Helicopter Design

MSAN 631 - Design of Experiments

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Abstract

Paper helicopter design has long been viewed simply as an art, and as a result is often looked down upon by snobby scientific elitists. Thus, it has lacked the sufficient quantitative inquiry needed to find an optimal design. In this report, we propose, design, and implement multiple construction methods for paper helicopters, rigorously analyzing each approach using statistical techniques. We then identify the best construction method and justify this choice quantitatively.

Approach

The PROBLEM at hand is finding the ideal design that maximizes *flight time* of a paper helicopter, where *flight time* is defined as the number of seconds elapsed between release and first contact with ground. In reality, there are infinitely many possible designs, so we restrict our experiment to the specific design shown in Figure 1. This design was chosen as it embodies the latest advances in paper helicopter aerodynamics, spearheaded by the Stevens Institute at the University of San Francisco.

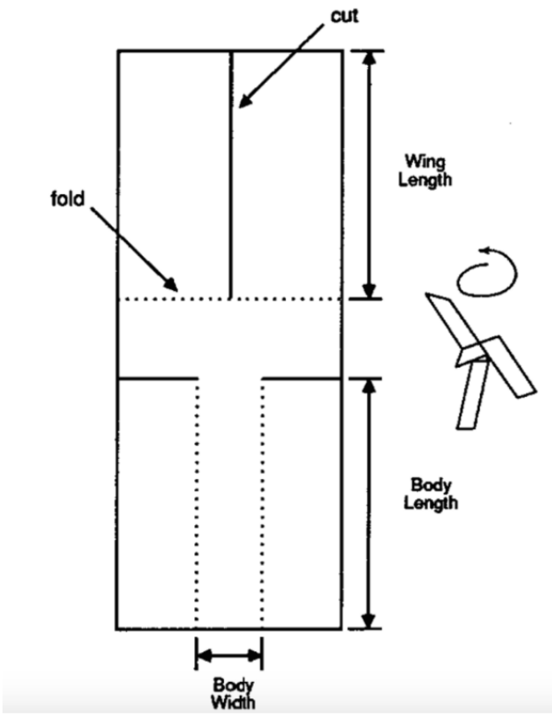

Figure 1: Helicopter Design Used in Experiment

Some things to consider in designing the experiment are controllable and uncontrollable factors. A list of controllable factors include

- paper thickness, paper size (length × width)
- paper type (some materials may be more or less prone to air friction)
- wing length, body length, body width
- drop height

A list of uncontrollable factors include

- wind speed, potential mid-air collisions (bugs!)
- drop style (angle, initial velocity)
- folding precision
- stopwatch accuracy

The experimental units are the helicopters themselves, so there is no need for randomization in this aspect. By design, any helicopter characteristics relevant to the experiment are factors that can be controlled. In particular, we are only interested in wing length, body length, and body width; these will be the design factors. The first three nuisance factors, paper thickness, paper size, and paper type, can be eliminated by consistently using standard printer paper. Drop height can be accounted for by using a consistent height: the distance from the floor of the agora to the ledge of the barrier on the second level.

The effects of our uncontrollable factors can be mitigated by assigning group members to specific jobs and paying careful attention. Designating one group member as dropper, one as builder, and another as timekeeper will ensure consistency with respect to drop style, folding precision, and stopwatch accuracy. The effects of wind speed and potential mid-air collisions can be minimized by dropping the helicopters indoor from a consistent position. The hope is that any remaining variation not controlled for by this procedural design are negligible, or will be adequately randomized through replication.

Power analysis has not been considered for this experiment, so we have not specified an ideal number of trials needed for each treatment. Using intuition, however, an amount of five trials per treatment has been chosen, under the assumption that it will provide an adequate estimate of flight time.

The PLAN is to parameterize the design with the following factors:

- wing length
- body length
- body width

where the first three are defined as shown in Figure 1. Our response/measurement of interest will be the *flight time*.

Four treatments will be used. The levels that will be used across these treatments are shown in Table 1.

	Wing Length	Body Length	Body Width
Treatment 1	Long	Short	Wide
Treatment 2	Short	Long	Wide
Treatment 3	Short	Long	Thin
Treatment 4	Long	Short	Thin

Table 1: Factors Across Treatments

where

- wide body width = $1/2$ paper width, thin body width = $1/4$ paper width
- short body length = $3/8$ paper length, long body length = $1/2$ paper length
- short wing length = $3/8$ paper length, long wing length = $1/2$ paper length

We will limit the scope of our experiment to paper helicopter designs that utilize 4.25" × 11" of standard printer paper, which is part of our blocking scheme described above.

Procedure

The experiment followed the design principles outlined above and was conducted in the following manner:

- Helicopter was dropped from second floor of agora, at shoulder height of dropper
- Time elapsed was measured between moment of release and moment of ground impact

Each treatment underwent 5 trials.

Results and Conclusion

The data collected from the experiment is shown in Table 2.

	Sample Mean	Sample Variance	Sample Min	Sample Max
Treatment 1	2.224	0.057	1.87	2.5
Treatment 2	2.068	0.332	1.31	2.88
Treatment 3	2.222	0.066	2.00	2.65
Treatment 4	3.11	0.049	2.88	3.38

Table 2: Results Across Treatments (seconds)

In this context, it makes sense to compare the means of each treatment in order to ANALYZE which treatment results in longer flight time. We could attempt to compare all combinations of means, but this would result in an unreasonable number of hypotheses. Instead, we note that treatment 4 yields the largest sample mean, and so we will limit our tests to the hypotheses that are related to this treatment. Specifically, we test the following:

- $H_0 : \mu_4 \leq \mu_1$; $H_A : \mu_4 > \mu_1$
- $H_0 : \mu_4 \leq \mu_2$; $H_A : \mu_4 > \mu_2$
- $H_0 : \mu_4 \leq \mu_3$; $H_A : \mu_4 > \mu_3$

We will assume $\sigma_1 \neq \sigma_2 \neq \sigma_3 \neq \sigma_4$ since it is unknown whether the true population variances are equal, and our sample variances seem to indicate that they are not. Using the `t.test` function in R, we obtain the following results:

H_0	H_A	t	p	Reject H_0
$\mu_4 \leq \mu_1$	$\mu_4 > \mu_1$	6.042	0.0002	yes
$\mu_4 \leq \mu_2$	$\mu_4 > \mu_2$	3.769	0.0061	yes
$\mu_4 \leq \mu_3$	$\mu_4 > \mu_3$	5.843	0.0002	yes

Table 3: t-Test Results Using $\alpha = 0.05$

In all three tests, we reject the null hypothesis. This suggests that the true population mean of treatment 4 is indeed larger than those in our other treatments. Given this result, we CONCLUDE that the helicopter design corresponding to treatment 4 is superior with respect to flight time. The details of how to construct this helicopter can be found on the following page.

Cut standard 8x11 printer paper exactly in half (lengthwise), so you have a 4.25x11 sheet. Utilizing this entire sheet, make the following cuts and folds: