

DESIGN DESCRIPTION OF THE MODEL

REV 0.1.0

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1 Vessel model

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \mathbf{R}(\psi)\boldsymbol{\nu} \\ (\mathbf{M}_{RB} + \mathbf{M}_A)\dot{\boldsymbol{\nu}} + \mathbf{D}\boldsymbol{\nu} &= \boldsymbol{\tau}_{thr} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{current} + \boldsymbol{\tau}_{ext} \end{aligned} \quad (1)$$

The matrix \mathbf{M}_{RB} can be defined as

$$\mathbf{M}_{RB} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_z \end{bmatrix}, \quad (2)$$

where m is the displacement, i.e. the mass of the displaced fluid, or the mass of the vessel, and I_z is the moment of inertia about the z_b -axis. The elements that are not on the diagonal of the matrix are ignored.

The added-mass matrix, \mathbf{M}_A , is calculated in the origin of the coordinate system, CO. This matrix can be written as

$$\mathbf{M}_A = \begin{bmatrix} -X_{\dot{u}} & 0 & 0 \\ 0 & -Y_{\dot{v}} & 0 \\ 0 & 0 & -N_{\dot{r}} \end{bmatrix}, \quad (3)$$

in SNAME notation. $X_{\dot{u}}$ is added mass in surge, $Y_{\dot{v}}$ is added mass in sway og $N_{\dot{r}}$ is added mass in yaw. The elements that are not on the diagonal of the matrix are ignored.

The matrix \mathbf{D} can, in SNAME notation, be written as

$$\mathbf{D} = \begin{bmatrix} -X_u & 0 & 0 \\ 0 & -Y_v & 0 \\ 0 & 0 & -N_r \end{bmatrix}, \quad (4)$$

where X_u is drag in surge, Y_v is drag in sway and N_r is drag in yaw. The elements that are not on the diagonal of the matrix are ignored.

$\boldsymbol{\tau}_{thr} = [\tau_{thr,X}, \tau_{thr,Y}, \tau_{thr,N}]^\top$ are forces from thrusters in surge, sway and yaw. $\boldsymbol{\tau}_{wind} = [\tau_{wind,X}, \tau_{wind,Y}, \tau_{wind,N}]^\top$ and $\boldsymbol{\tau}_{ext} = [\tau_{ext,X}, \tau_{ext,Y}, \tau_{ext,N}]^\top$ are wind forces and external forces (pipe, winch, etc.) that affects the vessel.

Rotation from vessel coordinates (BODY) to Earth coordinates (NED) can be done with a rotation matrix, $\mathbf{R}(\psi)$. For three degrees of freedom, this can be written as

$$\mathbf{R}(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (5)$$

where ψ is the heading of the vessel. $\boldsymbol{\eta} = [N, E, \psi]^\top$ is the position in North, East and heading. $\boldsymbol{\nu} = [u, v, r]^\top$ is velocity in surge, sway and yaw.

2 Thruster model

The force that a single thruster can use can be written as

$$T = K_T \rho D^4 n^2, \quad (6)$$

where T is the thruster force, ρ is the density of water, D is the diameter of the propeller and n is the rpm of the propeller. K_T is an empirical value that is dependent on water speed into the propeller and the pitch angle of the propeller. A simplified version can be written as

$$K_T = K \cdot \theta^\alpha, \quad (7)$$

where K is a constant, θ is pitch angle normalized to $0 \rightarrow 1$ ($0\% \rightarrow 100\%$) and α is a constant. Because K , ρ og D are constants they can be merged together into one constant. n can also be normalized such that it's between 0 and 1: $n_n = K_n \cdot n$. The final expression for T is then

$$\begin{aligned} T &= K_T \rho D^4 n^2 \\ &= K \theta^\alpha D^4 (K_n n_n)^2 \\ &= K \theta^\alpha D^4 K_n^2 n_n^2 \\ &= T_K \cdot \theta^\alpha n_n^2, \end{aligned} \quad (8)$$

where $T_K = K D^4 K_n^2$. This model is suitable for bollard pull condition, i.e. zero water speed through the propeller other than the self-induced speed.

3 Wind model

τ_{wind} are wind forces that influence the vessel. The forces can be written as:

$$\tau_{wind} = \begin{bmatrix} q \cdot C_X \cdot A_f \\ q \cdot C_Y \cdot A_l \\ q \cdot C_N \cdot A_f \cdot Loa \end{bmatrix}, \quad (9)$$

where C_X , C_Y og C_N are wind coefficients (drag coefficients) in surge, sway og yaw, A_f is projected frontal area and A_l is projected lateral area. $q = \frac{1}{2} \cdot \rho_{air} \cdot V_{w,r}^2$, where ρ_{air} is the density of air and V_r is relative wind velocity. C_X , C_Y og C_N are typically found with empirical methods such as Blendermann. These are functions of relative wind velocity.

4 Current model