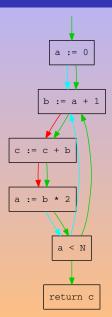
Akim Demaille Étienne Renault Roland Levillain first.last@lrde.epita.fr

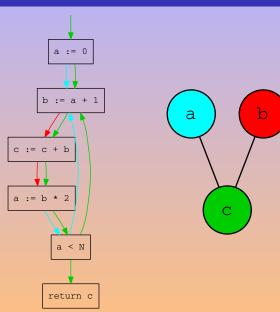
EPITA — École Pour l'Informatique et les Techniques Avancées

May 19, 2018

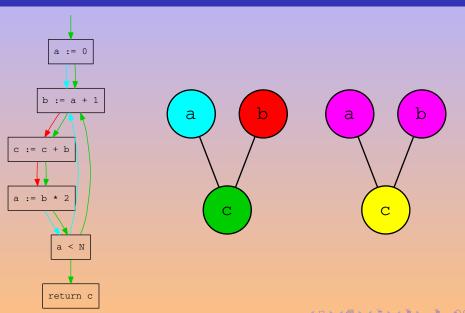
- 1 Interference Graph
- 2 Coloring by Simplification
- 3 Alternatives to Graph Coloring

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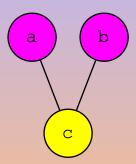


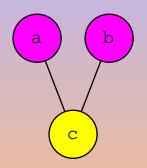


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```
a := 0
L1: b := a + 1
c := c + b
a := b * 2
if a < N goto L1
return c
```





```
r1 := 0
L1: r1 := r1 + 1
r2 := r2 + r1
r1 := r1 * 2
if r1 < N goto L1
return r2
```

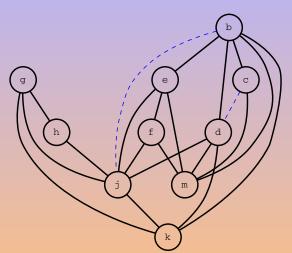
Coloring by Simplification

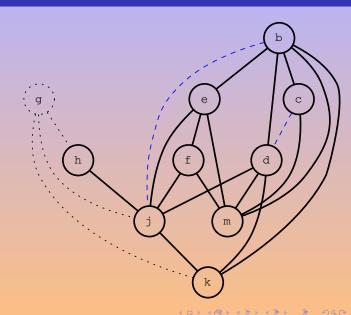
- Interference Graph
- Coloring by Simplification
 - Spilling
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 - Precolored Nodes
 - Implementation
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Interference Graph [Appel, 1998]

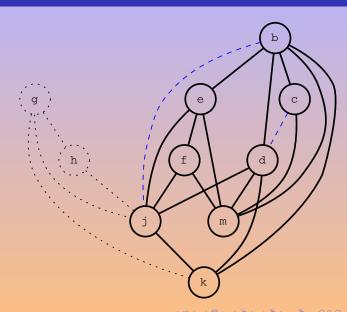
Four registers: r1, r2, r3, r4.

