ASSIGNMENT 3

GROUP 5

SECTION 03 - SEM 1, 2024/2025

SECI1013 *(DISCRETE STRUCTURE)*

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DATE SUBMITTED : 29th DECEMBER 2024

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"Crafted for logical clarity"

ANSWER

CHAPTER 3 (3.5) : PROBABILITY

QUESTION 1

1. Let M = Math Book, C = Chemistry Book, B = Biology Book & S = Physics Book,

i. $P(M) = 3P(C)$

$P(C) = 2P(B)$

$P(B) = P(S)$

$P(M) + P(C) + P(B) + P(S) = 1$

$3P(C) + 2P(B) + P(S) + P(S) = 1$

$3(2P(B)) + 2P(S) + 2P(S) = 1$

$6P(B) + 4P(S) = 1$

$6P(S) + 4P(S) = 1$

$10P(S) = 1$

$P(S) = \frac{1}{10}$

$P(B) = P(S)$

$= \frac{1}{10}$

$P(C) = 2P(B)$

$= 2\left(\frac{1}{10}\right)$

$= \frac{1}{5}$

$P(M) = 3P(C)$

$= 3\left(\frac{1}{5}\right)$

$= \frac{3}{5}$

$\therefore P(M) = \frac{3}{5}$, $P(C) = \frac{1}{5}$, $P(B) = \frac{1}{10}$, $P(S) = \frac{1}{10}$.

ii. $P(M \cup B) = P(M) + P(B)$

$= \frac{3}{5} + \frac{1}{10}$

$= \frac{7}{10}$

QUESTION 2

2. If A and B are events of mutually exclusive and $P(A)=0.4$ and $P(B)=0.5$, find;

i) $P(A \cup B)$

ii) $P(A^c)$

iii) $P(A^c \cap B)$

Answer:

i) $P(A \cup B) = P(A) + P(B)$

$= 0.4 + 0.5$

$= 0.9$

ii) $P(A^c) = P(S) - P(A)$

$= 1 - 0.4$

$= 0.6$

iii) Since B is mutually exclusive with A, then all B must contained in A'
which means,

$P(A' \cap B) = P(B)$

$P(A' \cap B) = 0.5$

QUESTION 3

3. Assumed that there are 100 participants in a lucky draw competition. There are 3 prizes being offered which are the grand prize, second prize and third prize. The winners are randomly selected. What is the probability that Anis can win one of the prizes, if she participates in the competition?

Possible ways to select 3 winners $= {}^{100}C_3$

$= 161700$

If Anis does not win $= {}^{99}C_3$

$= 156849$

Probability that Anis can win one of the prizes $= 1 - \frac{156849}{161700}$

$= 1 - 0.97$

$= 0.03$

QUESTION 4

Question 4

$P(A) = 0.40$ choose male randomly that has pneumonia

$P(B) = ?$ male that hasn't pneumonia

$P(C) =$ smokers

$P(C \mid A) = 0.80$ ~~have pneumonia problem~~ are a smoker ~~smokers given~~ given pneumonia

$P(C \mid B) = 0.30$ ~~do not have pneumonia~~ are a smoker ~~smokers.~~ given no pneumonia

i) male doesn't has pneumonia

$P(B) = 1 - P(A)$

$= 1 - 0.40$

$P(B) = 0.60$

ii) selected male has pneumonia given he is smoker.

$$P(A \mid C) = \frac{P(C \mid A)\,P(A)}{P(C)}$$

$P(C) = P(C \mid A)\,P(A) + P(C \mid B)\,P(B)$

$= (0.80)(0.40) + (0.30)(0.60)$

$P(C) = 0.32 + 0.18 = 0.50$

$$P(A \mid C) = \frac{(0.80)(0.40)}{0.50}$$

$P(A \mid C) = 0.64$

QUESTION 5

5. Let B = Black coloured boot, C = chocolate coloured boot & Y = yellow coloured boot,

Total boots $= 2+2+2$
$= 6$

$P(B) = \frac{2}{6} = \frac{1}{3}$ $P(C) = \frac{2}{6} = \frac{1}{3}$ $P(Y) = \frac{2}{6} = \frac{1}{3}$

$P(B \cap B) = P(B) \times P(B)$
$= \frac{1}{3} \times \frac{1}{3}$
$= \frac{1}{9}$

CHAPTER 4 (4.1 TO 4.6): GRAPH THEORY

QUESTION 1

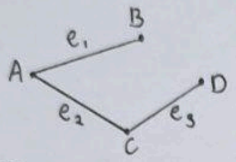
1. Explain the given keyword using your own word and represent your understanding by drawing the graph.

a. Vertices
b. Edges
c. Adjacent Vertices
d. Incident Edge
e. Isolated Vertex
f. loop
g. Parallel Edges

Answer:

a. Vertices

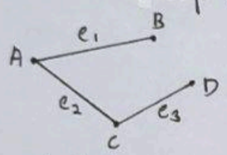
represent point/nodes in a graph. Vertices connected by edges that will represent relation.



