ICCS208: Assignment 6

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1: Task 1: Mathematical Truth

"Every binary tree on n nodes where each has either zero or two children has precisely $\frac{n+1}{2}$ leaves."

(1) Base case

$$P(1) = \frac{1+1}{2} = 1 \tag{1}$$

A binary tree with only one node, which therefore has no subtrees, has one leaf, which is also the root. Therefore, this base case holds true.

(2) Inductive step

First and foremost, if a tree on k nodes has zero children, then we can deduce that it will have $\frac{k+1}{2}$ leaves since we don't have to worry about children.

Now suppose we have two children. Suppose a k node binary tree has $\frac{k+1}{2}$ leaves (per IH). Let's assume that this k-node tree is part of a larger tree, with n nodes, such that the total amount of nodes in this tree will be $1 + k + (n - k - 1) = n$ nodes.

- Per strong induction, we know that since $k < n$ and $(n-k-1) < n$ it means that $k \rightarrow \frac{k+1}{2}$ leaves and $(n-k-1) \rightarrow \frac{n-k}{2}$ leaves.
- If we sum these together, we can get the total number of leaves:

$$Total = \frac{k+1}{2} + \frac{n-k}{2} = \frac{k+1+n-k}{2} == \frac{n+1}{2} \tag{2}$$

This aligns with our initial statement, therefore we've proven it.