1/10/24 Problem set 2

Collaborators: None

Sources: None, except Lecture Notes

Problem 1

a) Row 2 is already a violation of (P1) so this means it is not in row echelon form

b) Meets both the conditions required for row echelon form and reduced row echelon form.

c) All the conditions are satisfied. This meets both row echelon form + reduced row echelon form.

Problem 2

a) Not in row echelon form because Row 2 is a row of zeroes but it isn't on the bottom.

b) It is in row echelon form but not reduced row echelon form because the rightmost column has more than one non-zero element.

c) Row echelon form AND reduced row echelon form. It meets all the prerequisites.

Problem 3

a) Row echelon form. However, it is not reduced echelon form because two columns with the leading 1 do not meet (P4)

b) Row echelon form. Reduced row echelon form NOT met because column 3 does not meet (P4)

c) Meets all the conditions for row echelon and reduced row echelon form.

Problem 4

a) Not row echelon form as column #2 has two leading ones.

b) Not row echelon form because (P3) is violated. The below leading ones aren't to the right of the upper leading ones.

c) Satisfies both row echelon and reduced row echelon form.

Problem 5

a) Both row echelon and reduced row echelon. Pretty self explanatory.

b) Reduced row echelon form. Meets all the prerequisites.

c) Not reduced row echelon form but is row echelon. (P4) is violated in column #3

Problem Set 6 [RREF]

$$\begin{cases} x + 3y + 4z = 5 \\ 2x - y = 1 \\ 3x + y + 2z = 3 \end{cases}$$

a)
$$\begin{bmatrix} 1 & 3 & 4 & 5 \\ 2 & -1 & 0 & 1 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

b) $R_2 \Rightarrow R_2 + R_1$

1
$$\begin{matrix} 1 & 3 & 4 & 5 \\ 3 & 2 & 4 & 6 \\ 3 & 1 & 2 & 3 \end{matrix}$$

2 $R_2 \Rightarrow R_2 - R_3$
$$\begin{matrix} 1 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 \\ 3 & 1 & 2 & 3 \end{matrix}$$

3 $R_3 \Rightarrow R_3 - 3R_1$
$$\begin{matrix} 1 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & -8 & -10 & -12 \end{matrix}$$

4 $R_3 \Rightarrow R_3 + 8R_2$
$$\begin{matrix} 1 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 6 & 12 \end{matrix}$$

5 $R_3 \Rightarrow \frac{1}{6} R_3$
$$\begin{matrix} 1 & 3 & 4 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{matrix}$$

6 $R_1 = R_1 - 4R_3$
$$\begin{matrix} 1 & 3 & 0 & -3 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{matrix}$$

7 $R_2 = R_2 - 2R_3$
$$\begin{matrix} 1 & 3 & 0 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{matrix}$$

8 $R_1 = R_1 - 3R_2$
$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{matrix}$$

c) $x = 0$

$y = -1$

$z = 2$

Problem 7 [RREF]

$$\begin{aligned} 2x - y - z &= 1 \\ x + 2y - 3z &= 1 \\ -x \quad + z &= 1 \end{aligned}$$

a) $\begin{bmatrix} 2 & -1 & -1 & 1 \\ 1 & 2 & -3 & 1 \\ -1 & 0 & 1 & 1 \end{bmatrix}$

b) $R_2 \Leftrightarrow R_1$

$$\begin{matrix} 1 & 2 & -3 & 1 \\ 2 & -1 & -1 & 1 \\ -1 & 0 & 1 & 1 \end{matrix}$$

$R_2 \Rightarrow R_2 + 2R_3$

$$\begin{matrix} 1 & 2 & -3 & 1 \\ 0 & -1 & 1 & 3 \\ -1 & 0 & 1 & 1 \end{matrix}$$

$R_3 \Rightarrow R_3 + R_1$

$$\begin{matrix} 1 & 2 & -3 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 2 & -2 & 2 \end{matrix}$$

$R_1 = R_1 - R_3$

$$\begin{matrix} 1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 3 \\ 0 & 2 & -2 & 2 \end{matrix}$$

$R_2 = R_2 + R_3$

$$\begin{matrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 5 \\ 0 & 2 & -2 & 2 \end{matrix}$$

$R_3 = R_2 \div 2$ or $\frac{1}{2} R_2$

$$\begin{matrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 5 \\ 0 & 1 & -1 & 1 \end{matrix}$$

$R_3 = R_3 - R_2$

$$\begin{matrix} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & -4 \end{matrix}$$

c) Although it is in RREF we have a contradiction. where $0 = -4$

Therefore this system has <u>no solutions</u>.

