IN SLIDE EXERCISE FOR CHAPTER 1

GROUP 5

SECTION 03 - SEM 1, 2024/2025

SECI1013 *(DISCRETE STRUCTURE)*

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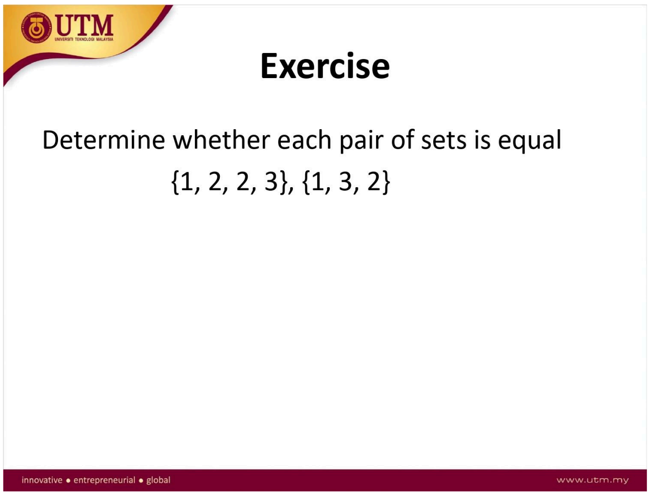
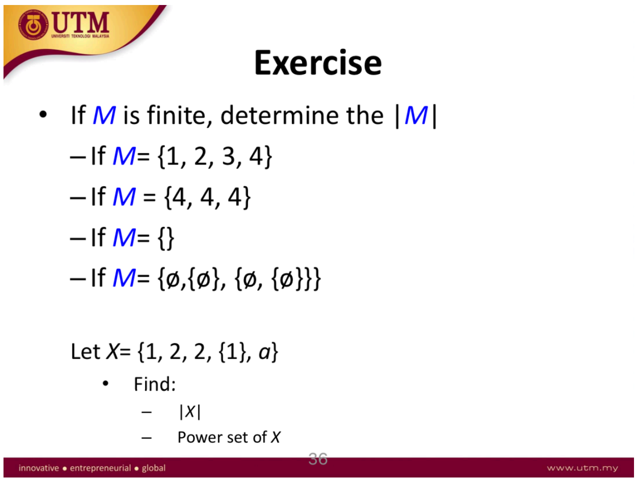
" Discrete elements, cohesive structure."



ANSWER:

Yes. They are equal because both sets contain the same elements, even if some element is duplicating.



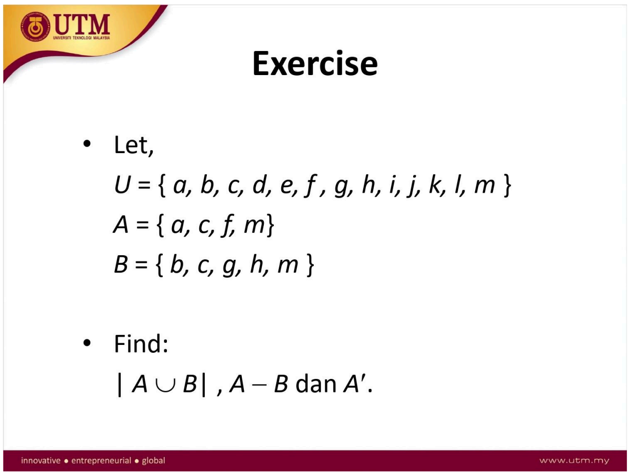
ANSWER:

$M = \{1, 2, 3, 4\}$	$M = \{4, 4, 4\}$	$M = \{\}$	$M = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$
$\lvert M\rvert = 4$	$\lvert M\rvert = 1$	$\lvert M\rvert = 0$	$\lvert M\rvert = 3$

$|X| = 4$

Power set of X $= 2^{|X|} = 2^4$

$|P(X)| = 16$

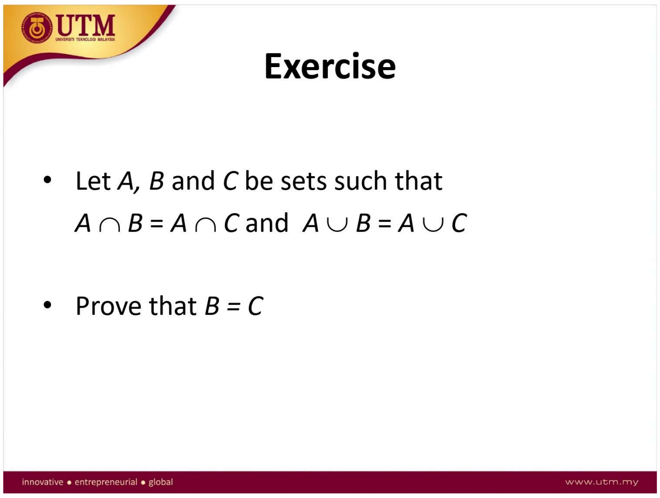


ANSWER:

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= 4 + 5 - 2$$
$$= 7$$

$$A - B = \{a, f\}$$

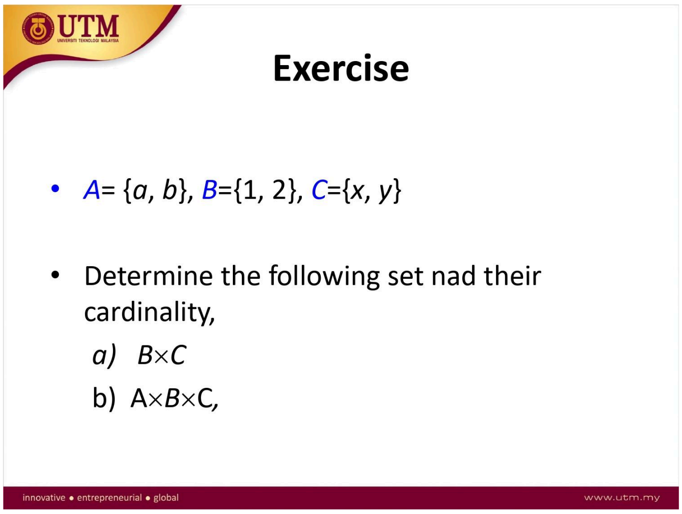
$$A' = \{b, d, e, g, h, i, j, k, l\}$$



ANSWER:

$$
\begin{aligned}
B &= B \cup \emptyset && : \textit{properties of empty set} \\
&= B \cup (A \cap \emptyset) && : \textit{properties of empty set} \\
&= (A \cup B) \cap (B \cup \emptyset) && : \textit{distributive law} \\
&= (A \cup C) \cap B && : \textit{properties of empty set} \\
&= (A \cap B) \cup (B \cap C) && : \textit{distributive law} \\
&= (A \cap C) \cup (B \cap C) && : \textit{by given conditions} \\
&= C \cap (A \cup B) && : \textit{distributive law} \\
&= C \cap (A \cup C) && : \textit{by given conditions} \\
&= C && : \textit{absorption law}
\end{aligned}
$$

$\therefore$ Proven, $B = C$.



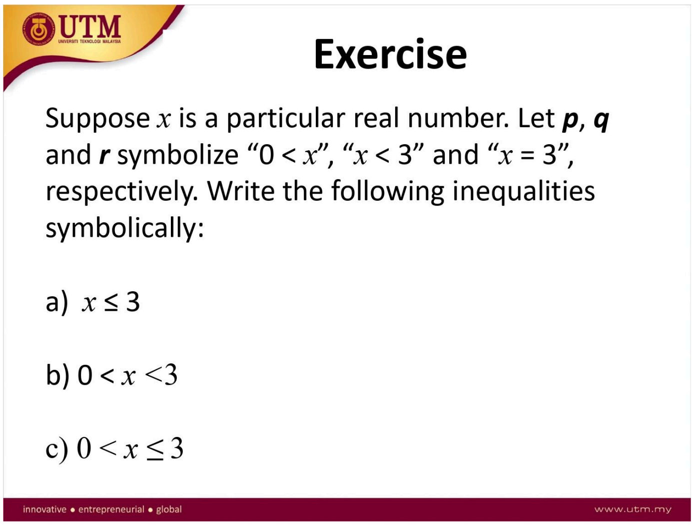
ANSWER:

(a) $B \times C = \{(1, x), (1, y), (2, x), (2, y)\}$

$$\begin{aligned} |B \times C| &= |B| \times |C| \\ &= 2 \times 2 \\ &= 4 \end{aligned}$$

(b) $A \times B \times C = \{(a, 1, x), (a, 1, y), (a, 2, x), (a, 2, y), (b, 1, x), (b, 1, y), (b, 2, x), (b, 2, y)\}$

$$\begin{aligned} |A \times B \times C| &= |A| \times |B| \times |C| \\ &= 2 \times 2 \times 2 \\ &= 8 \end{aligned}$$



ANSWER:

(a) $q \wedge r$

(b) $p \vee q$

(c) $p \vee (q \wedge r)$

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Exercise

Propositional functions ***p***, ***q*** and ***r*** are defined as follows:

p is "$n = 7$"
q is "$a > 5$"
r is "$x = 0$"

Write the following expressions in terms of ***p***, ***q*** and ***r***, and show that each pair of expressions is **logically equivalent**. State carefully which of the above laws are used at each stage.

