Design description of the model

Rev 0

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1 Vessel model

$$\begin{aligned}\dot{\boldsymbol{\eta}} &= \boldsymbol{R}(\psi)\boldsymbol{\nu} \\ (\boldsymbol{M}_{RB} + \boldsymbol{M}_A)\dot{\boldsymbol{\nu}}_r + \boldsymbol{D}\boldsymbol{\nu}_r|\boldsymbol{\nu_r}| &= \boldsymbol{\tau}_{thr} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{ext}\end{aligned} \tag{1}$$

The matrix $\boldsymbol{M}_{RB}$ can be defined as

$$\boldsymbol{M}_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix}, \tag{2}$$

where m is the displacement, i.e. the mass of the displaced fluid, or the mass of the vessel, and I_z is the moment of inertia about the z_b-axis. The elements that are not on the diagonal of the matrix are ignored.

The added-mass matrix, $\boldsymbol{M}_A$, is calculated in the origin of the coordinate system, CO. This matrix can be written as

$$\boldsymbol{M}_A = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & 0 \\ 0 & 0 & -N_{\dot{r}} \end{bmatrix}, \tag{3}$$

in SNAME notation. $X_{\dot{u}}$ is added mass in surge, $Y_{\dot{v}}$ is added mass in sway og $N_{\dot{r}}$ is added mass ini yaw. The elements that are not on the diagonal of the matrix are ignored.

The matrix $\boldsymbol{D}$ can, in SNAME notation, be written as

$$\boldsymbol{D} = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & 0 \\ 0 & 0 & -N_r \end{bmatrix}, \tag{4}$$

where X_u is drag in surge, Y_v is drag in sway and N_r is drag in yaw. The elements that are not on the diagonal of the matrix are ignored.

$\boldsymbol{\tau}_{thr} = [\tau_{thr,X}, \tau_{thr,Y}, \tau_{thr,N}]^\top$ are forces from thrusters in surge, sway and yaw. $\boldsymbol{\tau}_{wind} = [\tau_{wind,X}, \tau_{wind,Y}, \tau_{wind,N}]^\top$ and $\boldsymbol{\tau}_{ext} = [\tau_{ext,X}, \tau_{ext,Y}, \tau_{ext,N}]^\top$ are wind forces and external forces (pipe, winch, etc.) that affects the vessel.

Rotation from vessel coordinates (BODY) to Earth coordinates (NED) can be done with a rotation matrix, $\boldsymbol{R}(\psi)$. For three degrees of freedom, this can be written as

$$\boldsymbol{R}(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{5}$$

where ψ is the heading of the vessel. $\boldsymbol{\eta} = [N, E, \psi]^\top$ is the position in North, East and heading. $\boldsymbol{\nu} = [u, v, r]^\top$ is velocity in surge, sway and yaw. $\boldsymbol{\nu}_c = [u_c, v_c, r_c]^\top$ is current velocity in surge, sway and yaw. $\boldsymbol{\nu}_r = \boldsymbol{\nu} - \boldsymbol{\nu}_c$is then relative vessel velocity.

2 Thruster model

The force that a single thruster can give can be written as

$$T = K_T \rho D^4 n^2, \tag{6}$$

where T is the thruster force, ρ is the density of water, D is the diameter of the propeller and n is the rpm of the propeller. K_T is an empirical value that is dependent on water speed into the propeller and the pitch angle of the propeller. A simplified version can be written as

$$K_T = K \cdot \theta^\alpha, \tag{7}$$

where K is a constant, θ is pitch angle normalized to $0 \rightarrow 1$ ($0\% \rightarrow 100\%$) and α is a constant. Because K, ρ og D are constants they can be merged together into one constant. n can also be normalized such that it's between 0 and 1: $n_n = K_n \cdot n$. The final expression for T is then

$$\begin{aligned} T &= K_T \rho D^4 n^2 \\ &= K\theta^\alpha D^4 (K_n n_n)^2 \\ &= K\theta^\alpha D^4 K_n^2 n_n^2 \\ &= T_K \cdot \theta^\alpha n_n^2, \end{aligned} \tag{8}$$

where $T_K = KD^4K_n^2$.