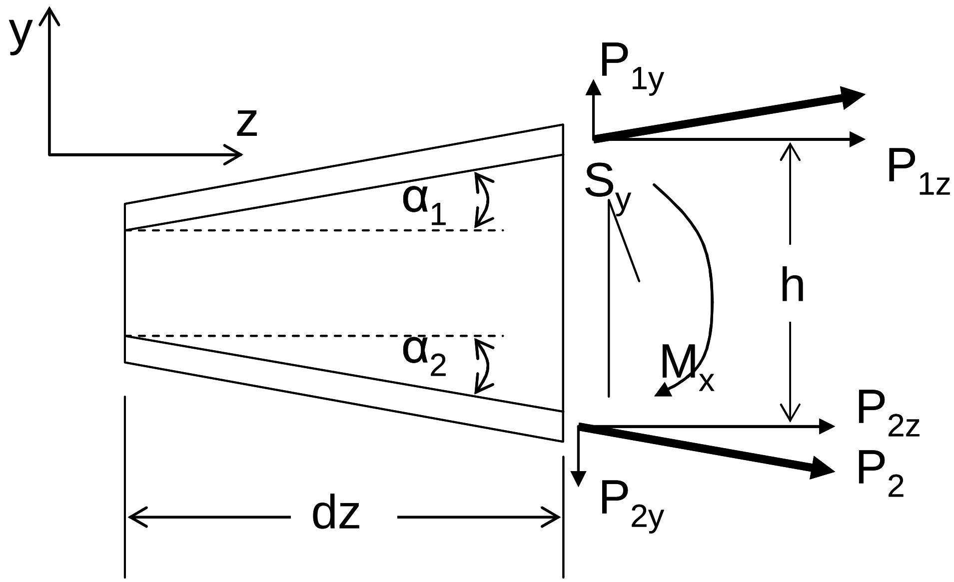
• if the moment M_x is carried by the flanges only (web is ineffective in carrying direct stresses),

$$P_{1z} = \frac{M_x}{h} \qquad (8.9)$$

$$P_{2z} = -\frac{M_x}{h} \qquad (8.10)$$



Tapered beam



$$P_1 = \frac{P_{1z}}{\cos\alpha_1}$$

$$P_{1y} = P_{1z}\tan\alpha_1$$

$$P_2 = \frac{P_{2z}}{\cos\alpha_2} \qquad (8.5)\text{-}(8.8)$$

$$P_{2y} = P_{2z}\tan\alpha_2$$

$$(neg \quad y \quad dir)$$

- if the web is fully effective in carrying direct stresses, then

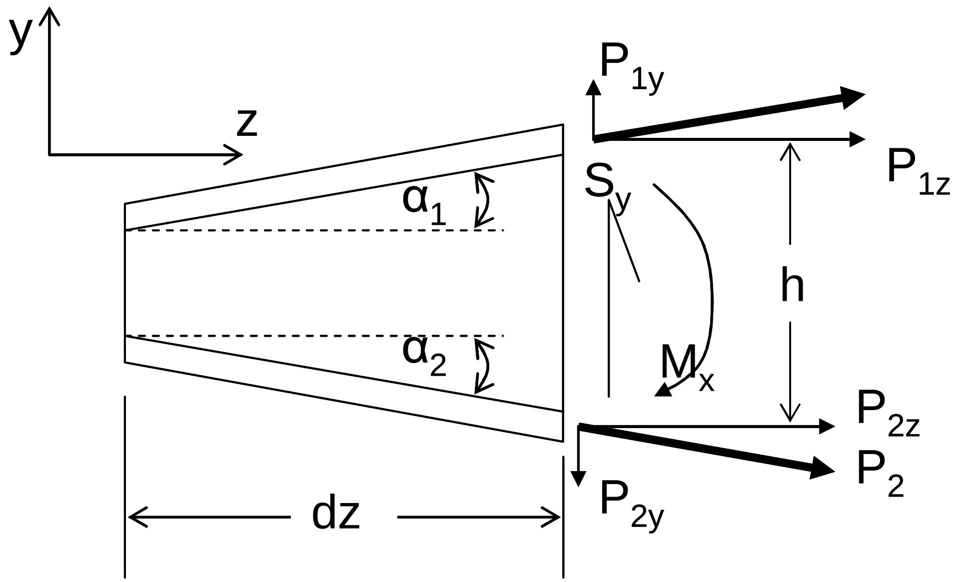
$$P_{1z} = B_1\sigma_{1z} \qquad (8.10)$$

$$P_{2z} = B_2\sigma_{2z} \qquad (8.11)$$

where B_1 and B_2 are the corresponding flange areas and for σ_{1z}, σ_{2z}, use beam theory; for example, for cross-section with one axis of symmetry,

$$\sigma_{1z} = \frac{M_x \frac{h}{2}}{I_{xx}} \qquad \sigma_{2z} = -\frac{M_x \frac{h}{2}}{I_{xx}}$$

Tapered beam



• the total shear force S_y is made up of the web shear force S_w and the two vertical forces P_{1y} and P_{2y}

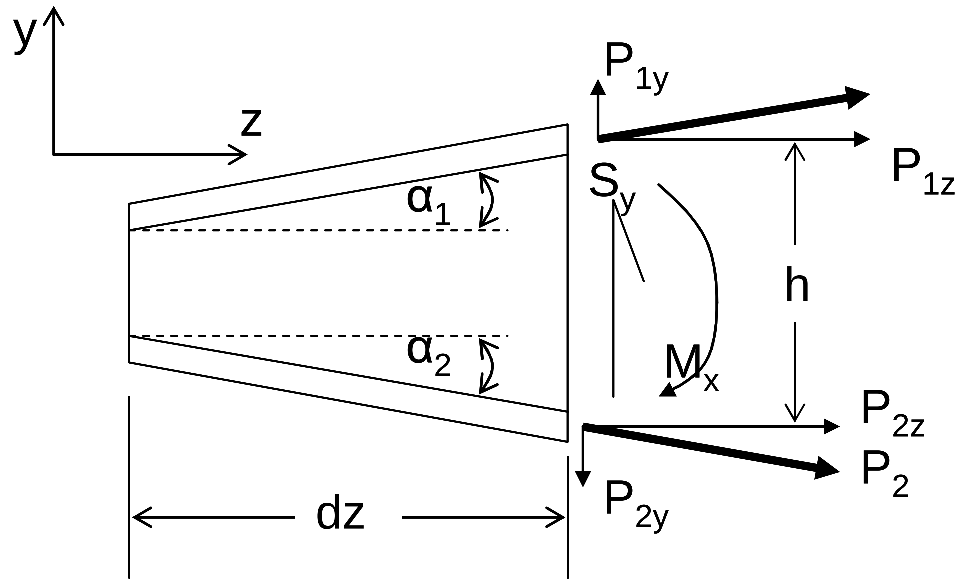
$$S_y = S_{wy} + P_{1y} - P_{2y} \tag{8.12}$$

• this can be solved for the web shear force S_{wy} using also the previously found expressions for P_{1y} and P_{2y}

$$S_{wy} = S_y - P_{1z} \tan \alpha_1 + P_{2z} \tan \alpha_2 \tag{8.13}$$

what is S_{wy}??

