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UNIVERSITI TEKNOLOGI MALAYSIA

IN SLIDE EXERCISE FOR CHAPTER 2

GROUP 5

SECTION 03 - SEM 1, 2024/2025

SECI1013 (*DISCRETE STRUCTURE*)


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DATE : 5th NOVEMBER 2024

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“ Finite sets, boundless utility. ”



Exercise

Define a relation R from \mathbf{Z} to \mathbf{Z} as follows: For all integer number m and n , $(m,n) \in \mathbf{Z} \times \mathbf{Z}$,

$$m R n \leftrightarrow m - n \text{ is even}$$

- i) Is $4 R 0$?
- ii) Is $2 R 6$?
- iii) Is $3 R (-3)$?
- iv) Is $5 R 2$?
- v) List 5 integers that are related by R to 1.

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ANSWER:


i) Yes

ii) Yes

iii) Yes

iv) No


v) 3, 5, 7, 9, 11



Exercise

An airline services the five cities c_1, c_2, c_3, c_4 and c_5 . Table below gives the cost (in dollars) of going from c_i to c_j . Thus the cost of going from c_1 to c_3 is RM100, while the cost of going from c_4 to c_2 is RM200

To from	c_1	c_2	c_3	c_4	c_5
c_1		140	100	150	200
c_2	190		200	160	220
c_3	110	180		190	250
c_4	190	200	120		150
c_5	200	100	200	150	



If the relation R on the set of cities $A = \{c_1, c_2, c_3, c_4, c_5\} : c_i R c_j$ if and only if the cost of going from c_i to c_j is defined and less than or equal to RM180.

i) Find R .


ii) Matrices of relations for R

ANSWER:

i) $R = \{(c_1, c_2), (c_1, c_3), (c_1, c_4), (c_2, c_4), (c_3, c_1), (c_3, c_2), (c_4, c_3), (c_4, c_5), (c_5, c_2), (c_5, c_4)\}$

ii)

$$R = \begin{matrix} & \begin{matrix} c_1 & c_2 & c_3 & c_4 & c_5 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix} \end{matrix}$$



Exercise

Let $A = \{1, 2, 3, 4\}$ and R is a relation from A to A .
 Suppose $R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

- What is R (represent)?
- What is matrix representation of R ?

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ANSWER:

R is a relation from A to A such that $x \in A, y \in A, (x, y) \in A \times A$ and $R \subseteq A \times A$.
 R can be defined by $x, y \in A, x R y \leftrightarrow y > x$.

Matrix representation of R :

$$R = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{matrix}$$

Exercise

Let $A = \{1, 4, 5\}$ and let R be given by the digraph shown below. list in-degrees and out-degrees of all vertices.


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graph TD
    1((1)) --> 1
    1 --> 4((4))
    4 --> 4
    4 --> 5((5))
    5 --> 5
    5 --> 4
  
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ANSWER:

	1	4	5
in - degree	1	3	2
out - degree	2	2	2



Exercise

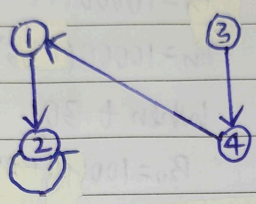
1. Let $A=\{1,2,3,4\}$ and let $R = \{(1,2), (2,2), (3,4), (4,1)\}$

Determine whether R symmetric, asymmetric or antisymmetric.

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ANSWER:

$R =$



$\therefore R$ is one-way digraph, R is antisymmetric.

$$M_R = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$


$$M_R^T = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

$\therefore M_R \neq M_R^T$, R is not symmetric.

\therefore For M_R , there are values 0 and 1 on diagonal and there is a loop at vertex 2. R is not reflexive. Although R is antisymmetric, R is not irreflexive but a not reflexive relation.

Hence, R is not asymmetric.

Conclusion: R is not antisymmetric.



Exercise

Determine which of the relations f are functions from the set X to the set Y . In case any of these relations are functions, determine if they are one-to-one, onto Y , and/or bijection.

a) $X = \{-2, -1, 0, 1, 2\}$, $Y = \{-3, 4, 5\}$ and
 $f = \{(-2, -3), (-1, -3), (0, 4), (1, 5), (2, -3)\}$

b) $X = \{-2, -1, 0, 1, 2\}$, $Y = \{-3, 4, 5\}$ and
 $f = \{(-2, -3), (1, 4), (2, 5)\}$

c) $X = Y = \{-3, -1, 0, 2\}$ and
 $f = \{(-3, -1), (-3, 0), (-1, 2), (0, 2), (2, -1)\}$

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ANSWER:

(a) f is a function from set X to set Y , not one-to-one function and it is onto Y .

(b) f is not a function. Not all elements in set X are mapped onto set Y .

(c) f is not a function. Many-to-many function is not proper.



Exercise 1

A depositor deposits RM 10,000 in a savings account at a bank yielding 5% per year with interest compounded annually. How much money will be in the account after 30 years? Let P_n denote the amount in the account after n years.

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ANSWER:

Compound interest formula: $A = P \left(1 + \frac{r}{n}\right)^{nt}$

$$P_n = 10000 \left(1 + \frac{0.05}{1}\right)^n$$

$$P_n = 10000 (1.05)^n$$

P_n = amount after n^{th} year

10000 = principal amount


0.05 = annual interest rate (decimal)

1 = number of times interest is compounded per year

n = time (year)

$$\begin{aligned} P_{30} &= 10000 (1.05)^{(30)} \\ &= 43219.42375 \end{aligned}$$

$$\therefore P_{30} = \text{RM}43219.42$$



Exercise 2

Consider the following sequence:

1, 5, 9, 13, 17

Find the recurrence relation that defines the above sequence.


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ANSWER:

Difference between terms = +4

Recurrence relation:

$$a_n = a_{n-1} + 4, n \geq 1, a_0 = 1$$



Exercise 3

A basketball is dropped onto the ground from a height of 15 feet. On each bounce, the ball reaches a maximum height 55% of its previous maximum height.

a) Write a recursive formula, a_n , that completely defines the height reached on the n_{th} bounce, where the first term in the sequence is the height reached on the ball's first bounce.

b) How high does the basketball reach after the 4_{th} bounce? Give your answer to two decimal places.

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ANSWER:

(a) $a_n = 0.55a_{n-1}, n \geq 1, a_0 = 8.25$

(b) 4^{th} bounce, $n = 3$;

$\begin{aligned} a_1 &= 0.55a_0 \\ &= 0.55(8.25) \\ &= 4.5375 \end{aligned}$	$\begin{aligned} a_2 &= 0.55a_1 \\ &= 0.55(4.5375) \\ &= 2.495625 \end{aligned}$	$\begin{aligned} a_3 &= 0.55a_2 \\ &= 0.55(2.495625) \\ &= 1.37259375 \end{aligned}$
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$\therefore 4^{\text{th}}$ bounce = 1.37 ft.