Comparing the LMC Complexity of Neural Networks with their Inference Capability

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Outline

Context: The Era of Large Language Models

- **Transformers (2017)**: Revolutionized NLP, enabling massive parallelization.
- **Rapid Adoption**: GPT-3.5 (ChatGPT) became the fastest-growing consumer app.
- **Scaling Laws** [?]:
  - Performance depends strongly on scale (N, D, C) and weakly on shape.
  - Follows **Power Laws**: $L(N) \approx (N_c/N)^\alpha$.
- **The Cost**: Exponential increase in resources (Compute, Data, Parameters) required for constant linear gains in performance.

Problem Statement and Thesis

The Problem:

- Current scaling is resource-intensive and showing signs of diminishing returns.
- Understanding the "learning" process is crucial for architectural improvements.

Work Thesis

"There exists a relationship between model complexity and its inference capability." [**?**]

Objectives:

1. Validate if LMC statistical complexity of weights correlates with inference performance.
2. Analyze the influence of other dimensions: Parameter count, Weight types, and Filtering.

Experimental Setup

Hardware Constraints:

- **RAM**: 512GB DDR4 (Crucial for loading large models).
- **GPU**: NVIDIA Quadro P5000 (16GB) - Insufficient for inference of ¿70B models.
- **CPU**: 2x Intel Xeon Gold 6130 (64 threads).

Implication:

- Models loaded in **Main Memory (CPU)** cast to `float32`.
- Inference not possible locally; reliance on reported benchmarks.

Model Selection Strategy

Source: Hugging Face (Open Weights).

Selection Criteria:

- Transformer-based, Text-only, Base models (no fine-tunes).
- Parameter count < 150 Billion.
- Supported by `AutoModel` utility.

Selected Models (35 Total):

- **Meta**: Llama 2, 3, 3.1, 3.2, 4 (Scout).
- **Google**: Gemma 1, 2, 3, RecurrentGemma.
- **Microsoft**: Phi-1, 1.5, 2, 4 (Mini/Reasoning).
- **OpenAI**: GPT-2 (Small to XL), GPT-OSS (120B, 20B).

LMC Statistical Complexity

Defined by Lopez-Ruiz, Mancini, and Calbet (1995) [?]:

$$C_{LMC} = H \times D$$

1. **Disequilibrium (D)**:
   - Measures distance from uniform distribution (Order).
   - $D = \sum_{i=1}^{n} (p_i - \frac{1}{n})^2$
2. **Shannon Entropy (H)**:
   - Measures uncertainty or randomness.
   - $H = -K \sum_{i=1}^{n} p_i \log p_i$

Interpretation: High complexity requires both structure (high D) and information content (high H).

Data Processing Pipeline

1. **Weight Extraction**: Flatten tensors from `named_parameters()`.
2. **Filtering**: Remove outliers caused by float32 casting.
   - Range: $\mu \pm \sigma_{\text{filter}} \cdot \sigma$.
   - Tested $\sigma_{\text{filter}} \in \{0.125, \ldots, 20, 40(\text{unfiltered})\}$.
3. **Discretization (Histogram)**:
   - **Freedman-Diaconis Rule**: $h = \frac{2 \times IQR}{N^{1/3}}$.
   - Adapts to distribution spread and sample size (N).
   - Crucial for stable probability (p_i) calculation.
4. **Calculation**: Compute C_{LMC} from histogram probabilities.

Inference Capability: Benchmarks

Used as proxies for Test Loss (Performance).

Benchmark	Description
MMLU	57 tasks, STEM/Humanities. Standard for LLMs.
MMLU-Pro	Enhanced MMLU, harder reasoning.
OpenLLM	Aggregated score of multiple datasets.
LMArena	Crowdsourced Elo ratings based on human preference.

Data Collection: Manually aggregated from Hugging Face, Papers, and Leaderboards.

Analysis Dimensions

We constructed a dataset of ≈ 5500 tuples to analyze:

1. **Filtering Setting**: How σ affects bin count and complexity.
2. **Weight Types**:
   - Categories: Bias, Norm, Embedding, Other.
   - Combinations: Power set (15 combinations).
3. **Parameter Count**: Relation to complexity.
4. **Performance**: Correlation between C_{LMC} and Benchmarks.

Statistical Tools:

- Pearson Correlation (r).
- Linear and Free Regression (Curve fitting).
- T-tests for statistical significance ($p < 0.05$).

Extraction Statistics

- **Total Parameters Processed**: 652,802,782,352 ($\approx$ 652 Billion).
- **Compute Time**: 228 hours ($\approx$ 9.5 days).
- **Dataset Size**: 5511 valid data points.

Note: Some models excluded due to exceeding 1 billion histogram bins.