```
live in: k j
  g := [j + 12]
  h := k - 1
  f := g * h
  e := [j + 8]
  m := [j + 16]
  b := \lceil f \rceil
  c := e + 8
  d := c
  k := m + 4
  j := b
live out: d k j
```

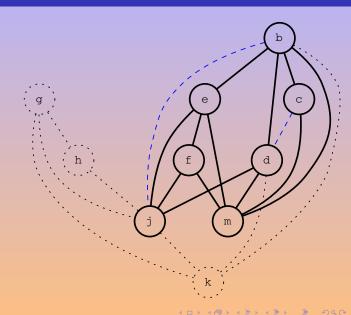




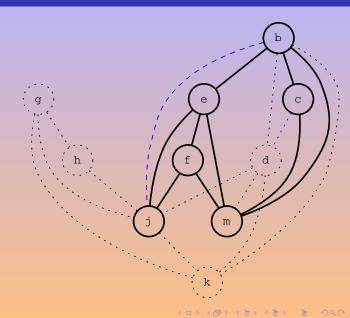
g



h g



k h



d k

g



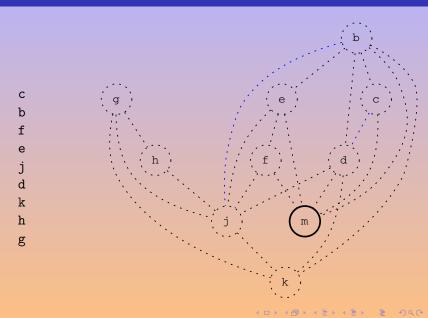
j d k h

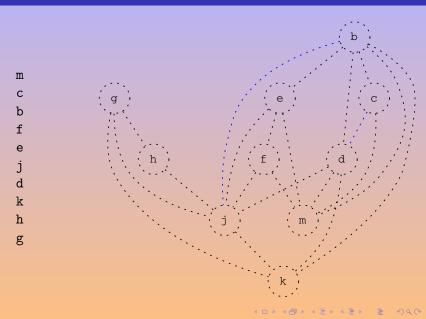


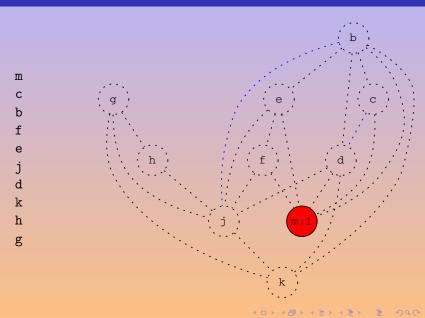
A. Demaille, E. Renault, R. Levillain

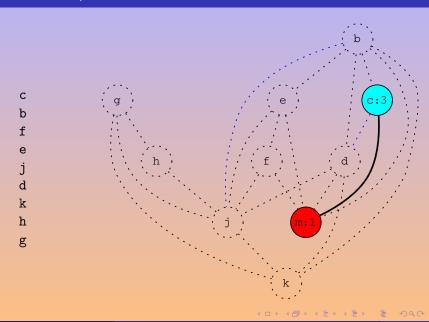


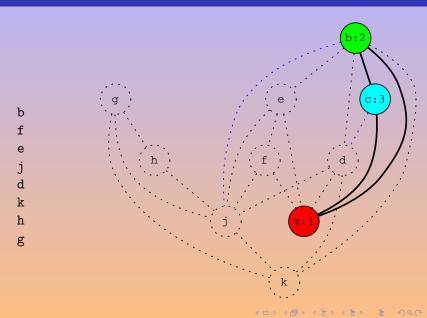


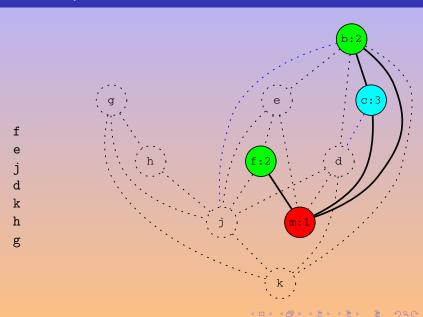


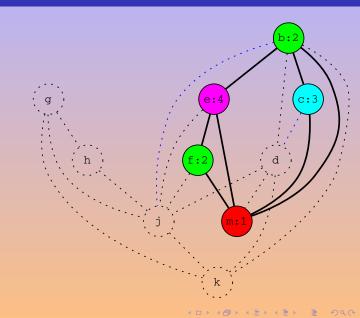






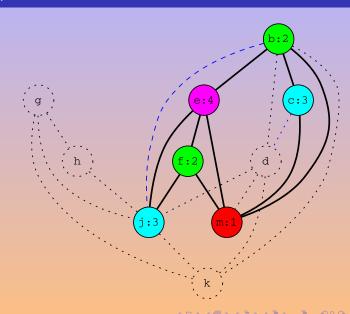




