ASSIGNMENT 1

GROUP 5

SECTION 03 - SEM 1, 2024/2025

SECI1013 *(DISCRETE STRUCTURE)*

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"Elements of structured thinking"

ANSWER

QUESTION 1

1. Let set $U = \{n \mid n \in \text{whole numbers}, 10 \le n \le 30\}$;
   set $G = \{g \mid g \in \text{even numbers}\}$;
   set $F = \{f \mid f \in \text{natural numbers}, f > 10 \text{ and } f < 30\}$;
   $G \subseteq F$;
   $F \subseteq U$.

   Answer all the questions for each of the following.

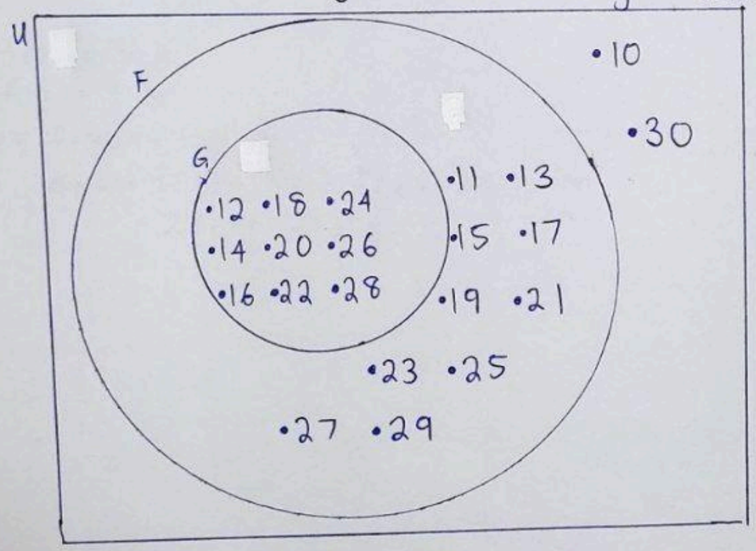
   a. Write down set F and find $|F|$

   $F = \{11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29\}$
   $|F| = 19$

   b. Write down set G and find $|G|$

   $G = \{12,14,16,18,20,22,24,26,28\}$
   $|G| = 9$

   c. Construct a Venn diagram based on the given sets.



   d. Find cardinality of symmetric difference of set G and set F

   $G - F = \{\}$
   $F - G = \{11,13,15,17,19,21,23,25,27,29\}$
   $G \oplus F = (G-F) \cup (F-G)$
   $= \{11,13,15,17,19,21,23,25,27,29\}$
   $|G \oplus F| = 10$

QUESTION 2

2. Let set $A = \{s, u, b\}$; set $B = \{s, e, t\}$; set $C = \{n, e, t\}$.
Answer all questions for each of the following in ascending order. [5 marks]

a. Find $|P(A)|$

$|P(A)| = 2^{|A|}$
$= 2^3$
$= 8$

b. Find $A \cap B \cup C$

$A \cap B = \{s\}$

$A \cap B \cup C = \{s, n, e, t\}$

* ascending order:

$A \cap B \cup C = \{e, n, s, t\}$

c. Find $A - B$

$A - B = \{u, b\}$

* ascending order:

$A - B = \{b, u\}$

d. Find $B \times C$

$B \times C = \{(s,n), (s,e), (s,t), (e,n), (e,e), (e,t), (t,n), (t,e), (t,t)\}$

* ascending order:

$B \times C = \{(e,e), (e,n), (e,t), (s,e), (s,n), (s,t), (t,e), (t,n), (t,t)\}$

QUESTION 3

3. Which of the following statements are propositions? State true or false. [5 marks]

a. The discrete structure implements set theory, relations, and functions to solve computer science problems.

- It is not a proposition

b. In computer science, the Boolean data type defines 0 = false and 1 = true

- It is proposition
- True

c. Let A and B be the subsets of U. $A \cup (A \cap B) = A$ can be proved by distributive, idempotent, and commutative laws.