Tapered beam



- to determine S_{wy} we need to know the shear flow in the web

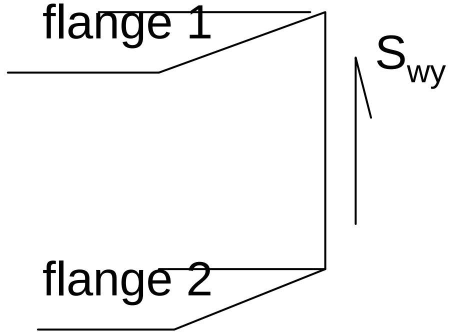
- if the web is idealized and carries only shear stresses, the shear flow is constant; then:

$$q_w = \frac{S_{wy}}{h} \Rightarrow S_{wy} = hq_w \qquad (8.14)$$

- if the web is fully effective in carrying direct stresses, the shear flow is no longer constant; we need to use eq (7.12)

$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D x ds + \sum_{r=1}^n B_r x_r\right] - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D y ds + \sum_{r=1}^n B_r y_r\right] \qquad (7.12)$$

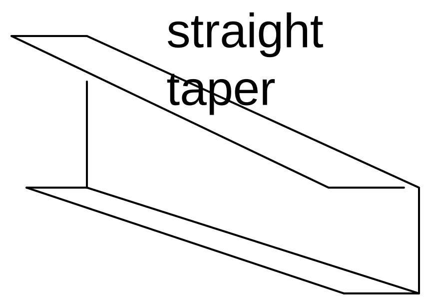
Tapered beam

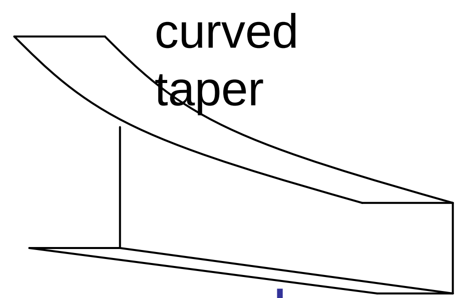


$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D x ds + \sum_{r=1}^{n} B_r x_r\right] - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D y ds + \sum_{r=1}^{n} B_r y_r\right] \quad (7.12)$$

• for the case of the C-section beam, with flanges 1 and 2, eq. (7.12) simplifies to:

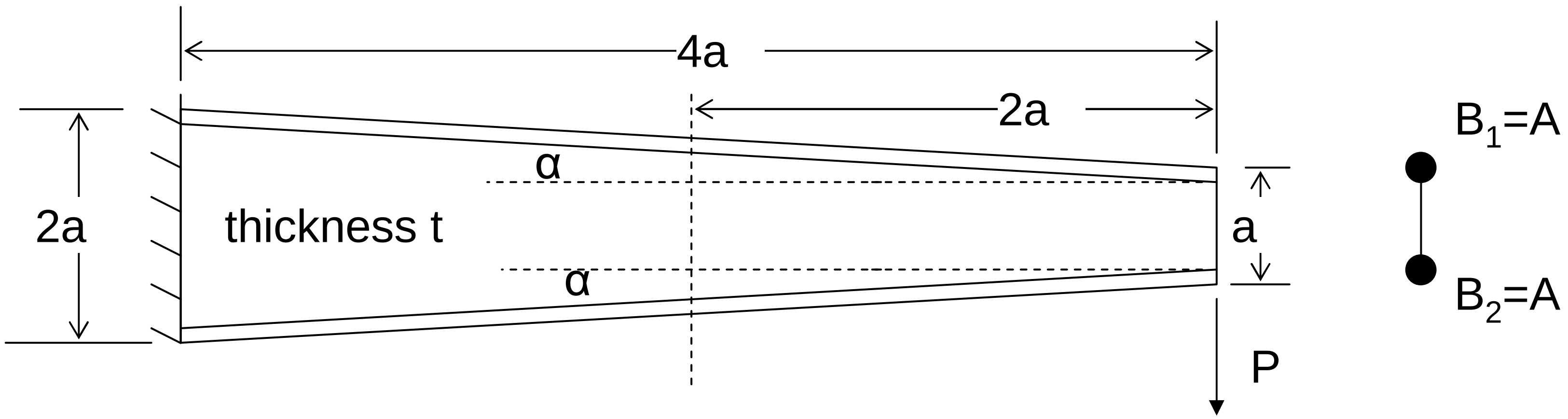
$$q_s = -\frac{S_{wy}}{I_{xx}}\left[\int_0^s t_D y ds + B_1 \frac{h}{2}\right]$$





Note that, in determining P_{1y}, P_{1z}, etc. it was assumed that the beam taper is straight and not curved; if it were curved, we would have to use derivatives instead of sines and tangents (see Megson 21.1)

Tapered beam - Example



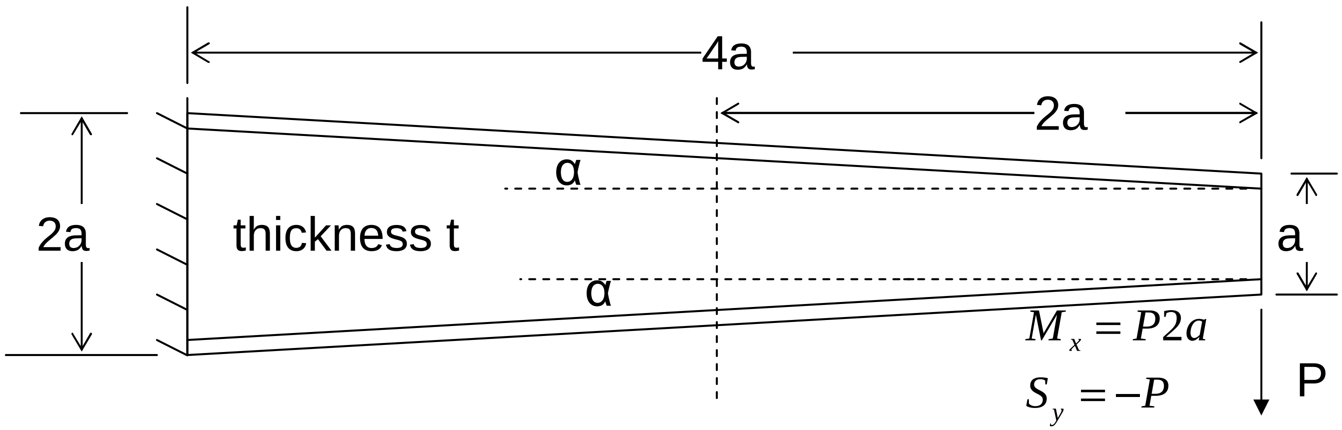
• determine the shear flow distribution in the web at the mid-point of the beam; note, the web is fully effective