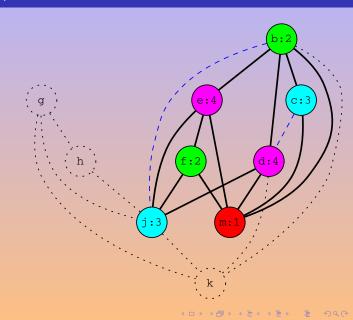


e j d k

g

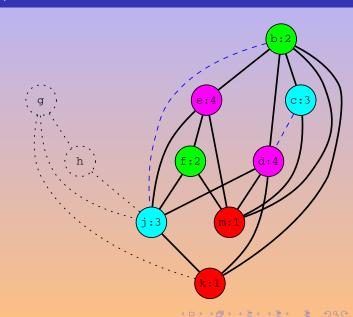


j d k h

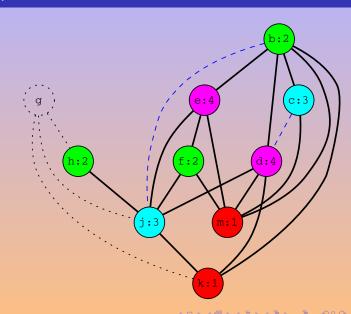


d k

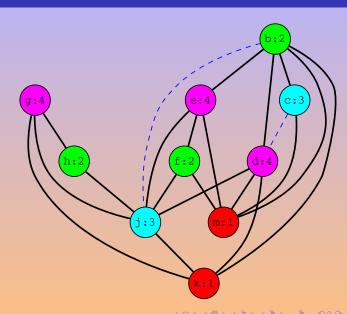
g



k h g



h g

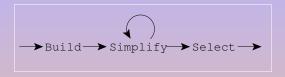


g

Result

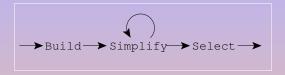
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  m := [j + 16]
  b := [f]
  c := e + 8
  d := c
 k := m + 4
  i := b
live out: d k j
```

```
live in: r1 r3
  r4 := [r3 + 12]
  r2 := r1 - 1
  r2 := r4 * r2
 r4 := [r3 + 8]
  r1 := [r3 + 16]
 r2 := [r2]
  r3 := r4 + 8
 r4 := r3
 r1 := r1 + 4
 r3 := r2
live out: r4 r1 r3
```



