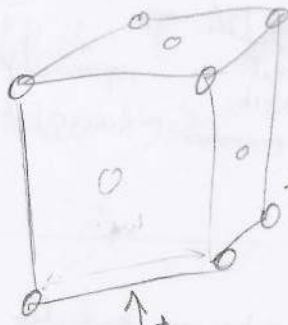
oblig 4

2). assume fcc-Si ($a = 5.43\,\text{Å}$)
"diamond cubic"

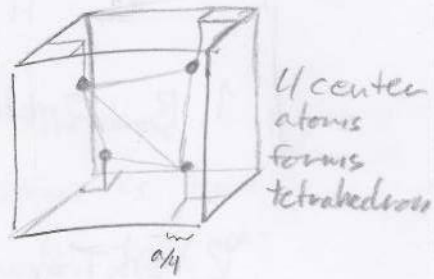


a) There are atoms at:

Corners ($8 \cdot 1/8$): 000 001 010 100 011 101 110 111

Faces ($6 \times 1/2$): 0 1/2 1/2, 1/2 0 1/2, 1/2 1/2 0, 1 1/2 1/2, 1/2 1 1/2, 1/2 1/2 1

Inside (4): 1/4 1/4 1/4, 3/4 3/4 1/4, 3/4 1/4 3/4, 1/4 3/4 3/4



Basis vectors:

$\vec{a} = (0\ 1/2\ 1/2)$ $\vec{b} = (1/2\ 1/2\ 0)$ $\vec{c} = (1/4\ 1/4\ 1/4)$

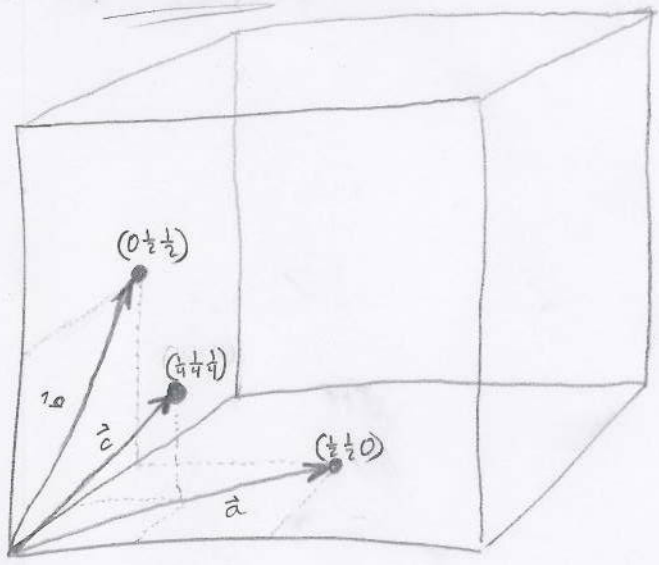
Summing any integer-scaled basis vectors (linear combination with integers as weights) gives the position of an atom. All atoms can be represented in this way.

$$\vec{x} = n_1\vec{a} + n_2\vec{b} + n_3\vec{c} = 2n_1(0\ 1/4\ 1/4) + 2n_2(1/4\ 0\ 1/4) + n_3(1/4\ 1/4\ 1/4)$$

c) $N_a = 8 \cdot 1/8 + 6 \cdot 1/2 + 1 \cdot 4 = \underline{8}$ eight atoms / unit cell

each Si-atom has 14 electrons. $8 \cdot 14 = \underline{112} = N_e$

b) $N_a = 8 \cdot 1/8 + 6 \cdot 1/2 = 4$ $N_e = 4 \cdot 14 = 56$



3. electron density is $\rho_e = {}^{Ne}/_{a^3} = {}^{112}/_{(0{,}543\ nm)^3} \approx 700\ nm^{-3}$

a) $\rho_e \cdot 1000\,nm^3 \approx \underline{7 \cdot 10^5 \text{ electrons}}$

b) $1\mu m^3 = (10^3 nm)^3 = 10^9 nm^3$

$\rho_e \cdot 10^9\,nm^3 = \underline{7 \cdot 10^{11} \text{ electrons}}$

c) ~~$1nm^3 = (10^3 \mu m)^3 = 10^9 \mu m^3$~~

$1mm^3 = (10^6 nm)^3 = 10^{18} nm^3$

$\rho_e \cdot 10^{18}\,nm^3 \approx \underline{7 \cdot 10^{20} \text{ electrons}}$

4. 100 petaFLOPS (10^{17} FLOPS)

a) A person weighing 100 kg, with approximately the same density as water will have a volume of approximately:

$V = 100\,dm^3 = 100\,(10^8 nm)^3 = 10^{26}\,nm^3$

the number of electrons is then:

$\rho_e \cdot V \approx \underline{7 \cdot 10^{28} \text{ electrons}}$

To calculate all electrons would take

$7 \cdot 10^{28} / 10^{17} = 7 \cdot 10^{11}$ seconds.

That is $7 \cdot 10^{11} / (3600 \cdot 24 \cdot 365) \approx 22197$ years

In a lifetime of 80 years it would be able to calculate:

$100 \cdot \frac{80}{22197} \approx 0{,}36\%$ of the person

b) The total number of terms is the product of terms in the first sum and the number in the second.

$\sum_{i=1}^{Ne}$ has Ne terms, while $\sum_{j>i}^{Ne}$ has

The number of terms is $\sum_{i}^{Ne} \sum_{j>i}^{Ne} 1 = \overbrace{1+2+3+\ldots+(Ne-2)+(Ne-1)}^{Ne \text{ terms}}$

(assuming i starting at 1)

reordering the terms: $\sum_{i}^{Ne} \sum_{j>i}^{Ne} 1 = \overbrace{1+(Ne-1)+2(Ne-2)+\ldots\frac{Ne}{2}+(Ne-\frac{Ne}{2})}^{Ne \text{ terms}}$

$= \sum_{i}^{Ne/2} Ne = \frac{Ne}{2} \cdot Ne = Ne^2/2 = \frac{(7 \cdot 10^{20})^2}{2} = 2{,}45 \cdot 10^{41}$

$2.45 \cdot 10^{40}$ cycles (terms)

With 1000 FLOPS pr. cycle it is $2.45 \cdot 10^{43}$ FLOPS

That will take $2.45 \cdot 10^{40} \cdot 10^{-14} s = 2.45 \cdot 10^{26}$ seconds

Which is $\frac{2.45 \cdot 10^{26}}{3600 \cdot 24 \cdot 365} \approx 7.8 \cdot 10^{18}$ years

Which is quite long considering the age of the universe is approximately $13.8 \cdot 10^{9}$ years