(A,B,C,D) are vertices

b. Edges

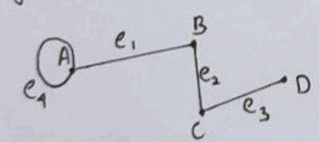
represent lines/loop that connects two vertices.



(e_1,e_2,e_3) are edges

c. Adjacent Vertices

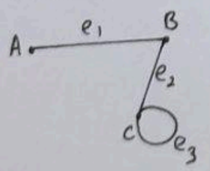
two vertices connected by edges directly. For loop, vertex that is an endpoint is adjacent vertices to itself



C and D are adjacent vertices
A and A are adjacent vertices

d. Incident edges

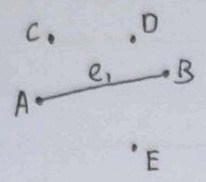
refer to edges connected to vertex in graph. In other words, edges that touch the vertex



e_1 is incident to vertex A and B
e_2 is incident to vertex B and C
e_3 is incident to vertex C

e. Isolated vertex

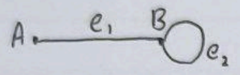
vertex that not connected with any edges.



(C, D, E) are isolated vertex

f. Loop

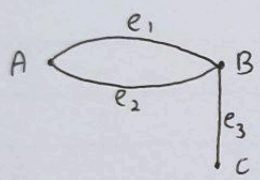
edge that have only one endpoint or connect to itself.



e_2 is loop

g. Paralled edges

when two or more edges in graph connected with same pair of vertex

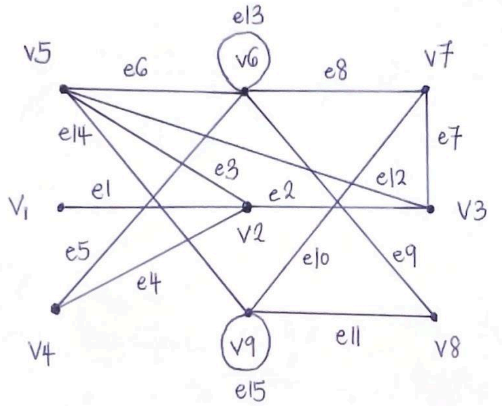


e_1 and e_2 are paralled edges

QUESTION 2

2. Let G = {V,E} be a graph. Draw a graph with the following specified properties

a. An undirected graph having V = { v1, v2, v3, v4, v5, v6, v7, v8, v9 } and E = { e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12, e13, e14, e15 }. Where e1 = (v1, v2), e2 = (v2, v3), e3 = (v2, v5), e4 = (v2, v4), e5 = (v4, v6) and e6 = (v5, v6), e7 = (v3, v7), e8 = (v6, v7), e9 = (v6, v8), e10 = (v7, v9), e11 = (v8, v9), e12 = (v5, v3), e13 = (v6, v6), e14 = (v5, v9) and e15 = (v9, v9).



i. Find the degree of each vertex.

v1 = 1 (e1)

v2 = 4 (e1, e2, e3, e4)

v3 = 3 (e2, e7, e12)

v4 = 2 (e4, e5)

v6 = 6 (e5, e6, e8, e9, e13, e13)

v7 = 3 (e7, e8, e10)

v8 = 2 (e9, e11)

v9 = 5 (e10, e11, e14, e15, e15)