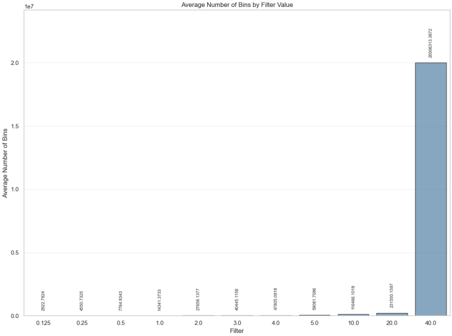
Filter Dimension Analysis

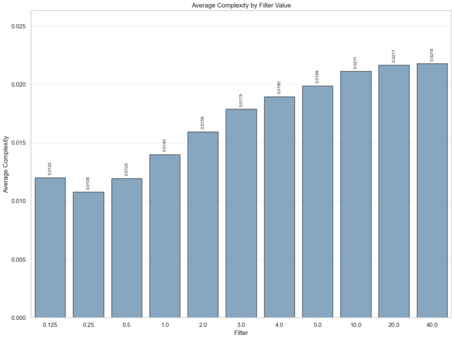
Histogram Bins:

- Follows **Exponential Decay**.
- Max bins explode without filtering.

Complexity:

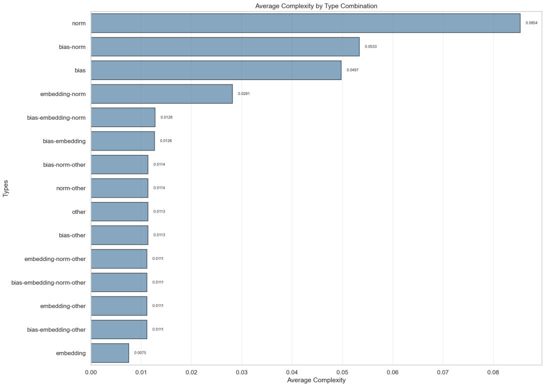
- Follows **Logarithmic Trend**.
- Spike at 0.125 σ (Global Min at 0.25).





Decision: **20** σ chosen (Significant bin reduction, minimal complexity loss).

Weight-Type Analysis



Ranking:

1. **Norm**: Highest complexity.
2. **Bias/Other**: Medium.
3. **Embedding**: Near zero.

Decision: Use **Bias** + **Norm** + **Other** (No Embeddings).

- Embeddings dilute complexity.
- Aligns with Kaplan et al. (2020) methodology.

Complexity vs. Parameter Count

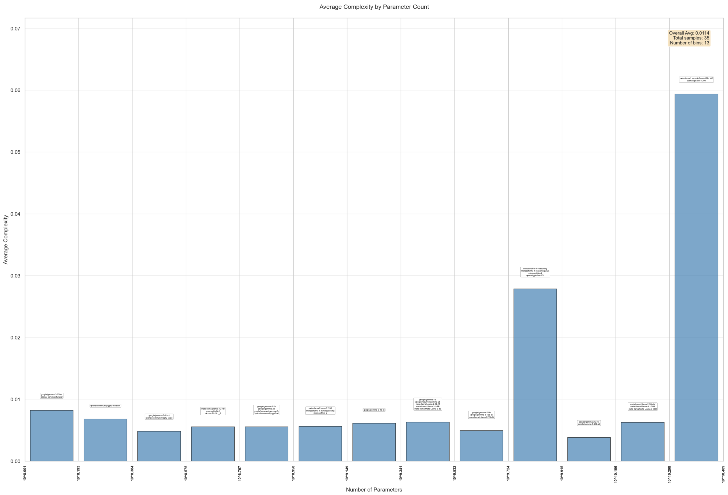
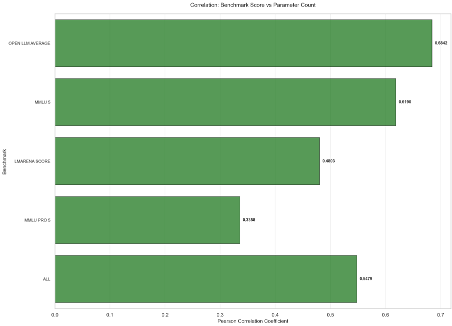


Figure: Average complexity vs number of parameters.

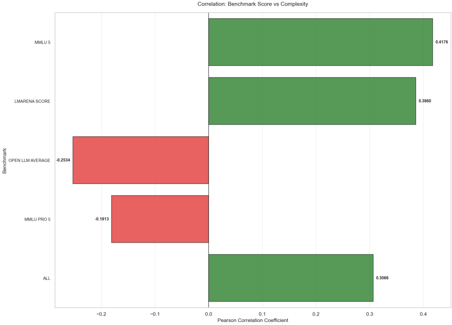
- **Trend**: Mostly flat/stable

Control: Parameters vs. Benchmarks



- **Validation**: All benchmarks show positive correlation with parameter count.
- **Expected**: Confirms scaling laws.
- **Baseline**: R^2 values indicate non-linear relationship.

Complexity vs. Benchmarks: Overview



Inconsistency:

- **Positive**: MMLU, LMArena, All.
- **Negative**: MMLU-Pro, OpenLLM.

Comparison:

- Lower correlations than Control.
- Lower R^2 values.

Statistical Significance (t-test)

Benchmark	r	n	**p-value**	**Sig. (< 0.05)**
LMArena	0.3860	21	0.0839	No
MMLU	**0.4176**	**26**	**0.0338**	**Yes**
MMLU-Pro	-0.1813	15	0.5179	No
OpenLLM	-0.2534	24	0.2322	No
All (Aggregated)	**0.3066**	**86**	**0.0041**	**Yes**

- **MMLU** and **Aggregated (All)** show statistically significant positive correlations.
- Negative correlations (MMLU-Pro, OpenLLM) are **not** significant.

Regression Analysis: MMLU

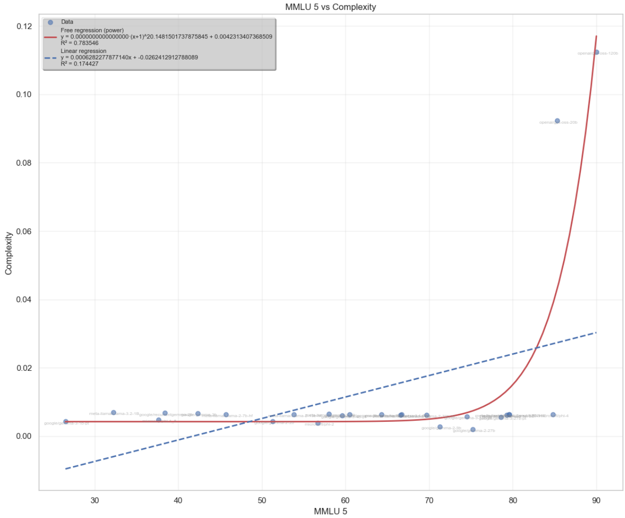


Figure: LMC complexity vs MMLU benchmark.

Regression Analysis: OpenLLM

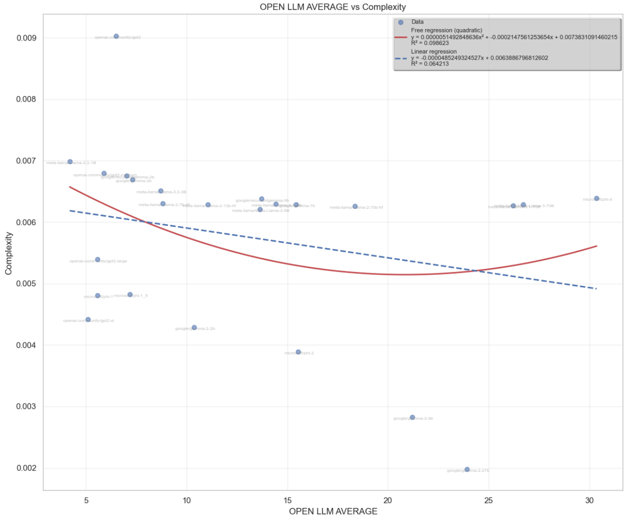


Figure: LMC complexity vs OpenLLM benchmark.

Top 20 Correlations

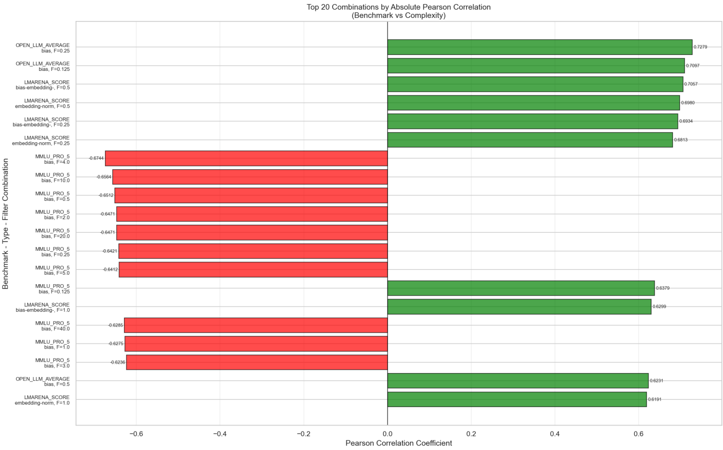


Figure: Top 20 configurations by Pearson correlation.

- Dominated by **High Filtering** (0.25 σ).
- **Bias** weights appear in almost all top configurations.

Conclusion

Main Finding

A general correlation between LMC Complexity and Inference Capability cannot be confirmed.

Evidence For Hypothesis:

- Statistically significant positive correlation in aggregated data ($r \approx 0.31$).
- MMLU shows significant positive correlation ($p < 0.05$).
- Positive correlations are stronger/more significant than negative ones.

Evidence Against Hypothesis:

- Positive trends are heavily driven by outliers (GPT-OSS).
- Inconsistent trends across benchmarks (some negative).
- Low R^2 values compared to parameter count control.

Future Work

1. **Controlled Training**:
   - Train models from scratch.
   - Compare Test Loss vs Complexity directly (removes benchmark noise).
2. **Self-Comparison**:
   - Track complexity evolution of a *single* model during training.
3. **Optimization**:
   - Can maximizing LMC complexity (e.g., of Norm layers) during training improve efficiency or performance?

Thank You!

Questions?