Problem 8

$$\begin{cases} w + 2x + 3y - z = 7 \\ 2w - 3x - y - 2z = 0 \\ w + y - z = 3 \\ -w + 3x + 2y + z = 3 \end{cases}$$

a)

$$\begin{bmatrix} 1 & 2 & 3 & -1 & 7 \\ 2 & -3 & -1 & -2 & 0 \\ 1 & 0 & 1 & -2 & 3 \\ -1 & 3 & 2 & 2 & 3 \end{bmatrix}$$

b) $R_2 \Rightarrow R_2 - 2R_3$

$$\begin{matrix} 1 & 2 & 3 & -1 & 7 \\ 0 & -3 & -3 & 2 & -6 \\ 1 & 0 & 1 & -2 & 3 \\ -1 & 3 & 2 & 2 & 3 \end{matrix}$$

$R_3 \Rightarrow R_3 + R_4$

$$\begin{matrix} 1 & 2 & 3 & -1 & 7 \\ 0 & -3 & -3 & 2 & -6 \\ 0 & 3 & 3 & 0 & 6 \\ -1 & 3 & 2 & 2 & 3 \end{matrix}$$

$R_4 = R_4 + R_1$

$$\begin{bmatrix} 1 & 2 & 3 & -1 & 7 \\ 0 & -3 & -3 & 2 & -6 \\ 0 & 3 & 3 & 0 & 6 \\ 0 & 5 & 5 & 1 & 10 \end{bmatrix}$$

$R_4 = R_4 - R_3$

$$\begin{matrix} 1 & 2 & 3 & -1 & 7 \\ 0 & -3 & -3 & 2 & -6 \\ 0 & 3 & 3 & 0 & 6 \\ 0 & 2 & 2 & 1 & 4 \end{matrix}$$

$R_3 = R_3 - R_4$

$$\begin{matrix} 1 & 2 & 3 & -1 & 7 \\ 0 & -3 & -3 & 2 & -6 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 2 & 2 & 1 & 4 \end{matrix}$$

$R_2 \Leftrightarrow R_3$

$$\begin{matrix} 1 & 2 & 3 & -1 & 7 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & -3 & -3 & 2 & -6 \\ 0 & 2 & 2 & 1 & 4 \end{matrix}$$

$R_4 = R_4 - R_2$

$$\begin{matrix} 1 & 2 & 3 & -1 & 7 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & -3 & -3 & 2 & -6 \\ 0 & 1 & 1 & 1 & 2 \end{matrix}$$

$R_3 = R_3 + 3R_4$

$$\begin{matrix} 1 & 2 & 3 & -1 & 7 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 1 & 1 & 1 & 2 \end{matrix}$$

$R_4 = R_4 - R_2$

$$\begin{matrix} 1 & 2 & 3 & -1 & 7 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{matrix}$$

$R_1 = R_1 - 2R_2$

$$\begin{matrix} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{matrix}$$

$R_3 = R_3 \circ \frac{1}{5}$

$$\begin{matrix} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{matrix}$$

$R_4 = R_4 - R_3$

$$\begin{matrix} 1 & 0 & 1 & -1 & 3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

$R_1 = R_1 + R_3$

$$\begin{matrix} 1 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{matrix}$$

c) $w + y = 3$ let $y = t$ since y column has no leading ones

$x + y = 2$

$z = 0$

$\{3-t, \; 2-t, \; t, \; 0\}$

$w + t = 3 \quad w = 3 - t$

$x + t = 2 \quad x = 2 - t$

Problem 9

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 1$$
$$x_3 + x_5 + x_6 = 3$$
$$x_4 + x_6 = 5$$

a) $$\left[\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 0 & 1 & 5 \end{array}\right]$$

b) $R_1 = R_1 - R_2$

$$\begin{array}{ccccccc} 1 & 1 & 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 0 & 1 & 5 \end{array}$$

$R_1 = R_1 - R_3$

$$\begin{array}{ccccccc} 1 & 1 & 0 & 0 & 0 & -1 & -7 \\ 0 & 0 & 1 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 & 0 & 1 & 5 \end{array}$$

Free variables: x_2, x_5, x_6

p, q, r

$x_4 + x_6 = 5 \Rightarrow x_4 + r = 5$

$\boxed{x_4 = 5 - r}$

$x_3 + x_5 + x_6 = 3 \Rightarrow x_3 + q + r = 3$

$\boxed{x_3 = 3 - q - r}$

$x_1 + x_2 - x_6 = -7$

$\Rightarrow x_1 + p - r = -7$

$\boxed{x_1 = -7 + r - p}$

Solution set

$$\{-7+r-p,\ p,\ 3-q-r,\ 5-r,\ q,\ r\}$$