V5 = 4 (e3, e6, e12, e14)

deg (v1) = 1   deg (v4) = 2

deg (v2) = 4   deg (v5) = 4   deg (v7) = 3

deg (v3) = 3   deg (v6) = 6   deg (v8) = 2

deg (v9) = 5

ii. Find the adjecent matrix and incident matrix

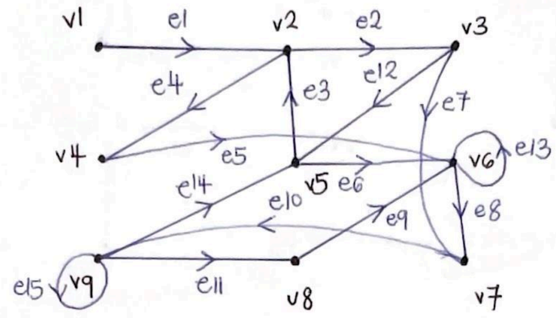
Adjacent matrix :

$$
\begin{array}{c} \\ v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \\ v7 \\ v8 \\ v9 \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} v1 & v2 & v3 & v4 & v5 & v6 & v7 & v8 & v9 \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1
\end{bmatrix}
\end{array}
$$

Incident matrix :

$$
\begin{array}{c} \\ v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \\ v7 \\ v8 \\ v9 \end{array}
\begin{array}{c}
\begin{array}{ccccccccccccccc} e1 & e2 & e3 & e4 & e5 & e6 & e7 & e8 & e9 & e10 & e11 & e12 & e13 & e14 & e15 \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 2
\end{bmatrix}
\end{array}
$$

b. A direct graph having $V = \{v1, v2, v3, v4, v5, v6, v7, v8, v9\}$ and $E = \{e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12, e13, e14, e15\}$. Where $e1 = (v1, v2)$, $e2 = (v2, v3)$, $e3 = (v5, v2)$, $e4 = (v2, v4)$, $e5 = (v4, v6)$, $e6 = (v5, v6)$, $e7 = (v3, v7)$, $e8 = (v6, v7)$, $e9 = (v8, v6)$, $e10 = (v7, v9)$, $e11 = (v9, v8)$, $e12 = (v3, v5)$, $e13 = (v6, v6)$, $e14 = (v9, v5)$, $e15 = (v9, v9)$



i. Find the degree of each vertex.

deg (v1) = 1 deg (v4) = 2 deg (v7) = 3

deg (v2) = 4 deg (v5) = 4 deg (v8) = 2

deg (v3) = 3 deg (v6) = 6 deg (v9) = 5

ii. Find the adjecent matrix and incident matrix.

Adjacent matrix :

$$
\begin{array}{c}
 \\ v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \\ v7 \\ v8 \\ v9
\end{array}
\begin{array}{c}
\begin{array}{ccccccccc} v1 & v2 & v3 & v4 & v5 & v6 & v7 & v8 & v9 \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1
\end{bmatrix}
\end{array}
$$

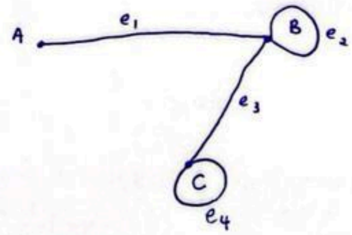
Incident matrix :

$$
\begin{array}{c}
 \\ v1 \\ v2 \\ v3 \\ v4 \\ v5 \\ v6 \\ v7 \\ v8 \\ v9
\end{array}
\begin{array}{c}
\begin{array}{ccccccccccccccc} e1 & e2 & e3 & e4 & e5 & e6 & e7 & e8 & e9 & e10 & e11 & e12 & e13 & e14 & e15 \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 1 & 1
\end{bmatrix}
\end{array}
$$

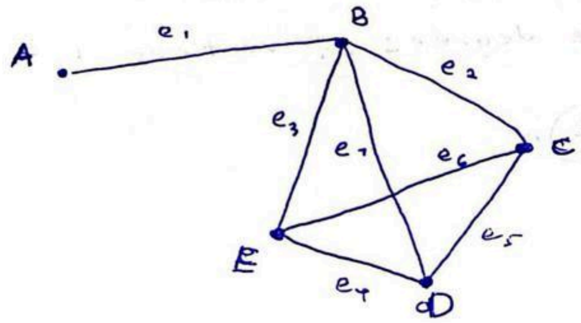
QUESTION 3

QUESTION 3

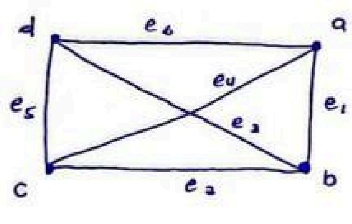
a) 3 vertices having the degrees of vertices 1, 3 and 4



b) 5 vertices having the degrees of vertices 1, 3, 3, 3 and 4.