build the conflict graph from the program

simplify the nodes with insignificant degree select (or color) while rebuilding the graph.

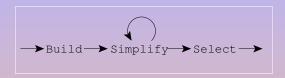


build the conflict graph from the program simplify the nodes with insignificant degree

select (or color) while rebuilding the graph.



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Based on:

A.B. Kempe. On the Geographical problem of the four colors, Am. J. Math 2, 193–200, 1879.

[Appel, 1998, Matz, 2003]

Yes, but What Color? [Matz, 2003]

- Usually, first-fit (registers are ordered).
- Trying caller save first helps.
- Biased Coloring. [Briggs, 1992]
 Use a color already unavailable to our neighbors.

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But for graph coloring, there is no reason for:

- this simple heuristics to always find a solution,
- a solution to always exist...

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Not enough registers

$$t1 := t1 + t2$$

So use the stack

$$[sp + 4] := [sp + 4] + [sp + 8]$$

But use temporaries to do so!

• Why should it solve the problem?

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• But use temporaries to do so!

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Register Allocation with Spills



spill when one cannot simplify, the (uses of the) temporary must be rewritten using the stack.

rebuild but then, the conflict graph is to be rewritten

[Appel, 1998, Matz, 2003]

- The simplification order does not matter
- The spilling order matters
- Spilling decreases the degree of the neighbors
- ... hence it enables additional simplifications
- ...so "first spilled, last served"
- ... therefore: spill cheap temporaries
 - few def/uses
- pay attention to loops

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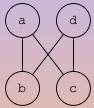
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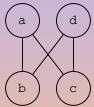
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• We miss many opportunities to avoid the stack



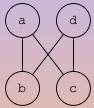
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- then try to color them
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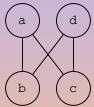
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- Some low-level form of copy propagation
- While building traces we tried to remove jumps
- While allocating registers, we try to remove moves

```
live-in: t2
t1 := ...
t2 := t1 + t2
t3 := t2
t4 := t1 + t3
t2 := t3 + t4
t1 := t2 - t4
```

- Some low-level form of copy propagation
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```
live-in: t2
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t2 := t1 + t2
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```

live-out: t1

- Some low-level form of copy propagation
- While building traces we tried to remove jumps
- While allocating registers, we try to remove moves

```
live-in: t2

t1 := ...

t2 := t1 + t2

t3 := t2

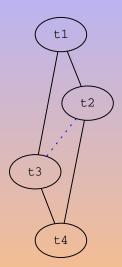
t4 := t1 + t3

t2 := t3 + t4

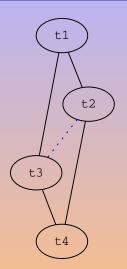
t1 := t2 - t4

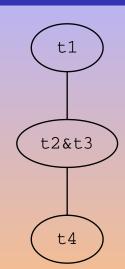
live-out: t1
```

Coalescing Improves the Coloralibility

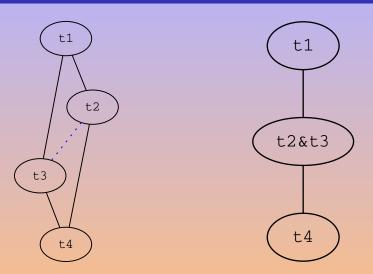


Coalescing Improves the Coloralibility





Coalescing Improves the Coloralibility



t1 and t4 have one neighbor less!

- Conservative Coalescing: don't make it harder.
- Coalesce a and b if

George's criterion is well suited for real registers

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Briggs ab has fewer than k neighbors of significant degree. George every neighbor of a is

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- already interfering with b
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Yes, But Coalesce Who?

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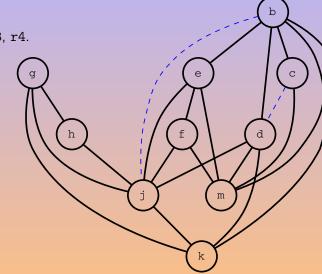
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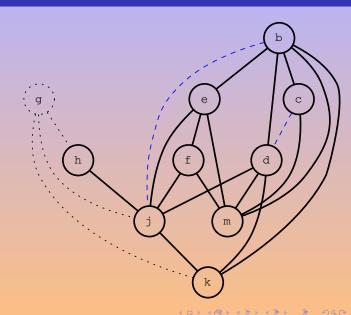
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Interference Graph [Appel, 1998]

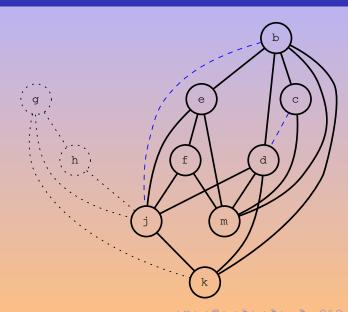
Four registers: r1, r2, r3, r4.