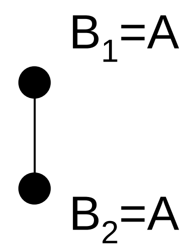
• from beam theory, the moment M_x and shear S_y at the mid-point are given by

$$M_x = P2a$$

$$S_y = -P$$

Tapered beam - Example



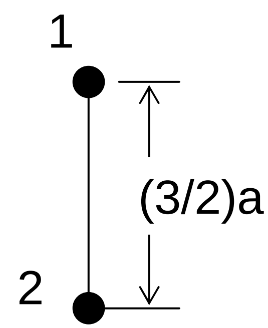


- the bending stress at the section of interest is given by

$$\sigma_z = \frac{M_x y}{I_{xx}}$$

- the moment of inertia is given by

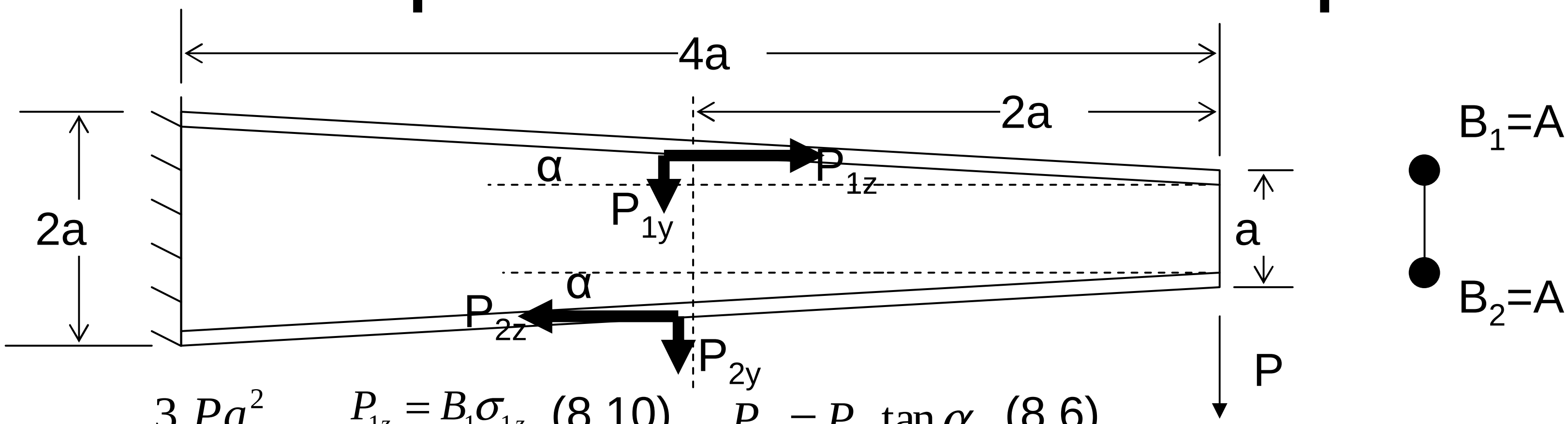
$$I_{xx} = I = t\frac{\left(\frac{3a}{2}\right)^3}{12} + 2A\left(\frac{3}{4}a\right)^2 = \frac{9}{32}ta^3 + \frac{9}{8}Aa^2$$



- combining, the stress in the flanges is given by

$$\sigma_{1,2} = \pm\frac{3}{2}\frac{Pa^2}{I}$$

Tapered beam - Example



$$\sigma_{1,2} = \pm \frac{3}{2}\frac{Pa^2}{I}$$

$P_{1z} = B_1\sigma_{1z}$ (8.10) $\quad P_{1y} = P_{1z}\tan\alpha_1$ (8.6)

$P_{2z} = B_2\sigma_{2z}$ (8.11) $\quad P_{2y} = P_{2z}\tan\alpha_2$ (8.8)

- at the section of interest, the axial forces P_{1z} and P_{2z} are given by eqs (8.10) and (8.11)

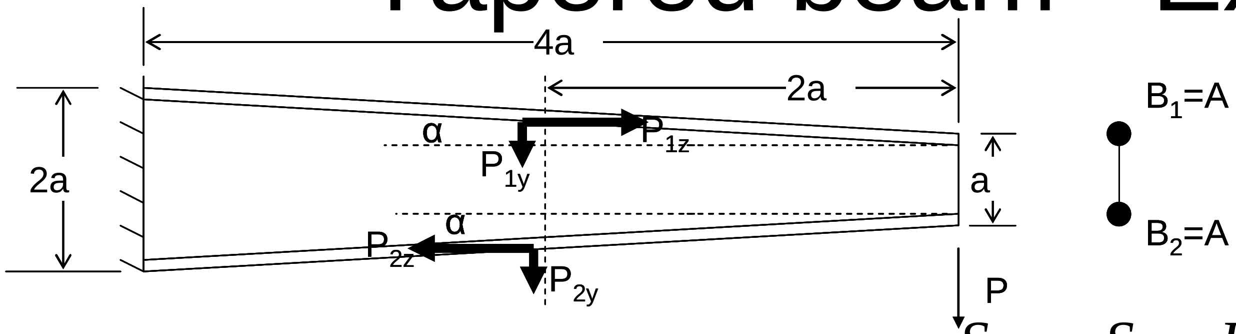
$$P_{1z} = -P_{2z} = \frac{3}{2}\frac{PAa^2}{I}$$

- the vertical forces P_{1y} and P_{2y} are then given by eqs (8.6) and (8.8)

- noting that $\tan\alpha_1 = -\tan\alpha_2 = \dfrac{\frac{2a-a}{2}}{4a} = \dfrac{1}{8}$

Important note: P_{1y}, P_{2y} are drawn with their actual orientations so the resultants P_1 and P_2 are in the right direction, tension in the upper and compr. in lower flange

Tapered beam - Example



$$P_{1y} = P_{1z}\tan\alpha_1 \quad (8.6)$$

$$P_{2y} = P_{2z}\tan\alpha_2 \quad (8.8)$$

$$P_{1z} = -P_{2z} = \frac{3}{2}\frac{PAa^2}{I}$$

$$S_{wy} = S_y - P_{1z}\tan\alpha_1 + P_{2z}\tan\alpha_2 \quad (8.13)$$

- leads to:

$$-P_{1y} = +P_{2y} = \frac{3}{16}\frac{PAa^2}{I}$$

(note that the orientations shown in the figure **already** account for the signs in the equation)

- the shear load in the web alone is then given by (8.13); accounting for the signs:

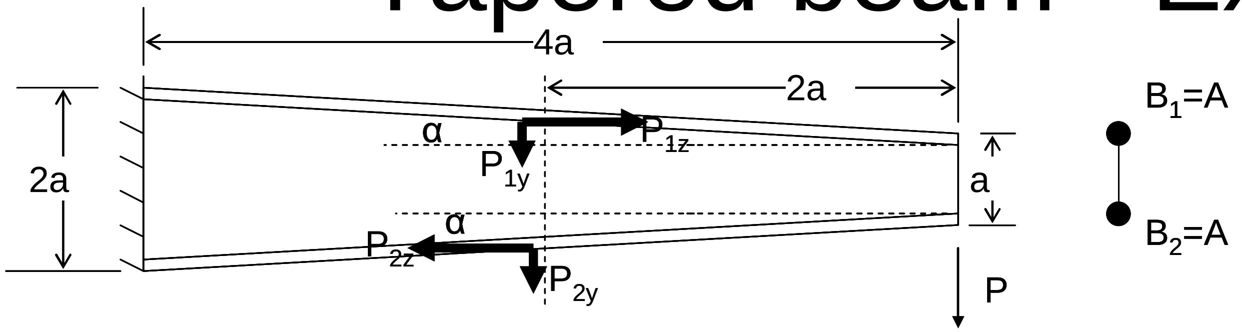
$$-P = S_y = S_{wy} - P_{1y} - P_{2y} \Rightarrow S_{wy} = -P + \frac{3}{16}\frac{PAa^2}{I} + \frac{3}{16}\frac{PAa^2}{I}$$

- the shear flow in the web is then obtained from (7.12)

