

1: Task 2: Oh My Magical Math (OHMM...)

(1) Subtask II

Consider the first n even numbers (including zero):

$$0, 2, 4, 6, 8, 10, \dots \quad (1)$$

Consider that the common difference is 2, and comparing it with the sequence $2n$, which is $\{2, 4, 6, 8, 10\}$. We can see that the general pattern is $2n - 2$ or $2(n - 1)$ for the n th term of this sequence. The sum of said sequence can be written like the following:

$$S = 0 + 2 + 4 + 6 + 8 + \dots + 2(n - 1) \quad (2)$$

Writing the same sequence backwards looks like:

$$S = 2(n - 1) + 2(n - 2) + 2(n - 3) + \dots + 2 + 0 \quad (3)$$

Adding these two variations of the sums together gives us:

$$2S = (0 + 2(n - 1)) + (2 + 2(n - 2)) + (4 + 2(n - 3)) + \dots + (2(n - 2) + 2) + (2(n - 1) + 0) \quad (4)$$

This simplifies to:

$$2S = (2n - 2) + (2n - 2) + (2n - 2) + \dots + (2n - 2) + (2n - 2) \quad (5)$$

Considering that there will be n occurrences of $2n - 2$, so:

$$2S = n(2n - 2) \quad (6)$$

$$\therefore S = \frac{n(2n - 2)}{2} \quad (7)$$

$$= \frac{(n)(2)(n - 1)}{2} \quad (8)$$

$$= n(n - 1) \quad (9)$$

Therefore, the sum of the first n even numbers, starting from 0, is $n(n - 1)$.

To prove that it works, say $n = 5$. The first 5 even numbers would then be 0, 2, 4, 6, 8 with the sum of that being $0+2+4+6+8=20$. Putting $n = 5$ in said formula would be $5(5 - 1) = 5(4) = 20$.

(2) Subtask III

Suppose we want to use the formula $n(n - 1)$ instead of our Java function, the function would always take the same amount of steps regardless of what n is. This gives us a time complexity of $O(1)$.