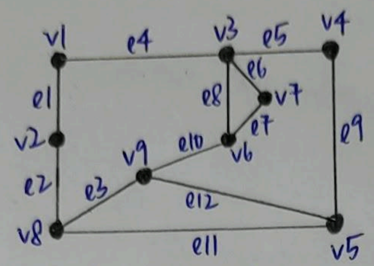


c) each vertex has degrees 3 and 6 edges



QUESTION 4

4. Graph:



i. Possible Paths from v1 to v9:

1. (v1,e1,v2,e2,v8,e3,v9)
2. (v1,e1,v2,e2,v8,e11,v5,e12,v9)
3. (v1,e4,v3,e8,v6,e10,v9)
4. (v1,e4,v3,e6,v7,e7,v6,e10,v9)
5. (v1,e4,v3,e5,v4,e9,v5,e12,v9)
6. (v1,e1,v2,e2,v8,e11,v5,e9,v4,e5,v3,e8,v6,e10,v9)
7. (v1,e1,v2,e2,v8,e11,v5,e9,v4,e5,v3,e6,v7,e7,v6,e10,v9)
8. (v1,e4,v3,e5,v4,e9,v5,e11,v8,e3,v9)

ii. Possible Trails from v1 to v9:

1. (v1,e1,v2,e2,v8,e3,v9)
2. (v1,e1,v2,e2,v8,e11,v5,e12,v9)
3. (v1,e4,v3,e8,v6,e10,v9)
4. (v1,e4,v3,e6,v7,e7,v6,e10,v9)
5. (v1,e4,v3,e5,v4,e9,v5,e12,v9)
6. (v1,e1,v2,e2,v8,e11,v5,e9,v4,e5,v3,e8,v6,e10,v9)
7. (v1,e1,v2,e2,v8,e11,v5,e9,v4,e5,v3,e6,v7,e7,v6,e10,v9)
8. (v1,e4,v3,e5,v4,e9,v5,e11,v8,e3,v9)
9. (v1,e4,v3,e8,v6,e7,v7,e6,v3,e5,v4,e9,v5,e12,v9)
10. (v1,e4,v3,e8,v6,e7,v7,e6,v3,e5,v4,e9,v5,e11,v8,e3,v9)
11. (v1,e4,v3,e6,v7,e7,v6,e8,v3,e5,v4,e9,v5,e11,v8,e3,v9)
12. (v1,e4,v3,e6,v7,e7,v6,e8,v3,e5,v4,e9,v5,e12,v9)

iii. Shortest Path: (v1,e4,v3,e8,v6,e10,v9)

Longest Path: (v1,e1,v2,e2,v8,e11,v5,e9,v4,e5,v3,e6,v7,e7,v6,e10,v9)

iv. Shortest Trail: (v1,e4,v3,e8,v6,e10,v9)

Longest Trail: (v1,e1,v2,e2,v8,e11,v5,e9,v4,e5,v3,e6,v7,e7,v6,e10,v9)

QUESTION 5

5. Given is the map of Miri Town in Sarawak. There is 6 main district that is connected by the main road in Miri

a. Find the possible Euler Path for this map?

b. Find the possible Euler Circuit for this map?

c. Find the possible Hamilton Circuit for this map?

d. Explain what the difference between Euler Circuit and Hamilton Circuit is?

Answer:

a. Euler Path

There's no Euler path for this map because every degrees are even.

Vertex	A	B	C	D	E	F
Degree	4	2	4	2	2	4

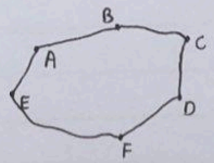
b. Euler Circuit

1. (A, B, C, A, F, C, D, F, E, A)
2. (A, B, C, F, A, C, D, F, E, A)
3. (A, B, C, D, F, C, A, F, E, A)
4. (A, C, B, A, F, C, D, F, E, A)
5. (A, F, C, B, A, C, D, F, E, A)
6. (A, E, F, A, C, F, D, C, B, A)
7. (A, E, F, D, C, F, A, C, B, A)
8. (A, E, F, C, D, F, A, C, B, A)
9. (A, E, F, C, A, F, D, C, B, A)
10. (A, E, F, C, B, A, F, D, C, A)

Vertex	A	B	C	D	E	F
Degree	4	2	4	2	2	4

c. Hamilton Circuit

1. (A, B, C, D, F, E, A)
2. (A, E, F, D, C, B, A)



d. Euler circuit uses all edges exactly once while Hamilton circuit does not matter use all or not. Euler circuit can use every vertex more than 1 time while Hamilton circuit can use every vertex exactly once.