- It is proposition
- True

d. $a^2 - 2a + 1 = 0$; when $a \neq 1$

- It is a proposition
- False

e. $a^2 - b^2 = 0$; when $a = b$ or $a = -b$

- It is a proposition
- True

QUESTION 4

4. Construct a truth table for each of the following conditional statements. [4 Marks]

(a) $(p \to q) \wedge (\neg p \leftrightarrow \neg q)$

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg p \leftrightarrow \neg q$	$(p \to q) \wedge (\neg p \leftrightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

(b) $(p \leftrightarrow q) \vee (\neg p \to \neg q)$

p	q	$\neg p$	$\neg q$	$p \leftrightarrow q$	$\neg p \to \neg q$	$(p \leftrightarrow q) \vee (\neg p \to \neg q)$
T	T	F	F	T	F	T
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

QUESTION 5

5. Given, $A = \neg p \wedge (\neg q \vee \neg r)$ and $B = p \vee (q \wedge r)$. State whether $A \equiv B$ or not

A =

p	q	r	$\neg p$	$\neg q$	$\neg r$	$\neg q \vee \neg r$	$\neg p \wedge (\neg q \vee \neg r)$
T	T	T	F	F	F	F	F
T	T	F	F	F	T	T	F
T	F	T	F	T	F	T	F
T	F	F	F	T	T	T	F
F	T	T	T	F	F	F	F
F	T	F	T	F	T	T	T
F	F	T	T	T	F	T	T
F	F	F	T	T	T	T	T

B =

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

$\therefore \neg p \wedge (\neg q \vee \neg r) \not\equiv p \vee (q \wedge r)$

$A \not\equiv B$

QUESTION 6

6. Given, $A = p \wedge (p \vee q)$ and $B = p \vee (p \wedge q)$. State whether $A \equiv B$ or not. [2.5 marks]

p	q	$p \vee q$	$p \wedge q$	$p \wedge (p \vee q)$	$p \vee (p \wedge q)$	
T	T	T	T	T	T	✓
T	F	T	F	T	T	✓
F	T	T	F	F	F	✓
F	F	F	F	F	F	✓

$p \wedge (p \vee q) \equiv p \vee (p \wedge q)$. $\therefore A \equiv B$

QUESTION 7

7. Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a student," "x is smart," and "x is shy." respectively.
Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists of all people. [2 marks]

a. Some students are shy.

$P(x)$: "x is a student"

$Q(x)$: "x is smart"

$R(x)$: "x is shy"

$$\exists x \, (P(x) \land R(x))$$

b. All smart people are not shy

$$\forall x \, (Q(x) \rightarrow \neg R(x))$$

QUESTION 8

8. Give direct proof to show a square of any negative numbers is positive. [3.5 Marks]

Let $P(x) = x$ is a negative number, and $Q(x) = x^2$ is positive.

If x is a negative number, then x^2 is positive. $P(x) \rightarrow Q(x)$

When $x=-1$, $Q(-1) = (-1)^2 = 1$

When $x=-2$, $Q(-2) = (-2)^2 = 4$

When $x=-3$, $Q(-3) = (-3)^2 = 9$

When $x=-4$, $Q(-4) = (-4)^2 = 16$

Let $x = -m$,

$Q(x) = x^2$

$Q(-m) = (-m)^2$

$= m^2$ (positive number)

$\therefore$ Through the proving,

$\forall x\, Q(x)$ is positive if x is negative number. Hence, it is true that a square of any negative numbers is positive.

QUESTION 9

9. Give a proof by contradiction to show if C and D are sets, then $C \cap (D \cap C') = \{\}$

Answer:

assume

$C \cap (D \cap C') \neq \{\}$ means there's at least one element belongs to $C \cap (D \cap C')$

assume element is x

$x \in C \cap (D \cap C')$ also means $x \in C$ and $x \in D \cap C'$ because it's intersection

For $x \in D \cap C'$,

means $x \in D$ and $x \in C'$

To conclude it is impossible to have $x \in C$ and $x \in C'$, since x should be only in either C or C'.

This is contradiction because if C contain x, C' can not contain x and $C \cap (D \cap C') = \{\}$ is true

QUESTION 10

10. Determine whether the relation R on the set $\mathbb{Z}$ (set of integer number) is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive

[6 marks]

$a \, R \, b$ if and only if $|a-b| = 2$

$R = \{(1,3), (3,1), (2,0), (0,2)\}$

$$M_R = M_R^T = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 & 0 \end{array}$$

Reflexive if aRa holds for all $a \in \mathbb{Z}$. $a \in \mathbb{Z}, (a,a) \in R$.