$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D x ds + \sum_{r=1}^{n} B_r x_r\right] - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D y ds + \sum_{r=1}^{n} B_r y_r\right] \quad (7.12)$$

with $S_x = I_{xy} = 0$ and the coordinate system centered at the mid-point of the web so that the flanges are at ±3a/4 from the origin

Tapered beam - Example



$$q_{sw} = -\frac{S_{wy}}{I}\left[\int_0^s tyds + B_2 y_2\right]$$

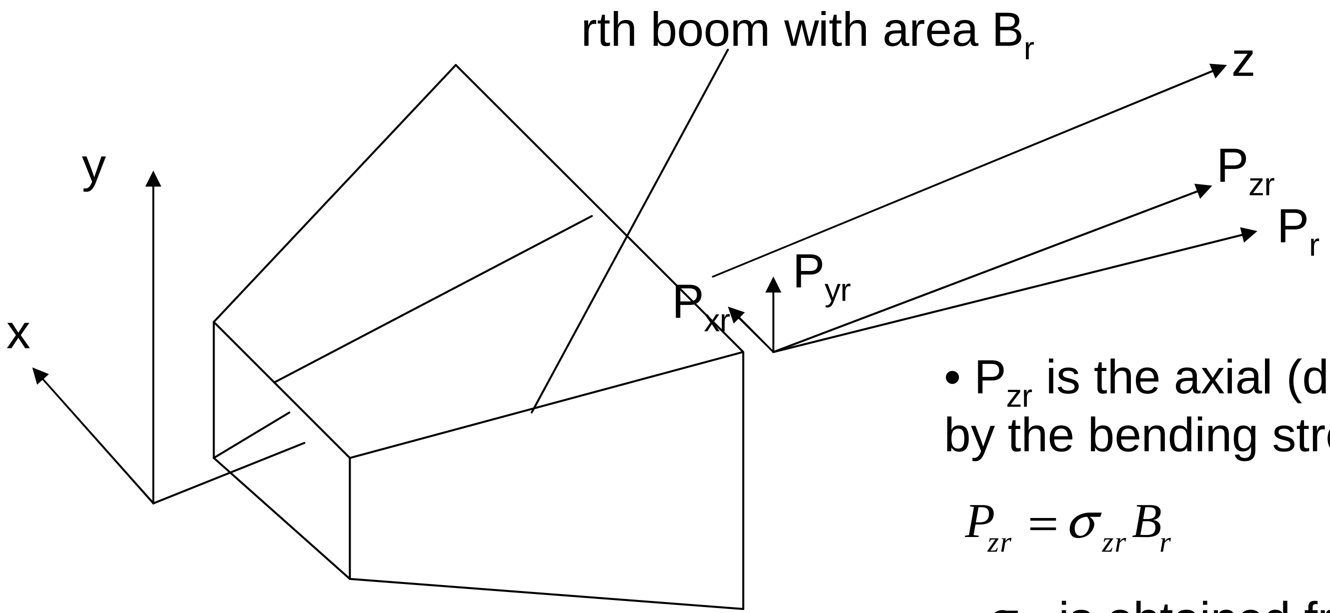
• using $y=(s-3a/4)$ and $y_2=-3a/4$

$$q_{sw} = -\frac{S_{wy}}{I}\left[t\left(\frac{s^2}{2} - \frac{3as}{4}\right) - \frac{3Aa}{4}\right]$$

• note that at the top and bottom flanges (s=0 and s=3a/2) the shear flows are the same and equal to

$$q_{s1} = q_{s2} = \frac{3S_{wy}Aa}{4I}$$

Taper in two directions (wing, aft or fwd fuselage)



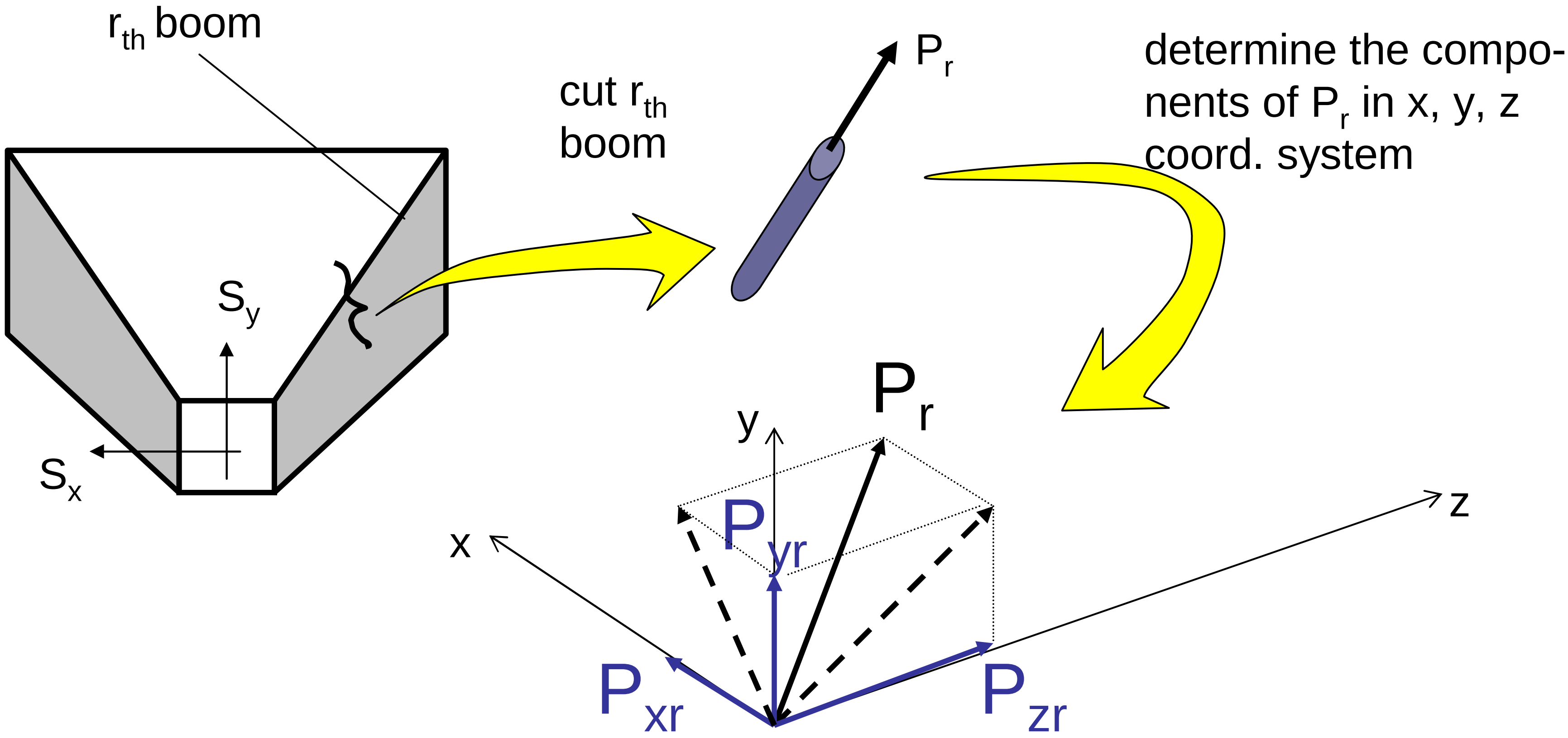
• P_{zr} is the axial (direct) load caused by the bending stress; so

$$P_{zr} = \sigma_{zr} B_r \tag{8.15}$$

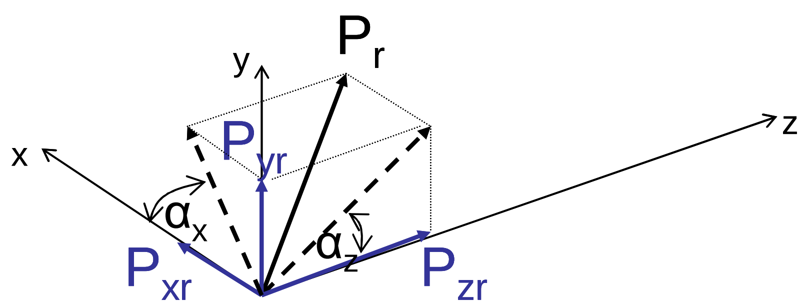
• σ_{zr} is obtained from standard bending theory, eq. (2.5)