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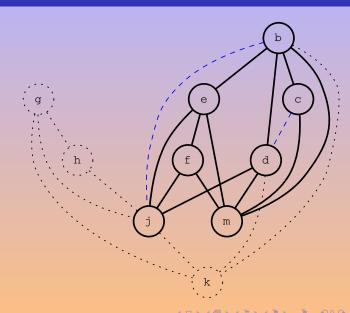




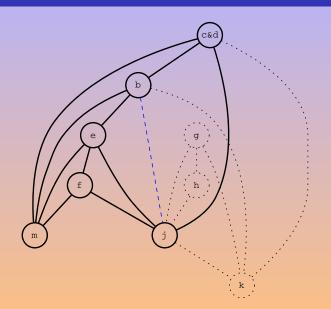
g



h g

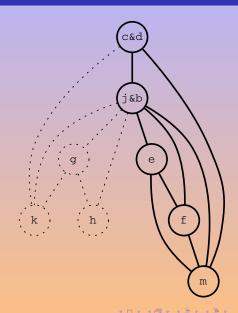


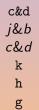
k h

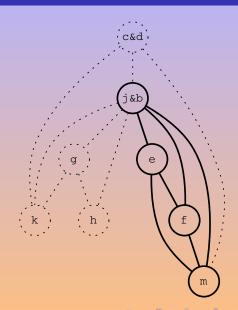


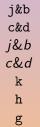
c&d k h

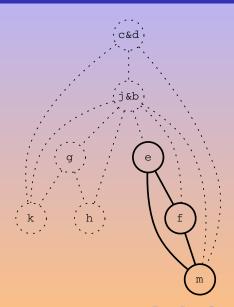


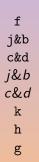


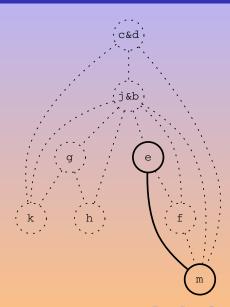


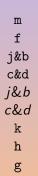


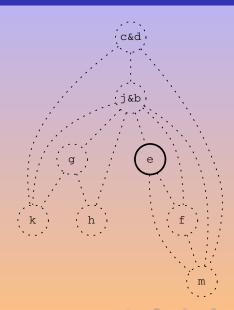


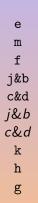


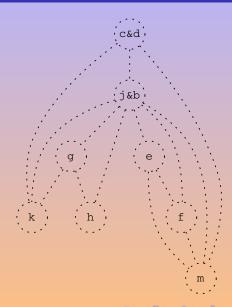


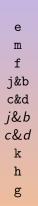


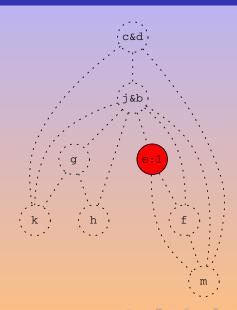


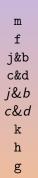


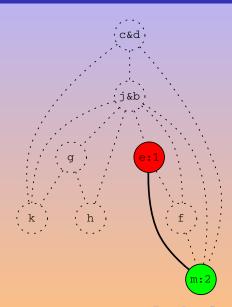


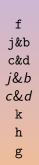


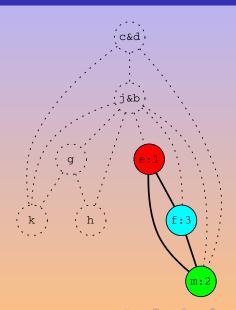


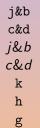


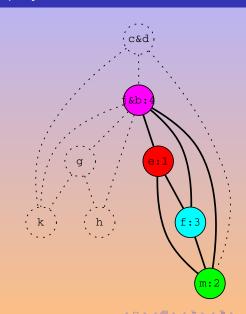


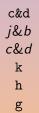


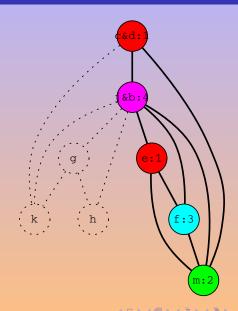


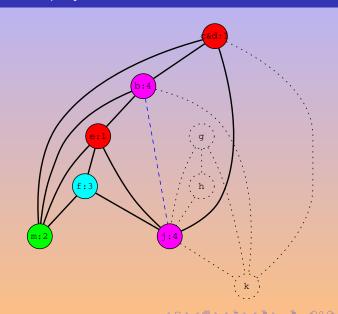




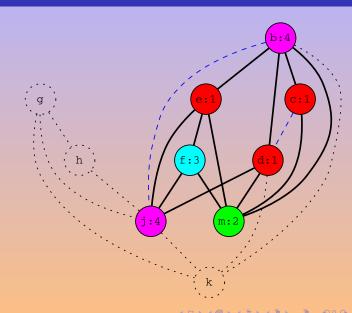




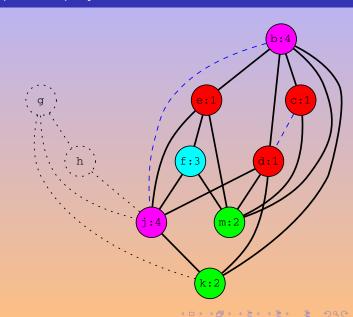




j&b c&d k h

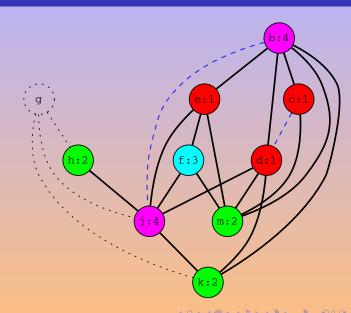


c&d k h

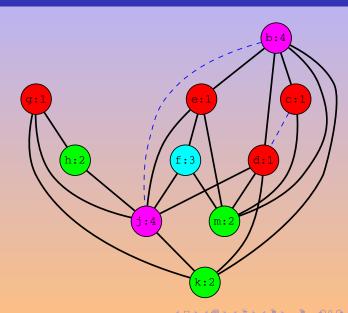


h g

k



h g



g

Interference Graph: Result

```
live in: k j
  g := [j + 12]
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  f := g * h
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  m := [j + 16]
  b := [f]
  c := e + 8
  d := c
 k := m + 4
  j := b