$|a-a| = 2$, but $|a-a| = 0$.

$\therefore$ R is not reflexive.

Irreflexive if aRa does not hold for any $a \in \mathbb{Z}$. $(a,a) \notin R$ for $a \in \mathbb{Z}$

$|a-a| = 0 \neq 2$.

$\therefore$ R is irreflexive

Symmetric if aRb implies bRa, $(a,b) \in R \rightarrow (b,a) \in R$.

$|a-b| = 2$, $|b-a| = 2$. Let $a = 6$, $b = 4$. $|6-4| = 2$, $|4-6| = 2$.

$\therefore$ R is symmetric

Antisymmetric if aRb and bRa imply $a = b$. $\forall a,b \in \mathbb{Z}, (a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

Let $a = 6$, $b = 4$. $|6-4| = 2$, $|4-6| = 2$, but $a \neq b$

$\therefore$ R is not antisymmetric

Transitive if aRb and bRc imply aRc. $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

Let $a = 6$, $b = 4$, $c = 2$. $|6-4| = 2$, $|4-2| = 2$, but $|6-2| = 4 \neq 2$. $(a,c) \notin R$.

$\therefore$ R is not transitive.

Asymmetric if $\forall a,b \in \mathbb{Z}, (a,b) \in R \rightarrow (b,a) \notin R$

aRb implies bRa.

$\therefore$ R is not asymmetric.

QUESTION 11

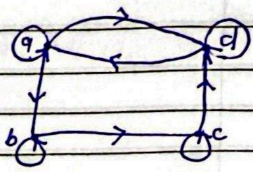
No.: .... Date: ....

11. Given a relation, R on A = {a, b, c, d} on as follows:
R = {(a,a), (a,b), (a,d), (b,b), (b,c), (c,c), (c,d), (d,a), (d,d)}
Show the matrix of relation, M_R and determine whether the relation, R is an equivalence relation.
[7 marks]

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \qquad M_R^T = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

∴ There is value 1 on diagonal and there is a loop at all vertices. The relation is reflexive.

Digraph for R:



∴ $M_R \neq M_R^T$ and the digraph is not symmetric. The relation is not symmetric.

$$M_R \otimes M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \times \begin{array}{c} \\ a \\ b \\ c \\ a \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$= \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

∴ $M_R \otimes M_R \neq M_R$. The relation is not transitive

∴ In a nutshell, since the relation is reflexive, not symmetric and not transitive, the relation is not an equivalence relation.

QUESTION 12

12. Let $f(x,y) = (2x-y, x-2y)$; $(x,y) \in R \times R$, (R is set of real numbers.)

(a) Show that f is one-to-one.

Since $(x,y) \in R \times R$

$(x_1, y_1) = (x_2, y_2)$

$(2x_1 - y_1, x_1 - 2y_1) = (2x_2 - y_2, x_2 - 2y_2)$

$2x_1 - y_1 = 2x_2 - y_2$

$2x_1 - 2x_2 = y_1 - y_2$

$y_1 - y_2 = 2(x_1 - x_2)$ — ①

$x_1 - 2y_1 = x_2 - 2y_2$

$x_1 - x_2 = 2y_1 - 2y_2$

$2(y_1 - y_2) = x_1 - x_2$ — ②

Substitute ① into ②:

$2[2(x_1 - x_2)] = x_1 - x_2$

$4(x_1 - x_2) = x_1 - x_2$

$4x_1 - 4x_2 = x_1 - x_2$

$4x_1 - x_1 = 4x_2 - x_2$

$3x_1 = 3x_2$

$x_1 = x_2$ — ③

Substitute ③ into ①:

$y_1 - y_2 = 2(x_2 - x_2)$

$y_1 - y_2 = 2(0)$

$y_1 - y_2 = 0$

$y_1 = y_2$

∴ Since $x_1 = x_2$ and $y_1 = y_2$. It is proven that $(x_1, y_1) = (x_2, y_2)$. Thus, f is one-to-one.

