MAT 119A - Homework 8

Due Date: 12/2/22

Problems 1, 2, and 3 will be graded for correctness, and the problems from Strogatz will be graded for completeness.

Problem 1. Use a Lyapunov function of the form $V(x, y) = ax^2 + by^2$ to show that

(a) $(0, 0)$ is an asymptotically stable fixed point of the system

$$\dot{x} = -x^3 + 2xy^2$$
$$\dot{y} = -y^3$$

(b) $(0, 0)$ is an unstable fixed point of the system

$$\dot{x} = x^3 - y^3$$
$$\dot{y} = xy^2 + 4x^2y + 2y^3$$

Problem 2. Determine whether the following systems are Hamiltonian. If they are, find the Hamiltonian.

(a)

$$\dot{x} = 14x + y + x^2 - 2xy + 3y^2$$
$$\dot{y} = x - 14y + x^2 - 2xy + y^2$$

(b)

$$\dot{x} = -x + 4y - y^2$$
$$\dot{y} = x - 4x^2y$$

Problem 3. Consider the system

$$\ddot{x} = x^3 - x^5 \tag{1}$$

(a) Find a conserved quantity for this system.

(b) Rewrite equation (1) as a first order system of equations. Then determine the fixed points and classify the ones at $(1, 0)$ and $(-1, 0)$.

Problems from Strogatz
7.2.10, 7.2.12, 7.3.6b