live out: d k j
```

```
live in: r2 r4
  r1 := [r4 + 12]
 r2 := r2 - 1
  r3 := r1 * r2
  r1 := [r4 + 8]
  r2 := [r4 + 16]
 r4 := [r3]
  r1 := r1 + 8
# r1 := r1
r2 := r2 + 4
# r4 := r4
live out: r1 r2 r4
```

Precolored Nodes

- Interference Graph
- Coloring by Simplification
 - Spilling
 - Coalescing
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Some nodes are precolored: the real registers

```
the stack pointer ($sp)
the frame pointer ($fp)
the argument registers ($a0, $a1, etc.)
the return value ($v0, $v1)
the return address ($ra)
etc.
```

- They all interfere with each other
- They cannot be simplified (infinite degree)

- Some nodes are precolored: the real registers
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Conflicts

```
Minimize the conflicts ("pressure") with hard regs. Source and sink.
# Routine: fact
                       # def $s0, $s1...
10:
                       # def: $x11 use: $s0
  move $x11, $s0
                    # def: $x12 use: $s1
  move $x12, $s1
  . . .
16:
  move $s0, $x11
                     # def: $s0 use: $x11
  move $s1, $x12
                       # def: $s1 use: $x12
  . . .
                       # use: $fp, $ra, $sp,
                       # ... $v0. $zero
```

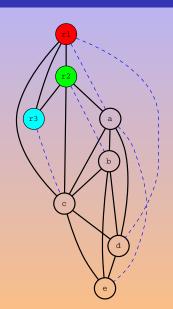
Example [Appel, 1998]

```
int
f (int a, int b)
 int d = 0;
 int e = a;
  do
   d += b;
   --e;
 } while (e > 0);
 return d;
```

```
enter:
  c := r3
  a := r1
  b := r2
  d := 0
  e := a
loop:
  d := d + b
  e := e - 1
  if e > 0 goto loop
  r1 := d
  r3 := c
  return
# liveout: r1, r3
```