(b) Find f^{-1} [7 Marks]

$f(x,y) = (2x-y, x-2y)$

Let $a = 2x - y$ & $b = x - 2y$,

$a = 2x - y$ $\quad$ $b = x - 2y$

$y = 2x - a$ — ① $\quad$ $2y = x - b$ — ②

Substitute ① into ②:

$2(2x - a) = x - b$

$4x - 2a = x - b$

$4x - x = 2a - b$

$3x = 2a - b$

$x = \frac{2a-b}{3}$ — ③

Substitute ③ into ①:

$y = 2\left(\frac{2a-b}{3}\right) - a$

$= \frac{4a-2b}{3} - \frac{3a}{3}$

$= \frac{a-2b}{3}$

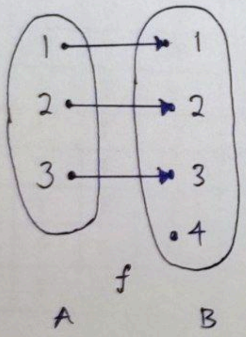
Let $f^{-1}(u,v)$ be the inverse function,

$f^{-1}(u,v) = \left(\frac{2a-b}{3}, \frac{a-2b}{3}\right)$

QUESTION 13

13. Let a set $A=\{1,2,3\}$, $B=\{1,2,3,4\}$ and $C=\{1,2\}$

a. Draw the arrow diagram to define function $f: A \to B$ that is one-to-one but not onto



$\therefore$ function $f: A \to B$ is one to one because every elements in A paired with B.
It is not onto because there's an element in B that has no domain.

b. List the three ordered pairs to define function $g: A \to C$ that is onto but not one-to-one

Three ordered pairs is $(1,1),(2,1),(3,2)$.

QUESTION 14

14. Function f and g are defined by formulas as shown below.

$f(x) = x^3$ and $g(x) = x-1$, for all real number x.

i) Find $g \circ f$ and $f \circ g$. [4 marks]

$$g \circ f = g[f(x)] = (x^3) - 1 = x^3 - 1$$

$$f \circ g = f[g(x)] = (x-1)^3 = x^3 - 3x^2 + 3x - 1$$

ii) Determine whether $g \circ f$ equals $f \circ g$. [2 marks]

Let $x = 2$,

$$g \circ f(x) = x^3 - 1$$
$$g \circ f(2) = (2)^3 - 1 = 8 - 1 = 7$$

$$f \circ g(x) = x^3 - 3x^2 + 3x - 1$$
$$f \circ g(2) = (2)^3 - 3(2)^2 + 3(2) - 1 = 8 - 3(4) + 6 - 1 = 8 - 12 + 6 - 1 = 1$$

$\therefore$ $g \circ f$ is not equal to $f \circ g$.

QUESTION 15

No.: Date:

15 Let $B = \{0, 1\}$. Give a recurrence relation for the strings of length n in B^* that do not contain 01. (B^* is the set of all string over B) [3 Marks]

when $n = 1$, $a_n = 2 \Rightarrow (0, 1)$
when $n = 2$, $a_n = 3 \Rightarrow (00, 10, 11)$
when $n = 3$, $a_n = 4 \Rightarrow (000, 111, 110, 100)$

In order to get strings that do not contain 01, 2 cases is discussed;

case 1: end with zero
→ either string consists of all zero → 1 way
→ or string consists of all one before all 0 → $(n-1)$ ways
eg (1110, 1100, 1000)

case 2: end with one
→ only when string consists of all one → 1 way

$a_n = n + 1$
$n = a_n - 1 \quad (1)$
$a_{n-1} = n - 1 + 1$
$a_{n-1} = n \quad (2)$
$a_n - 1 = a_{n-1}$
$a_n = a_{n-1} + 1, \quad n \geq 2, \quad a_1 = 2$

QUESTION 16

16. A game is played by moving a marker ahead either 2 or 3 steps on a linear path. Let c_n be the number of different ways a path of length n can be covered. Given,

$$c_n = c_{n-2} + c_{n-3}, \quad c_1 = 0, \; c_2 = 1, \; c_3 = 1$$

write a recursive algorithm to compute c_n. [3 Marks]

Input: n
Output: c(n)

```
Cn {
  if (n=2 or n=3)
       return 1
  else if (n=1)
       return 0
  return c(n-2)+c(n-3)
}
```