• then, the three forces P_{zr}, P_{yr}, and P_{xr} are obtained as cartesian components of the resultant vector P_r

Taper in two directions (wing, aft or fwd fuselage)



Taper in two directions (wing, aft or fwd fuselage)



- the procedure is very similar to the case of taper in one direction examined earlier

$$P_{zr} = \sigma_{zr} B_r \qquad (8.15)$$

- considering the projection of P_r on the yz plane:

$$P_{yr} = \left(\tan\alpha_z\right) P_{zr} \qquad (8.16)$$

- considering the projection of P_r on the xy plane:

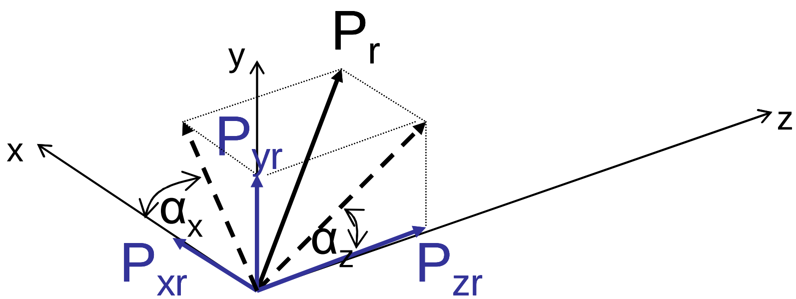
$$P_{xr} = \frac{P_{yr}}{\tan\alpha_x} \qquad (8.17)$$

- or, combining with (8.16),

$$P_{xr} = \frac{\tan\alpha_z}{\tan\alpha_x} P_{zr} \qquad (8.18)$$

(note that Megson uses derivatives instead of tangents but it is the same thing)

Taper in two directions



• from basic vector theory:

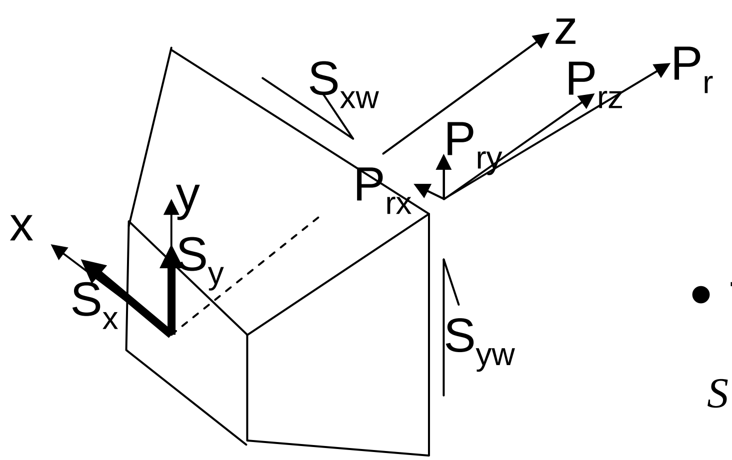
$$P_r = \sqrt{P_{xr}^{\ 2} + P_{yr}^{\ 2} + P_{zr}^{\ 2}} \qquad (8.19)$$

So P_{zr} is calculated first using the boom stress obtained from standard bending equations. Then, P_{xr} and P_{yr} are obtained from (8.16) and (8.18). Finally, (8.19) is used to determine the total force in the boom

• we still need to calculate the shear flows in the webs; to do that we need the portions S_{xw} of S_x and S_{yw} of S_y acting on the webs

• the total shear force in the x or y direction is equal to the web force in that direction plus the boom force in the same direction

Taper in two directions – web forces



• the total shear force in the x or y direction is equal to the web force in that direction plus the boom force in the same direction

• then,

$$S_x = S_{xw} + \sum_{r=1}^{m} P_{xr} \quad (8.20)$$

(m=No of booms)

$$S_y = S_{yw} + \sum_{r=1}^{m} P_{yr} \quad (8.21)$$

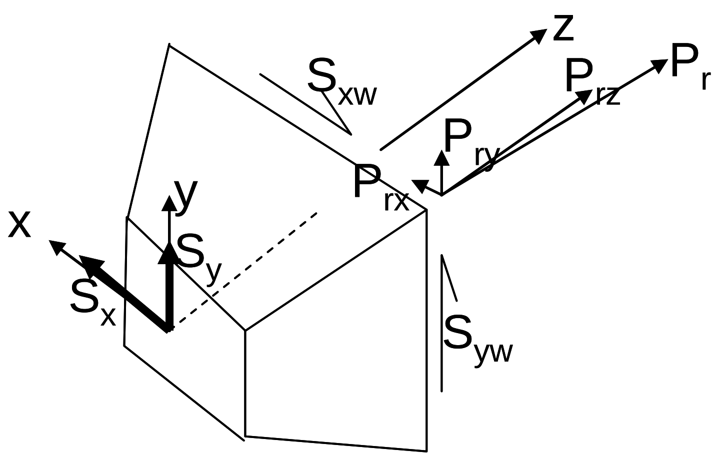
• from which we can solve for the web forces:

$$S_{xw} = S_x - \sum_{r=1}^{m} P_{xr} \quad (8.22)$$

$$S_{yw} = S_y - \sum_{r=1}^{m} P_{yr} \quad (8.23)$$

with P_{xr} and P_{yr} determined from (8.16) and (8.18)

Taper in two directions – shear flows



If the webs carry no normal stresses, (idealized structure), the shear flows are obtained by dividing the web forces by the web lengths h_x and h_y; see also eq. (8.14). If the webs also carry normal stresses, the full equation for shear flows must be used; for example, for open cross-sections, use eq. (7.12) with S_x and S_y replaced by S_{xw} and S_{yw} respectively.

