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Linear Algebra A

21/9/24 Problem Set 1

Collaborators: None

Sources consulted: Lecture Note #1

PROBLEM 1

(1) $\forall x \in \mathbb{R},\ x \in \mathbb{R}$

"for every x that is a real number, x is a real number."

Statement is TRUE as the two statements inside are the same.

(2) $\exists y \in \mathbb{R},\ y \notin \mathbb{R}$

"there exists a real number y, where y is not a real number"

Statement is FALSE as the two statements contradict each other.

(3) $\forall z \in \mathbb{R},\ z \in \mathbb{Q}$

"for every real number z, z is also a rational number."

Statement is FALSE as not every real number is rational. For example, π is a real number but isn't rational.

(4) $\exists w \in \mathbb{Q},\ 2w+1 \in \mathbb{Z}$

"there exists a rational number w, where $2w+1$ is an integer"

Statement is TRUE as for example, if $w = \frac{1}{2}$, which is rational, then $2(\frac{1}{2})+1 = 2$ which is an integer.

(5) $\forall (x,y) \in \mathbb{R}^2,\ 0 \cdot x = 0 \cdot y$

"For an ordered pair of real numbers (x,y), $0 \cdot x$ is equivalent to $0 \cdot y$"

This statement is TRUE as any number multiplied by 0 is 0.

(6) $\forall (x,y) \in \mathbb{R}^2,\ x+y=0$

"For an ordered pair of real numbers (x,y), $x+y$ is 0."

This statement is TRUE as long as x and y are additive inverses $(x=-y)$ so e.g $(2,-2)$, $(\pi,-\pi)$, etc.

Problem 2

①	a_1	a_2	3	a_3+1
②	2	7	a_4-1	a_1+a_2

③	a_1+a_2	a_3+a_4
④	$a_3+a_4+a_5$	$a_4+a_5+a_6+6$

From this information,

$\boxed{a_1=2}$

$\boxed{a_2=7}$

$a_3+1=a_1+a_2=2+7=9 \rightarrow a_3+1=9 \therefore \boxed{a_3=8}$

$a_4-1=3 \rightarrow \boxed{a_4=4}$

$a_1+a_2=a_3+a_4+a_5 \rightarrow 2+7=8+4+a_5$

$9=12+a_5 \rightarrow \boxed{a_5=-3}$

$a_3+a_4=a_4+a_5+a_6+6 \rightarrow 8+4=4-3+a_6+6$

$12=7+a_6 \rightarrow \boxed{a_6=5}$

(a) $a_1x+y-z=a_1+a_2$

$\hookrightarrow 2x+y-z=9$

(b) $-x+a_2y=a_2-a_4+a_5$

$\hookrightarrow -x+7y=7-4+(-3)=0$

(c) $a_3y-2z=-a_1+a_2-a_6$

$8y-2z=-2+7-5=0$

(d) $x+a_4z=a_2-a_3+a_4+a_5$

$x+4z=7-8+4-3=0$

Therefore, 3 of these linear equations are homogeneous.

Problem 3

$$\begin{cases} 2x - 3y = -1, \\ 6y + 5z = 11, \\ 4x + 5z = 9; \end{cases}$$

(a) $(1, 1, 1)$

$2(1) - 3(1) = -1$ ✓

$6(1) + 5(1) = 11$ ✓

$4(1) + 5(1) = 9$ ✓ $\therefore$ It is a solution

(b) $(4, 3, 2)$

$2(4) - 3(3) = 8 - 9 = -1$ ✓

$6(3) + 5(2) = 18 + 10 = 28$ ✗

$4(4) + 5(2) = 16 + 10 = 26$ ✗ $\therefore$ It is not a solution

(c) $(10, 7, -31)$

$2(10) - 3(7) = 20 - 21 = -1$ ✓

$6(7) + 5(-31) = 54 + 155 = -101$ ✗

$4(10) + 5(-31) = 40 - 155 = -115$ ✗ $\therefore$ not a solution

(d) $(-4, t, 5)$ is a solution

$\to 2(-4) - 3t = -1 \to -8 - 3t = -1$

$\therefore -7 - 3t = 0$

$3t = -7 \quad \therefore t = \frac{-7}{3}$

double check: $6t + 5(5) = 11 \to 6t + 25 = 11$

$6t = 11 - 25 = -14$

$t = \frac{-14}{6} = \frac{-7}{3}$

$\therefore \boxed{t = \frac{-7}{3}}$

Problem 4

$$\begin{cases} \frac{x}{2} + \frac{y}{3} = -1 \\ -6x - 4y = 12 \end{cases}$$

(a) $(0,-3)$

$$\frac{0}{2} + \frac{-3}{3} = 0 - 1 = -1 \checkmark$$

$$-6(0) - 4(-3) = 0 - (-12) = 12 \checkmark$$ $\therefore$ is a solution

(b) $(-2, 0)$

$$\frac{-2}{2} + \frac{0}{3} = -1 + 0 = -1 \checkmark$$

$$-6(-2) - 4(0) = 12 - 0 = 12 \checkmark$$ $\therefore$ is a solution

(c) Judging from how there are two solutions, we can assume that there are infinite sets of solutions for this linear system.

Problem 5

$$\begin{cases} -x+y+z = 6 \\ x-y+z = 4 \end{cases}$$

(a) Determine all $t \in \mathbb{R}$ where (t,t,t) is a solution

$\cancel{-t+t}+t = 6 \longrightarrow t=6$

$\cancel{t-t}+t = 4 \longrightarrow t=4$

Two different values of t, therefore there are no real numbers that satisfy the linear system.

(b) $(t-1, t, t+1)$

$-(t-1)+t+(t+1) = 6$

$\hookrightarrow \cancel{-t}+1\cancel{+t}+t+1 = 6 \longrightarrow t+2=6, \quad t=4$

$(t-1)-t+t+1 = 4$

$\hookrightarrow t\cancel{-1}\cancel{-t+t}\cancel{+1} \longrightarrow t=4$

Therefore, since $t=4$, the solution set will be $(3,4,5)$ so 4 is the only applicable real number.

(c) $(t, t+1, 5)$

$\cancel{-t+t}+1+5 = 6$

$6=6$

$t-(t+1)+5 = 4$ cancel out

$\cancel{t-t}-1+5 = 4$

$4=4$

This means that every real number is an approporiate solution for this triple. $(t \in \mathbb{R})$