(a) $((n = 7)$ or $(a > 5))$ *and* $(x = 0)$
$((n = 7)$ *and* $(x = 0))$ or $((a > 5)$ *and* $(x = 0))$

(b) $\neg((n = 7)$ *and* $(a \le 5))$
$(n \ne 7)$ or $(a > 5)$

(c) $(n = 7)$ or $(\neg((a \le 5)$ *and* $(x = 0)))$
$((n = 7)$ or $(a > 5))$ or $(x \ne 0)$

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ANSWER:

(a)

p	q	r	$p \vee q$	$(p \vee q) \wedge r$	$p \wedge r$	$q \wedge r$	$(p \wedge r) \vee (q \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	T	T	F	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

$\therefore (p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$

Distributive Law.

(b)

p	q	$\neg p$	$\neg q$	$\neg(p \wedge \neg q)$	$\neg p \vee q$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$\therefore \neg(p \wedge \neg q) \equiv \neg p \vee q$

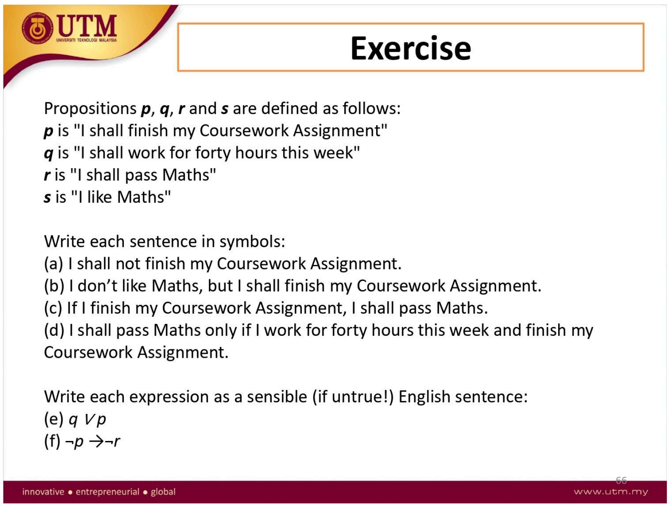
De Morgan's Law.

(c)

p	q	r	$\neg q$	$\neg r$	$\neg(\neg q \wedge r)$	$p \vee (\neg(\neg q \wedge r))$	$p \vee q$	$(p \vee q) \vee \neg r$
T	T	T	F	F	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	T	T	T
T	F	F	T	T	T	T	T	T
F	T	T	F	F	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	F	F	F	F	F
F	F	F	T	T	T	T	F	T

$\therefore p \vee (\neg(\neg q \wedge r)) \equiv (p \vee q) \vee \neg r$

De Morgan's Law & Distributive Law.



ANSWER:

(a) $\neg p$

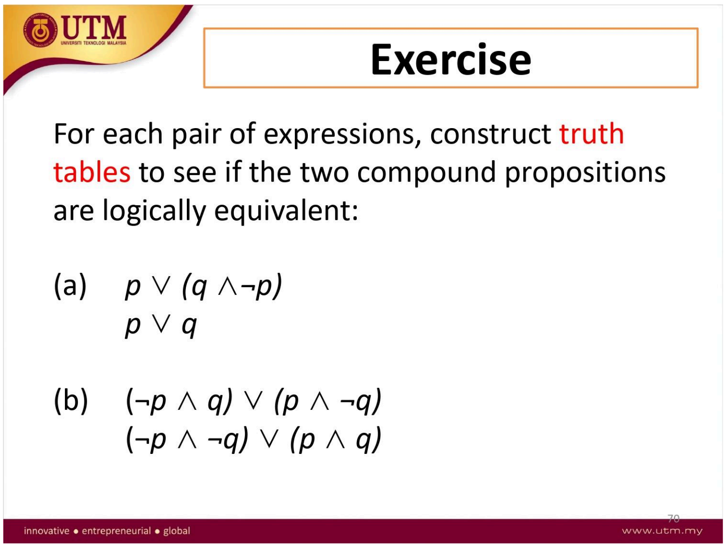
(b) $\neg s \wedge p$

(c) $p \rightarrow r$

(d) $r \leftrightarrow (q \wedge p)$

(e) I shall work for forty hours this week or finish my Coursework Assignment.

(f) If I don't finish my Coursework Assignment, then I will not pass Maths.