$$q_w = \frac{S_{wy}}{h} \Rightarrow S_{wy} = hq_w \tag{8.14}$$

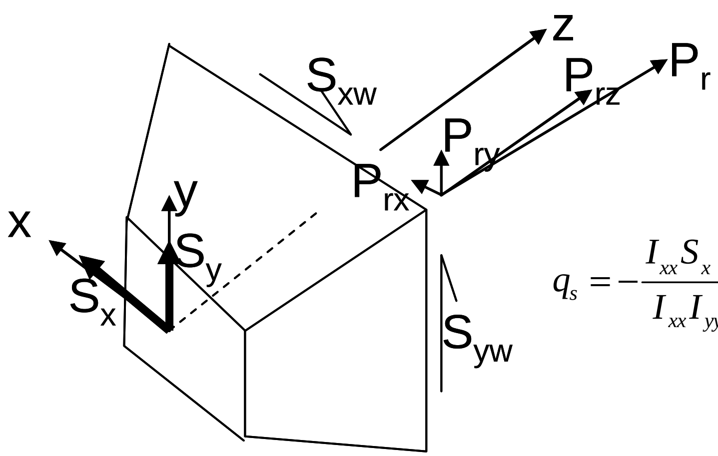
$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D x\,ds + \sum_{r=1}^{n} B_r x_r\right] - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D y\,ds + \sum_{r=1}^{n} B_r y_r\right] \tag{7.12}$$

- so for idealized structure

$$q_{wx} = \frac{S_{wx}}{h_x} \tag{8.24}$$

$$q_{wy} = \frac{S_{wy}}{h_y} \tag{8.25}$$

Taper in two directions – shear flows



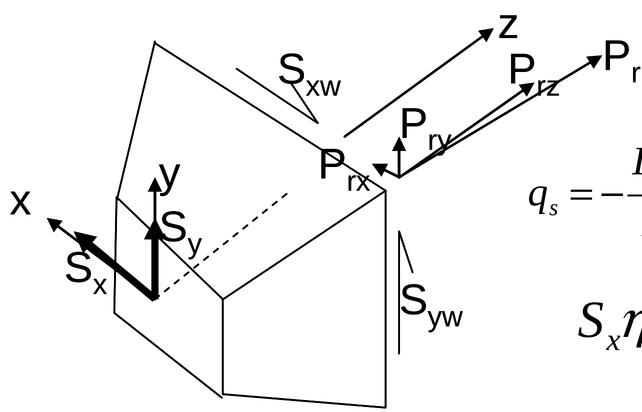
$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D x ds + \sum_{r=1}^{n} B_r x_r\right] - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D y ds + \sum_{r=1}^{n} B_r y_r\right] \qquad (7.12)$$

- for non-idealized structure (webs carry normal stresses also):

$$q_s = -\frac{I_{xx}S_{wx} - I_{xy}S_{wy}}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D x ds + \sum_{r=1}^{n} B_r x_r\right] - \frac{I_{yy}S_{wy} - I_{xy}S_{wx}}{I_{xx}I_{yy} - I_{xy}^2}\left[\int_0^s t_D y ds + \sum_{r=1}^{n} B_r y_r\right] \qquad (8.26)$$

valid for open cross-section!

Taper in two directions – shear flows



(7.17)

$$q_s = -\frac{I_{xx}S_x - I_{xy}S_y}{I_{xx}I_{yy} - I_{xy}^{\;2}}\left[\int_0^s t_D x\,ds + \sum_{r=1}^{n} B_r x_r\right] - \frac{I_{yy}S_y - I_{xy}S_x}{I_{xx}I_{yy} - I_{xy}^{\;2}}\left[\int_0^s t_D y\,ds + \sum_{r=1}^{n} B_r y_r\right] + q_{so}$$

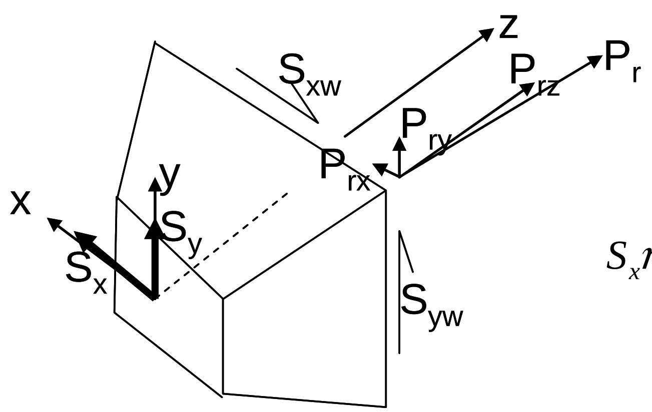
$$S_x\eta_o - S_y\xi_o = \oint_s p q_b\,ds + 2Aq_{s0} \qquad (5.9)$$

• if the cross-section is closed, there is a constant shear flow q_{s0} that is added once the cut is closed during solving (recall eq. (7.17); that shear flow is obtained by taking moments about a convenient point, which, for no taper, resulted in eq (5.9); here, the force components P_{rx} and P_{ry} also contribute to the moment eq

• therefore, use eq (7.17) with S_x, S_y changed to S_{wx}, S_{wy} and modify eq (5.9) to read:

$$S_x\eta_o - S_y\xi_o = \oint_s p q_b\,ds + 2Aq_{s0} - \sum_{r=1}^{m} P_{xr}\eta_r - \sum_{r=1}^{m} P_{yr}\xi_r \qquad (8.27)$$

Taper in two directions – shear flows



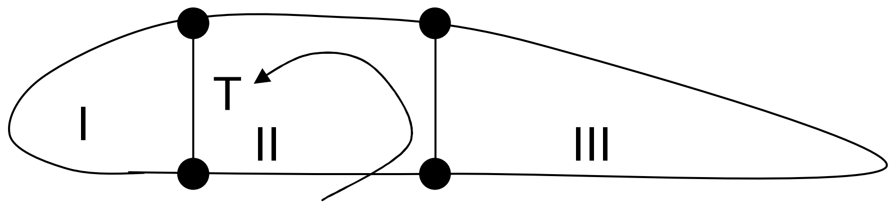
$$S_x\eta_o - S_y\xi_o = \oint_s pq_b ds + 2Aq_{s0} - \sum_{r=1}^{m} P_{xr}\eta_r - \sum_{r=1}^{m} P_{yr}\xi_r \quad (8.27)$$

- note that η_r and ξ_r are moment arms of P_{rx} and P_{ry} respectively from the (arbitrary) point about which moments are taken

- this is a nightmare to keep the signs correct

- see example in Megson at end of section 21.2 solved in tabular form

Torsion of multi-cell cross-sections



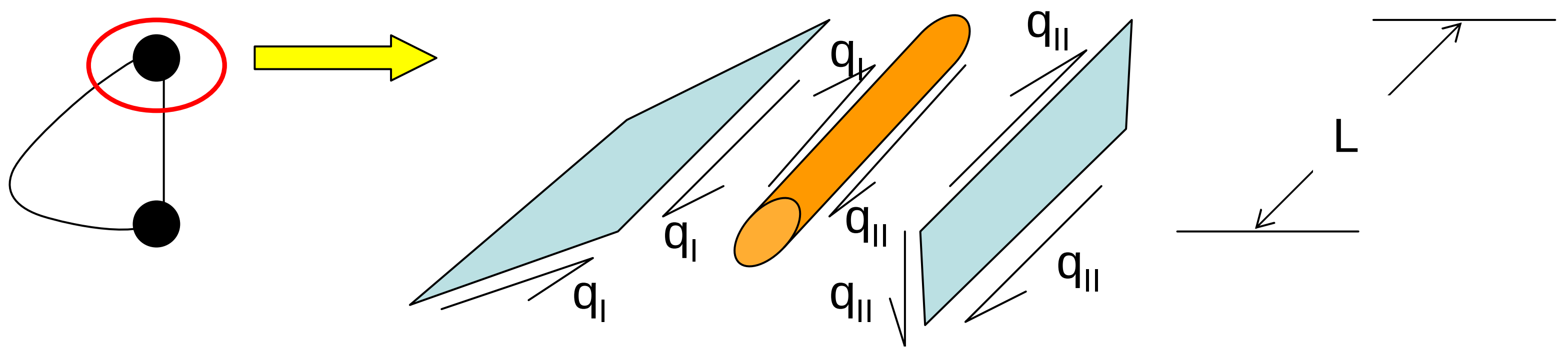
• pure torque applied, and beam is unrestrained

