Tutorial 4: Zero-Sum Games

1. Consider the following zero sum game:

	L	R
T	2	-3
B	-1	1

Suppose $(p, q) \in (0, 1)^2$ is a fully mixed strategy profile for the game. Let:

- $\pi_1(p, \sigma_2)$ denote the expected utility that player 1 would obtain by playing mixed strategy p against player 2's pure strategy σ_2

- $\pi_1(\sigma_1, q)$ denote the expected utility that player 1 would obtain by playing the pure strategy σ_1 against player 2's mixed strategy q.

(a) Write down expressions for $\pi_1(p, L)$, $\pi_1(p, R)$, $\pi_1(T, q)$, and $\pi_1(B, q)$ in terms of p and q.

(b) Plot the graph of the function $\pi_1(p, L)$ for values of p from 0 to 1 inclusive. On the same graph, plot the function $\pi_1(p, R)$ for values of p from 0 to 1 inclusive.

With reference to this graph, identify (e.g., emphasising with a boldface line) the minimum of these two functions for values of p from 0 to 1, and clearly identify the point at which the minimum of these functions takes its maximum value.

(c) On a new graph, plot the function $\pi_1(T, q)$ for values of q from 0 to 1 inclusive, and on the same graph, plot the function $\pi_1(B, q)$ for values of q from 0 to 1 inclusive.

With reference to this graph, identify (e.g., emphasising with a boldface line) the maximum of these two functions for all values of q from 0 to 1, and clearly identify the point at which the maximum of these functions takes its minimum value.

(d) Explain the significance of the two graphs and the points you have highlighted, with respect to the analysis of zero sum games. In your answer, clearly identify the optimal strategies for both players and the equilibrium outcome, and give the value of the game.