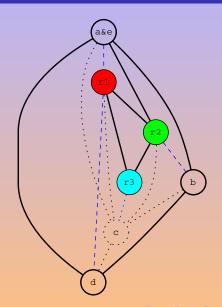
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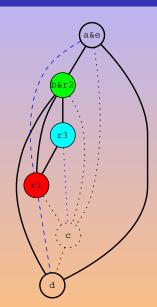


С

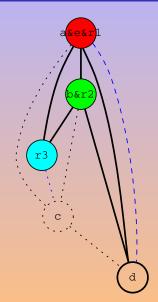




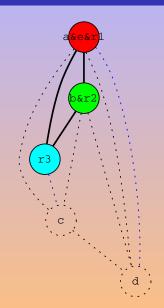
b&*r*2 *a*&*e* c



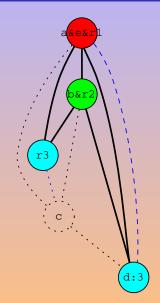




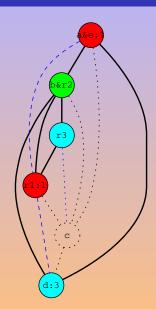
d a&e&r1 b&r2 a&e c



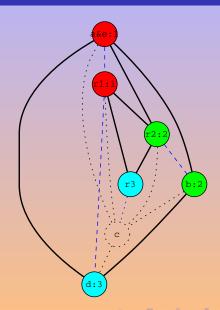
d a&e&r1 b&r2 a&e c



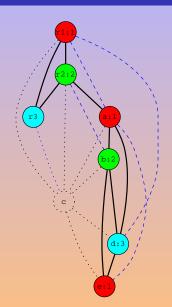
a&e&r1 b&r2 a&e c



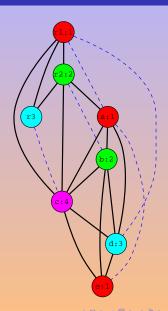












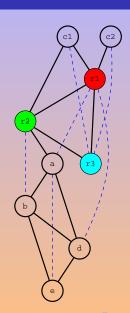
Spilling

```
enter:
  c := r3
  a := r1
  b := r2
  d := 0
  e := a
loop:
  d := d + b
  e := e - 1
  if e > 0 goto loop
  r1 := d
 r3 := c
  return
# liveout: r1, r3
```

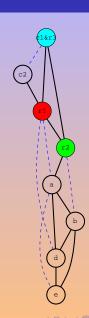
```
enter:
 c1 := r3
  [sp+8] := c1
 a := r1
 b := r2
 d := 0
 e := a
loop:
 d := d + b
 e := e - 1
 if e > 0
    goto loop
 r1 := d
 c2 := [sp+8]
 r3 := c2
 return
# liveout: r1, r3
```

Example

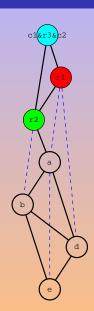
```
enter:
 c1 := r3
  [sp+8] := c1
     := r1
 b := r2
 d := 0
   := a
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 d := d + b
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 if e > 0
    goto loop
 r1 := d
 c2 := [sp+8]
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 return
 liveout: r1, r3
```