• cells are labelled I, II, and III

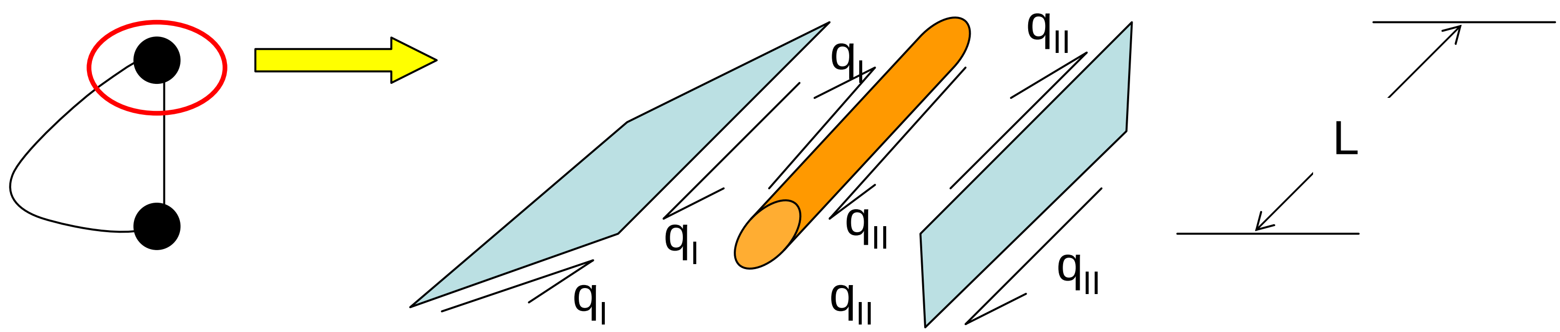
• it is required to determine the shear flows

• 1st observation: since this is under pure torque and the beam is unrestrained, there are no normal (direct) stresses in the booms

• if there were only a single cell, isolating the top boom:



Torsion of multi-cell cross-sections



- force equilibrium of the boom along its axis:

$$q_I L - F_{boom} - q_{II} L = 0 \qquad (8.27)$$

- but F_{boom}=0 in this case; so:

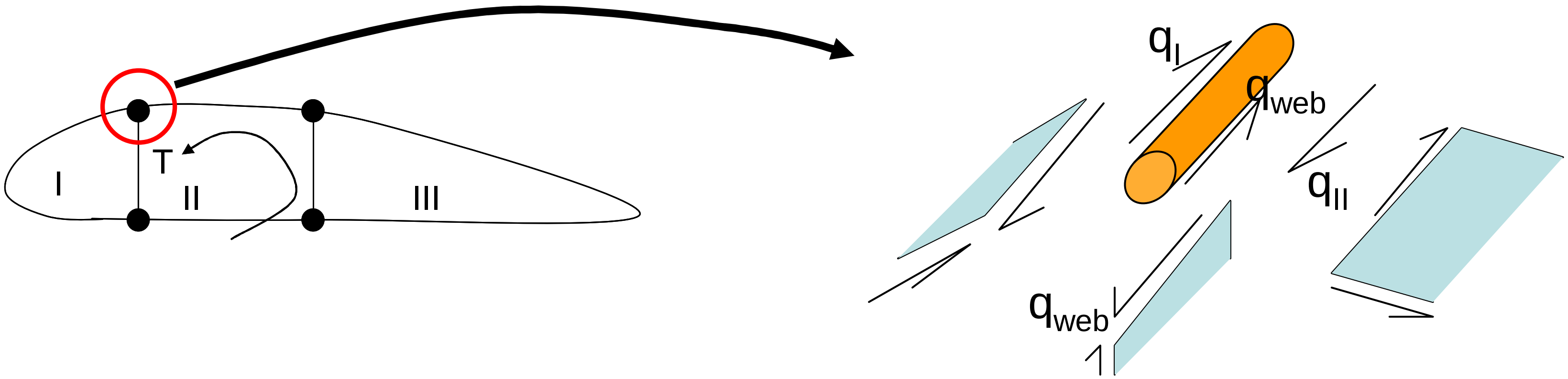
$$q_I L - q_{II} L = 0 \Rightarrow q_I = q_{II} \qquad (8.28)$$

- therefore, the presence of booms in a single cell under pure torsion has no effect on the shear flows; the shear flow around the cell is constant and given by the well known equation (3.44):

$$T = 2Aq \qquad (3.44)$$

with A the enclosed area of the cell

Torsion of multi-cell cross-sections



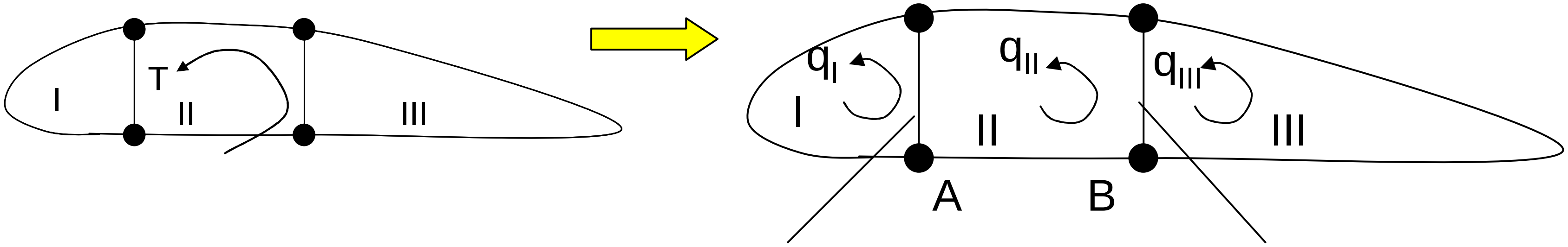
• for the case of multiple cells, at each boom location there are as many shear flows meeting each other as there are intersecting skins and webs

• force equilibrium of the top left boom (with F_{boom}=0) gives:

$$q_I L + q_{web} L = q_{II} L \Rightarrow q_I + q_{web} = q_{II} \tag{8.29}$$

• this is equivalent to saying that each cell has its own constant shear flow; these shear flows meet at the webs and add up to create a locally different shear flow

Torsion of multi-cell cross-sections



- for cell I, the torque about point A is

$$T_I = 2A_I q_I \qquad (8.30)$$

note this is torque equivalence and not moment equilibrium

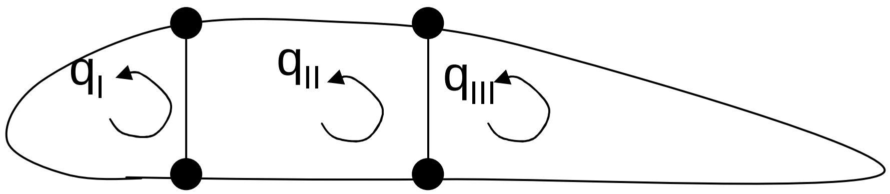
- similarly, for cells II and III, the torques about point B are

$$T_{II} = 2A_{II} q_{II} \qquad (8.31)$$

$$T_{III} = 2A_{III} q_{III} \qquad (8.32)$$

- then the total torque T is given by the sum of the individual torques for each cell

