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IN SLIDE EXERCISE FOR CHAPTER 1

GROUP 5

SECTION 03 - SEM 1, 2024/2025

SECI1013 (*DISCRETE STRUCTURE*)


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“ Discrete elements, cohesive structure.”




Exercise

Determine whether each pair of sets is equal
 $\{1, 2, 2, 3\}, \{1, 3, 2\}$

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ANSWER:

Yes. They are equal because both sets contain the same elements, even if some element is duplicating.



Exercise

- If M is finite, determine the $|M|$
 - If $M = \{1, 2, 3, 4\}$
 - If $M = \{4, 4, 4\}$
 - If $M = \{\}$
 - If $M = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$

Let $X = \{1, 2, 2, \{1\}, a\}$

- Find:
 - $|X|$
 - Power set of X

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ANSWER:

$$M = \{1, 2, 3, 4\}$$

$$|M| = 4$$

$$M = \{4, 4, 4\}$$

$$|M| = 1$$

$$M = \{\}$$

$$|M| = 0$$


$$M = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$$

$$|M| = 3$$

$$|X| = 4$$

$$\text{Power set of } X = 2^{|X|} = 2^4$$

$$|P(X)| = 16$$



Exercise

- Let,
 $U = \{ a, b, c, d, e, f, g, h, i, j, k, l, m \}$
 $A = \{ a, c, f, m \}$
 $B = \{ b, c, g, h, m \}$
- Find:
 $| A \cup B |$, $A - B$ dan A' .


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ANSWER:

$$\begin{aligned} |A \cup B| &= |A| + |B| - |A \cap B| \\ &= 4 + 5 - 2 \\ &= 7 \end{aligned}$$

$$A - B = \{a, f\}$$

$$A' = \{b, d, e, g, h, i, j, k, l\}$$



Exercise


- Let A , B and C be sets such that
 $A \cap B = A \cap C$ and $A \cup B = A \cup C$
- Prove that $B = C$

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ANSWER:

$$\begin{aligned}
 B &= B \cup \emptyset && : \text{properties of empty set} \\
 &= B \cup (A \cap \emptyset) && : \text{properties of empty set} \\
 &= (A \cup B) \cap (B \cup \emptyset) && : \text{distributive law} \\
 &= (A \cup C) \cap B && : \text{properties of empty set} \\
 &= (A \cap B) \cup (B \cap C) && : \text{distributive law} \\
 &= (A \cap C) \cup (B \cap C) && : \text{by given conditions} \\
 &= C \cap (A \cup B) && : \text{distributive law} \\
 &= C \cap (A \cup C) && : \text{by given conditions} \\
 &= C && : \text{absorption law}
 \end{aligned}$$

\therefore Proven, $B = C$.



Exercise

- $A = \{a, b\}$, $B = \{1, 2\}$, $C = \{x, y\}$
- Determine the following set and their cardinality,
 - a) $B \times C$
 - b) $A \times B \times C$

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ANSWER:

$$(a) B \times C = \{(1, x), (1, y), (2, x), (2, y)\}$$

$$\begin{aligned} |B \times C| &= |B| \times |C| \\ &= 2 \times 2 \\ &= 4 \end{aligned}$$

$$(b) A \times B \times C = \{(a, 1, x), (a, 1, y), (a, 2, x), (a, 2, y), (b, 1, x), (b, 1, y), (b, 2, x), (b, 2, y)\}$$

$$\begin{aligned} |A \times B \times C| &= |A| \times |B| \times |C| \\ &= 2 \times 2 \times 2 \\ &= 8 \end{aligned}$$



Exercise

Suppose x is a particular real number. Let p , q and r symbolize " $0 < x$ ", " $x < 3$ " and " $x = 3$ ", respectively. Write the following inequalities symbolically:

- a) $x \leq 3$
- b) $0 < x < 3$
- c) $0 < x \leq 3$


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ANSWER:

(a) $q \wedge r$

(b) $p \vee q$

(c) $p \vee (q \wedge r)$



Exercise

Propositional functions p , q and r are defined as follows:

p is " $n = 7$ "
 q is " $a > 5$ "
 r is " $x = 0$ "

Write the following expressions in terms of p , q and r , and show that each pair of expressions is **logically equivalent**. State carefully which of the above laws are used at each stage.

(a) $((n = 7) \text{ or } (a > 5)) \text{ and } (x = 0)$
 $((n = 7) \text{ and } (x = 0)) \text{ or } ((a > 5) \text{ and } (x = 0))$

(b) $\neg((n = 7) \text{ and } (a \leq 5))$
 $(n \neq 7) \text{ or } (a > 5)$

(c) $(n = 7) \text{ or } (\neg((a \leq 5) \text{ and } (x = 0)))$
 $((n = 7) \text{ or } (a > 5)) \text{ or } (x \neq 0)$

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ANSWER:

(a)

p	q	r	$p \vee q$	$(p \vee q) \wedge r$	$p \wedge r$	$q \wedge r$	$(p \wedge r) \vee (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	T	T	F	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

$$\therefore (p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$$

Distributive Law

(b)

p	q	$\neg p$	$\neg q$	$\neg(p \wedge \neg q)$	$\neg p \vee q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$$\therefore \neg(p \wedge \neg q) \equiv \neg p \vee q$$


De Morgan's Law

(c)

p	q	r	$\neg q$	$\neg r$	$\neg(\neg q \wedge r)$	$p \vee (\neg(\neg q \wedge r))$	$p \vee q$	$(p \vee q) \vee \neg r$
T	T	T	F	F	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	T	T	T
T	F	F	T	T	T	T	T	T
F	T	T	F	F	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	F	F	F	F	F
F	F	F	T	T	T	T	F	T

$$\therefore p \vee (\neg(\neg q \wedge r)) \equiv (p \vee q) \vee \neg r$$

De Morgan's Law & Distributive Law



Exercise

Propositions p , q , r and s are defined as follows:
 p is "I shall finish my Coursework Assignment"
 q is "I shall work for forty hours this week"
 r is "I shall pass Maths"
 s is "I like Maths"

Write each sentence in symbols:

- I shall not finish my Coursework Assignment.
- I don't like Maths, but I shall finish my Coursework Assignment.
- If I finish my Coursework Assignment, I shall pass Maths.
- I shall pass Maths only if I work for forty hours this week and finish my Coursework Assignment.

