Data Structures 2018
Exercise 2 (Week 38)

- **Notice that based on university's new regulations on degrees, you can have a degree fail from the course which is put to the register.**

  If a student does not participate in the course and does not cancel his/her enrollment, or if he/she discontinues the course, he/she will be assigned a fail grade for the course in question.

- **Students who participate in exercise group must be in place before the exercise group begins (12.15/14.15/16.15). Students who come late do not get the exercise points.**
- **Check the numbers of exercises made before you come to the exercise group. By this means we can save a lot of time when filling the exercise point list.**
- **Remember to registrate for the Data Structures course and to the exercise group at the course webpage (`http://www.sis.uta.fi/~tira/`).**

1. What are the asymptotic upper bounds (big-$\mathcal{O}$) of the following formulas:

   (a) $\sqrt{19n}$
   (b) $21n^2$
   (c) 2^{100000}
   (d) $\log(4n^7)$
   (e) $\log((n \log n^2)/(\log n))$
   (f) $\log(k^n)$
   (g) $7(n^3+1)(n+1)$
   (h) $\log(10n)$
   (i) $11n \log n$

   where k is a positive constant. (hint: brush up on the properties of the logarithm).

2. Sort the following functions into increasing order based on their asymptotic growth. Justify your answer.

   (a) $6n$
   (b) $3 \log n$
   (c) $\sqrt{n} \log n$
   (d) $5n^2$
   (e) $\sqrt{n}$
   (f) 42
   (g) $n!$
   (h) $7 \log \log n$
   (i) $n \log n$

3. Write a Java program which asks a positive integer n from a user (you can assume that the user will always give the right type of input) and prints the values of task 2 functions with given n.

4. In this task we will compare the time complexity of two sorting methods. Implement first bubble sort with Java and then measure how much time it will take to sort the datasets in the files data1.txt, data2.txt, data3.txt

ja data4.txt (present the elapsed time separately for each dataset)? Do the sorting again for each dataset using the sort method from the Arrays class (see Java API for more details) and examine now how much time it will take to sort the datasets. Read the dataset from a file into an array. Print the elapsed time and the number of elements in an array. Measure only the time what sorting will take. Do not take into account, for example, the time what reading the files will take.

5. Let $f(n)$ be a polynomial of degree d. Show that $f(n)$ is $\mathcal{O}(n^d)$.
6. Suppose that $d(n)$ is $\mathcal{O}(f(n))$ and $e(n)$ is $\mathcal{O}(g(n))$. Show that $d(n)e(n)$ is $\mathcal{O}(f(n)g(n))$.
7. Prove that $6n^4 + 12n^3 + n + \sqrt{5n} + 7$ is $\mathcal{O}(n^4)$. Use the definition of $\mathcal{O}$ in your proof.
8. Prove that $n^5 + \sqrt[3]{2}n^4 + n^3 + 5n\sqrt{n} + 2\log\log\log n + 6$ is $\mathcal{O}(n^5)$. Use the definition of $\mathcal{O}$ in your proof.