Torsion of multi-cell cross-sections



$T = T_I + T_{II} + T_{III}$

- therefore:

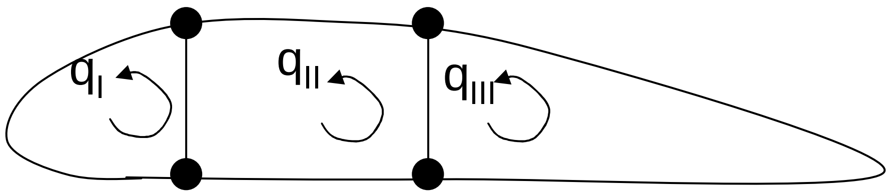
$$T = 2A_I q_I + 2A_{II} q_{II} + 2A_{III} q_{III} \quad (8.33)$$

where A_I, A_{II}, A_{III} are the enclosed areas for cells I, II, and III respectively

- this is one equation for the three unknowns q_I, q_{II}, and q_{III}
- we obtain the remaining two equations from the rate of twist relation
- recall from eq (5.12): $\frac{d\theta}{dz} = \frac{1}{2A}\oint \frac{q_s}{tG} ds$ (5.12)
- this equation can be written separately for each cell:

$$\left(\frac{d\theta}{dz}\right)_I = \frac{1}{2A_I}\oint \frac{q_I ds}{tG}$$ etc.

Torsion of multi-cell cross-sections



$$\frac{d\theta}{dz} = \frac{1}{2A}\oint \frac{q_s}{tG}ds \qquad (5.12)$$

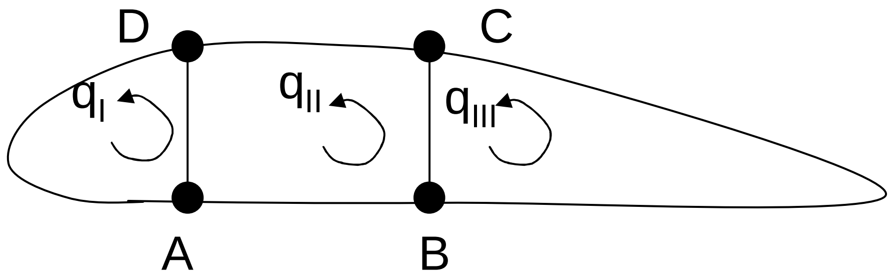
- since each cell is attached to the one before it and the one after it and it does not distort (only rotates), all cells rotate the same amount so they have the same rate of twist:

$$\left(\frac{d\theta}{dz}\right)_I = \left(\frac{d\theta}{dz}\right)_{II} = \left(\frac{d\theta}{dz}\right)_{III} = \frac{d\theta}{dz} \qquad (8.34)$$

- so our problem has four unknowns, q_I, q_{II}, q_{II}, and $d\theta/dz$

- eq (5.12) can be written separately for each of the three cells to obtain three additional equations; these, along with torque equivalence eq (8.33) form a system of four eqs in four unknowns

Torsion of multi-cell cross-sections



$$\frac{d\theta}{dz} = \frac{1}{2A}\oint \frac{q_s}{tG} ds \qquad (5.12)$$

$$T = 2A_I q_I + 2A_{II} q_{II} + 2A_{III} q_{III} \qquad (8.33)$$

• note that different portions (or skins and webs) of each cell may have different thickness t and may be made of different material (thus different G)

• this means the integral in the right hand side of eq (5.12) must be broken into segments according to thickness and material

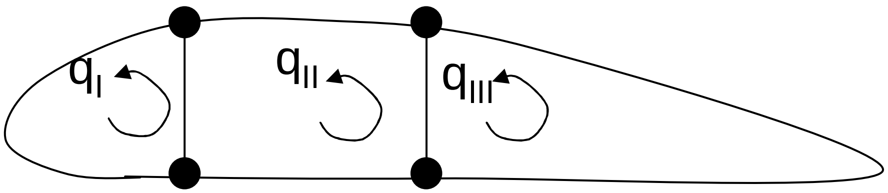
• for example, for cell II in our case, eq (5.12) becomes

$$\left(\frac{d\theta}{dz}\right)_{II} = \frac{1}{2A_{II}}\left[\int_A^B \frac{q_{II}\,ds}{t_{AB}G_{AB}} + \int_B^C \frac{(q_{II} - q_{III})\,ds}{t_{BC}G_{BC}} + \int_C^D \frac{q_{II}\,ds}{t_{CD}G_{CD}} + \int_D^A \frac{(q_{II} - q_{I})\,ds}{t_{AB}G_{AB}}\right] \qquad (8.35)$$

AD not AB!!

watch out for the q values used in each integral!

Torsion of multi-cell cross-sections



$$\left(\frac{d\theta}{dz}\right)_{II} = \frac{1}{2A_{II}}\left[\int_A^B \frac{q_{II}ds}{t_{AB}G_{AB}} + \int_B^C \frac{(q_{II}-q_{III})ds}{t_{BC}G_{BC}} + \int_C^D \frac{q_{II}ds}{t_{CD}G_{CD}} + \int_D^A \frac{(q_{II}-q_I)ds}{t_{AB}G_{AB}}\right] \tag{8.35}$$

• note also that the different G values can be replaced by a combination of a(n arbitrary) reference G value G_{ref} and a shear modulus-weighted thickness t^*:

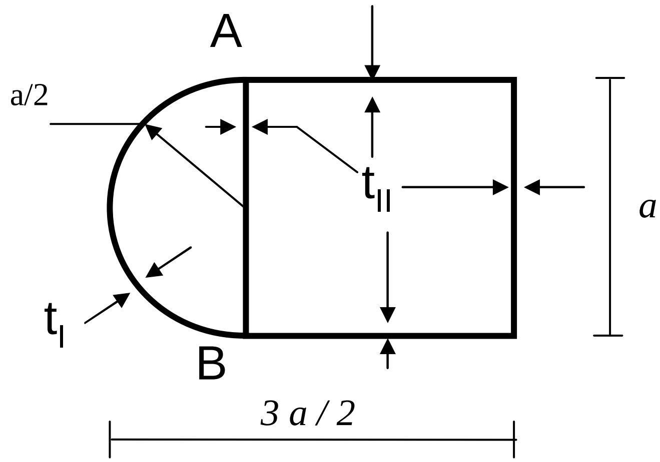
$$\frac{1}{Gt} = \frac{1}{G_{ref}\frac{G}{G_{ref}}t} = \frac{1}{G_{ref}t^*} \tag{8.36}$$

where

$$t^* = \frac{G}{G_{ref}}t \tag{8.37}$$

• this is substituted in the above integrals (with G_{ref}=const outside the integral sign)

Torsion of multi-cell cross-sections - Example



For the 2-cell cross-section shown under torque T, with thickness t_I for the curved portion and t_{II} everywhere else, find the relation between t_I and t_{II} so that she shear flow in the front straight web AB is zero

- first determine the shear flows q_I and q_{II} (the curved cell is I and the square cell is II)
- to determine the shear flows we use:
  - – torque equivalence
  - – equality of rates of twist

Application Session 2