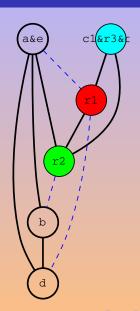
c1&r3



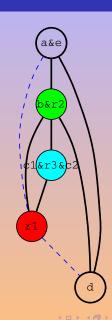
c1&r3&c2 c1&r3



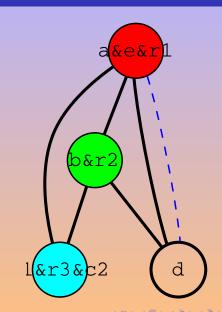
a&e c1&r3&c2 c1&r3



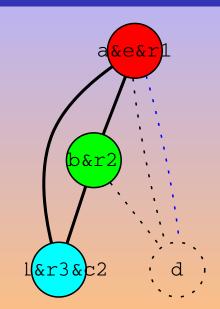
b&r2 a&e c1&r3&c2 c1&r3



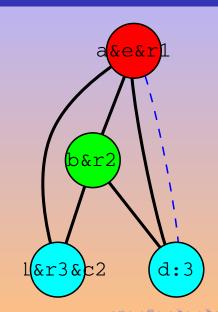
a&e&r1 b&r2 a&e c1&r3&c2 c1&r3



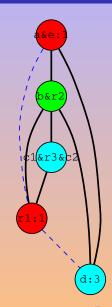
d
a&e&r1
b&r2
a&e
c1&r3&c2
c1&r3



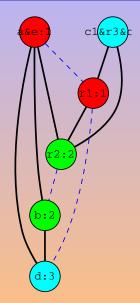
a&e&r1 b&r2 a&e c1&r3&c2 c1&r3



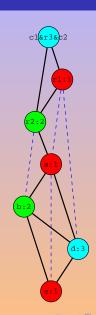
a&e&r1 b&r2 a&e c1&r3&c2 c1&r3



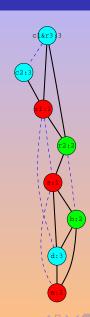
b&r2 a&e c1&r3&c2 c1&r3



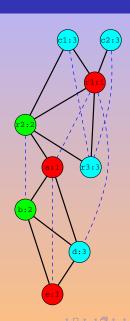
a&e c1&r3&c2 c1&r3



c1&r3&c2 c1&r3



c1&r3



Result

```
enter:
                     enter:
                                        enter:
 c1 := r3
                      r3 := r3
 [sp+8] := c1
                      [sp+8] := r3
                                          [sp+8] := r3
 a := r1
                      r1 := r1
 b := r2
                      r2 := r2
 d := 0
                      r3 := 0
                                         r3 := 0
                      r1 := r1
 e := a
loop:
                    loop:
                                        loop:
 d := d + b
                      r3 := r3 + r2
                                         r3 := r3 + r2
 e := e - 1
                      r1 := r1 - 1
                                         r1 := r1 - 1
 if e > 0
                      if r1 > 0
                                         if r1 > 0
  goto loop
                      goto loop
                                         goto loop
 r1 := d
                      r1 := r3
                                         r1 := r3
 c2 := [sp+8]
                      r3 := [sp+8]
                                         r3 := [sp+8]
 r3 := c2
                      r3 := r3
 return
                      return
                                         return
# liveout: r1, r3 # liveout: r1, r3 # liveout: r1, r3
```

- Interference Graph
- 2 Coloring by Simplification
 - Spilling
 - Coalescing
 - Precolored Nodes
 - Implementation
- 3 Alternatives to Graph Coloring

- Naive implementation is quadratic
- Lower with heavy use of worklists
- Queries on the conflict graph

• For more information, see [Appel, 1998]

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Alternatives to Graph Coloring

- 1 Interference Graph
- 2 Coloring by Simplification
- 3 Alternatives to Graph Coloring

Register Allocation for Trees

```
Can be done during instruction selection with maximal munch
function SimpleAlloc (t)
  for each nontrivial tile u child of t
    SimpleAlloc (u)
  for each nontrivial tile u child of t
    n := n - 1
  n := n + 1
  assign rn to (the root of) t
[Appel, 1998]
```

Bibliography I

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 Modern Compiler Implementation in C, Java, ML.

 Cambridge University Press.
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 Register Allocation via Graph Coloring.
 PhD thesis, Rice University, Houston, Texas.
- Matz, M. (2003).

 Design and Implementation of a Graph Coloring Register Allocator for gcc.

pages 151-169.