Write each expression as a sensible (if untrue!) English sentence:

- $q \vee p$
- $\neg p \rightarrow \neg r$

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ANSWER:

(a) $\neg p$


(b) $\neg s \wedge p$

(c) $p \rightarrow r$

(d) $r \leftrightarrow (q \wedge p)$

(e) I shall work for forty hours this week or finish my Coursework Assignment.

(f) If I don't finish my Coursework Assignment, then I will not pass Maths.



Exercise

For each pair of expressions, construct **truth tables** to see if the two compound propositions are logically equivalent:

(a) $p \vee (q \wedge \neg p)$
 $p \vee q$

(b) $(\neg p \wedge q) \vee (p \wedge \neg q)$
 $(\neg p \wedge \neg q) \vee (p \wedge q)$

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ANSWER:

(a)


p	q	$\neg p$	$q \wedge \neg p$	$p \vee (q \wedge \neg p)$	$p \vee q$	
T	T	F	F	T	T	✓
T	F	F	F	T	T	✓
F	T	T	T	T	T	✓
F	F	T	F	F	F	✓

$\therefore p \vee (q \wedge \neg p) \equiv p \vee q$

(b)

p	q	$\neg p$	$\neg q$	$\neg p \wedge q$	$p \wedge \neg q$	$\neg p \wedge \neg q$	$p \wedge q$	$(\neg p \wedge q) \vee (p \wedge \neg q)$	$(\neg p \wedge \neg q) \vee (p \wedge q)$	
T	T	F	F	F	F	F	T	F	T	✗
T	F	F	T	F	T	F	F	T	F	✗
F	T	T	F	T	F	F	F	T	F	✗
F	F	T	T	F	F	T	F	F	T	✗

$\therefore (\neg p \wedge q) \vee (p \wedge \neg q) \not\equiv (\neg p \wedge \neg q) \vee (p \wedge q)$



Exercise

1. Prove that if x is an even integer, then $x^2 - 6x + 5$ is odd
(Direct Proof)
2. Prove that if n is an integer and $n^3 + 5$ is odd, then n is even
(Indirect Proof)
3. Prove that if x is odd, then x^2 is odd (Contradiction)

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ANSWER:

1. Let;
 $P(x) = x$ is even integer.
 $Q(x) = x^2 - 6x + 5$ is odd.

Symbolically, $\forall x(P(x) \rightarrow Q(x))$ with domain of discourse is the set of all integers.
 Let $2n$ be an even integer.

$$\begin{aligned}
 Q(2n) &= (2n)^2 - 6(2n) + 5 \\
 &= 4n^2 - 12n + 5 \\
 &= 2(2n^2 - 6n) + 5 && : 2n^2 - 6n = m, \text{ where } m \text{ is an integer} \\
 &= 2m + 5 && : \text{odd integer}
 \end{aligned}$$

\therefore Proven, $\forall x(P(x) \rightarrow Q(x))$.

2. Let;
 $P(n) = n$ is an integer and $n^3 + 5$ is odd.
 $Q(n) = n$ is even.

$\forall n(P(n) \rightarrow Q(n)); \quad P(n) \rightarrow Q(n) \equiv \neg Q(n) \rightarrow \neg P(n)$
 $\neg Q(n) = n$ is odd. Suppose $\neg Q(n)$ is true, we need to show that $\neg P(n)$ is true.
 Let $2k + 1$ be an odd integer. $\neg P(n)$ is true if $n^3 + 5$ is even.

$$\begin{aligned}
 P(2k + 1) &= (2k + 1)^3 + 5 \\
 &= 8k^3 + 12k^2 + 6k + 6 \\
 &= 2(4k^3 + 6k^2 + 3k) + 6 && : 4k^3 + 6k^2 + 3k = t, \text{ where } t \text{ is an integer} \\
 &= 2t + 6 && : \text{even integer}
 \end{aligned}$$

\therefore Proven, $\forall x(\neg Q(n) \rightarrow \neg P(n))$.

3. Let;
 $P(x) = x$ is odd.
 $Q(x) = x^2$ is odd.

Originally, $\forall x(P(x) \rightarrow Q(x))$ is true but it is also true if the contradiction,
 $\forall x(P(x) \rightarrow \neg Q(x))$ is false. Suppose x is odd and x^2 is not odd.
 Let $2n + 1$ be an odd integer. $\neg Q(x)$ is false if x^2 is odd.

$$\begin{aligned}
 Q(2n+1) &= (2n+1)^2 \\
 &= 4n^2 + 4n + 1 \\
 &= 2(2n^2 + 2n) + 1 && : 2n^2 + 2n = m, \text{ where } m \text{ is an integer} \\
 &= 2m + 1 && : \text{odd integer}
 \end{aligned}$$

$\neg Q(x)$ is false since x^2 is odd. Thus, the statement $\forall x(P(x) \rightarrow \neg Q(x))$ is false.
 \therefore Proven, $\forall x(P(x) \rightarrow Q(x))$.