ANSWER:

(a)

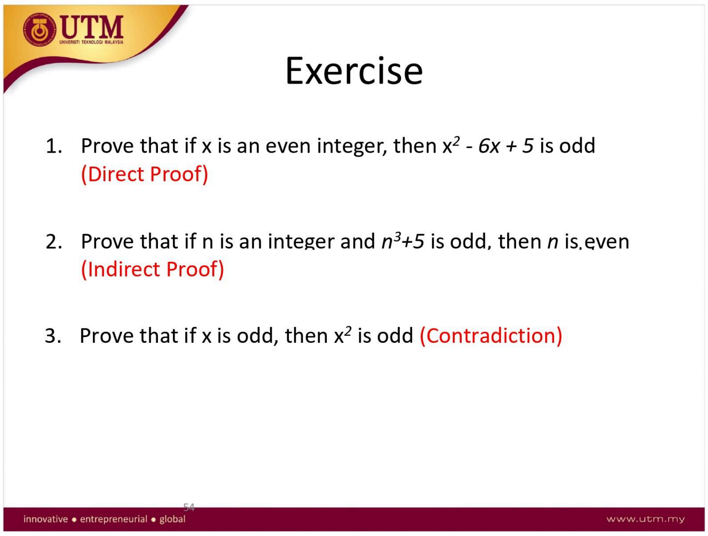
p	q	$\neg p$	$q \wedge \neg p$	$p \vee (q \wedge \neg p)$	$p \vee q$	
T	T	F	F	T	T	✓
T	F	F	F	T	T	✓
F	T	T	T	T	T	✓
F	F	T	F	F	F	✓

$\therefore p \vee (q \wedge \neg p) \equiv p \vee q$

(b)

p	q	$\neg p$	$\neg q$	$\neg p \wedge q$	$p \wedge \neg q$	$\neg p \wedge \neg q$	$p \wedge q$	$(\neg p \wedge q) \vee (p \wedge \neg q)$	$(\neg p \wedge \neg q) \vee (p \wedge q)$	
T	T	F	F	F	F	F	T	F	T	✗
T	F	F	T	F	T	F	F	T	F	✗
F	T	T	F	T	F	F	F	T	F	✗
F	F	T	T	F	F	T	F	F	T	✗

$\therefore (\neg p \wedge q) \vee (p \wedge \neg q) \not\equiv (\neg p \wedge \neg q) \vee (p \wedge q)$



ANSWER:

1. Let;
$\mathrm{P}(x) = x$ is even integer.
$\mathrm{Q}(x) = x^2 - 6x + 5$ is odd.

Symbolically, $\forall x(\mathrm{P}(x) \rightarrow \mathrm{Q}(x))$ with domain of discourse is the set of all integers.
Let $2n$ be an even integer.

$$
\begin{aligned}
\mathrm{Q}(2n) &= (2n)^2 - 6(2n) + 5 \\
&= 4n^2 - 12n + 5 \\
&= 2(2n^2 - 6n) + 5 && : 2n^2 - 6n = m\text{, where } m \text{ is an integer} \\
&= 2m + 5 && : \text{odd integer}
\end{aligned}
$$

$\therefore$ Proven, $\forall x(\mathrm{P}(x) \rightarrow \mathrm{Q}(x))$.

2. Let;
$\mathrm{P}(n) = n$ is an integer and $n^3 + 5$ is odd.
$\mathrm{Q}(n) = n$ is even.

$\forall n(\mathrm{P}(n) \rightarrow \mathrm{Q}(n))$; $\mathrm{P}(n) \rightarrow \mathrm{Q}(n) \equiv \neg\mathrm{Q}(n) \rightarrow \neg\mathrm{P}(n)$
$\neg\mathrm{Q}(n) = n$ is odd. Suppose $\neg\mathrm{Q}(n)$ is true, we need to show that $\neg\mathrm{P}(n)$ is true.
Let $2k + 1$ be an odd integer. $\neg\mathrm{P}(n)$ is true if $n^3 + 5$ is even.

$$
\begin{aligned}
\mathrm{P}(2k + 1) &= (2k + 1)^3 + 5 \\
&= 8k^3 + 12k^2 + 6k + 6 \\
&= 2(4k^3 + 6k^2 + 3k) + 6 && : 4k^3 + 6k^2 + 3k = t\text{, where } t \text{ is an integer} \\
&= 2t + 6 && : \text{even integer}
\end{aligned}
$$

$\therefore$ Proven, $\forall x(\neg\mathrm{Q}(n) \rightarrow \neg\mathrm{P}(n))$.

3. Let;

   $P(x) = x$ is odd.

   $Q(x) = x^2$ is odd.

   Originally, $\forall x(P(x) \to Q(x))$ is true but it is also true if the contradiction, $\forall x(P(x) \to \neg Q(x))$ is false. Suppose x is odd and x^2 is not odd.
   Let $2n + 1$ be an odd integer. $\neg Q(x)$ is false if x^2 is odd.

   $$\begin{aligned} Q(2n+1) &= (2n+1)^2 \\ &= 4n^2 + 4n + 1 \\ &= 2(2n^2 + 2n) + 1 && : 2n^2 + 2n = m, \text{ where } m \text{ is an integer} \\ &= 2m + 1 && : \text{odd integer} \end{aligned}$$

   $\neg Q(x)$ is false since x^2 is odd. Thus, the statement $\forall x(P(x) \to \neg Q(x))$ is false.
   $\therefore$ Proven, $\forall x(P(x) \to Q(x))$.