



# IN SLIDE EXERCISE FOR CHAPTER 4

*GROUP 5*

SECTION 03 - SEM 1, 2024/2025

**SECI1013 (*DISCRETE STRUCTURE*)**

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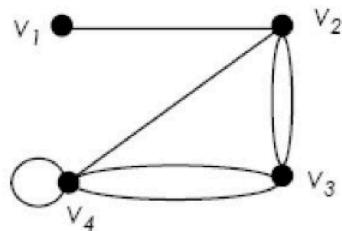
*“Order between complexity”*

## PART 1



### Exercise 1

- Find the degree of each vertex in the graph.



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### ANSWER:

Exercise 1

$$\deg(v_1) = 1$$

$$\deg(v_2) = 4$$

$$\deg(v_3) = 4$$

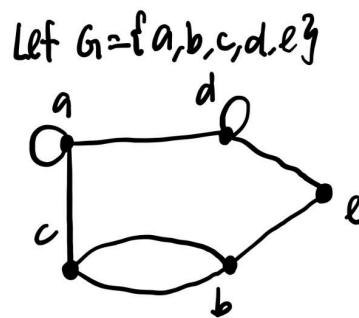
$$\deg(v_4) = 5$$

## Exercise 2

Draw the graph based on the following matrix:

$$A_G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

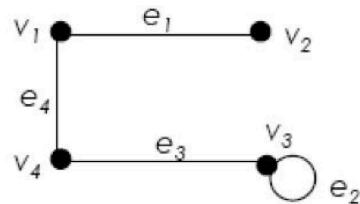
**ANSWER:**





## Exercise 3

- Find the adjacency matrix and the incidence matrix of the graph.



**ANSWER:**

Ex 3

adjacency matrix:

$$A_G = \begin{bmatrix} v_1 & v_2 & v_3 & v_4 \\ v_1 & 0 & 1 & 0 & 1 \\ v_2 & 1 & 0 & 0 & 0 \\ v_3 & 0 & 0 & 1 & 1 \\ v_4 & 1 & 0 & 1 & 0 \end{bmatrix}$$

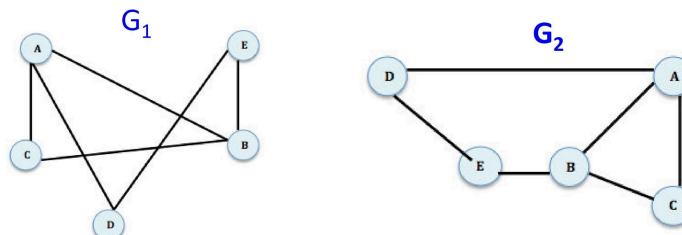
incidence matrix:

$$I_G = \begin{bmatrix} v_1 & e_1 & e_2 & e_3 & e_4 \\ v_2 & 1 & 0 & 0 & 1 \\ v_3 & 0 & 1 & 1 & 0 \\ v_4 & 0 & 0 & 1 & 1 \end{bmatrix}$$



## Exercise 4

Q: Show that the following two graphs are isomorphic.



**ANSWER:**

Both graphs are simple and have the same number of vertices and the same number of edges.

Vertices of both graphs have equivalent number of degree.

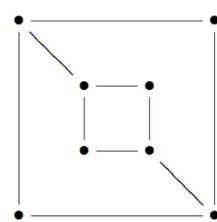
$$\begin{aligned} \mathbf{A}_{G_1} &= \begin{bmatrix} A & B & C & D & E \\ A & 0 & 1 & 1 & 1 & 0 \\ B & 1 & 0 & 1 & 0 & 1 \\ C & 1 & 1 & 0 & 0 & 0 \\ D & 1 & 0 & 0 & 0 & 1 \\ E & 0 & 1 & 0 & 1 & 0 \end{bmatrix} & \mathbf{A}_{G_2} &= \begin{bmatrix} A & B & C & D & E \\ A & 0 & 1 & 1 & 1 & 0 \\ B & 1 & 0 & 1 & 0 & 1 \\ C & 1 & 1 & 0 & 0 & 0 \\ D & 1 & 0 & 0 & 0 & 1 \\ E & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{aligned}$$

$\mathbf{A}_{G_1}$  and  $\mathbf{A}_{G_2}$  are the same, thus  $G_1$  and  $G_2$  are isomorphic.

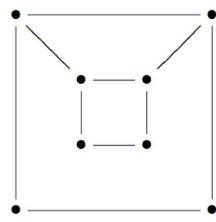
## Exercise 5

Q: Is these two graphs are isomorphic?

$G:$



$H:$



**ANSWER:**

Exercise 5

$G = \{U_1, U_2, U_3, U_4, U_5, U_6, U_7, U_8, U_9\}$

$G = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9\}$

$f(G) = H$

$f(U_1) = V_1$

$f(U_2) \neq V_2$

$f(U_3) = V_3$

$f(U_4) \neq V_4$

Since the degrees of corresponding vertices not equal for both graph, then it is not isomorphic.

**Exercise 6**

Tell whether the following is either a trail, path, circuit, simple circuit or none of these.

- $(V_2, e_2, V_3, e_3, V_4, e_4, V_3)$
- $(V_4, e_7, V_5, e_6, V_1, e_1, V_2, e_2, V_3, e_3, V_4)$
- $(V_4, e_4, V_3, e_3, V_4, e_5, V_2, e_1, V_1, e_6, V_5, e_7, V_4)$

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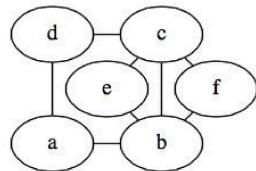
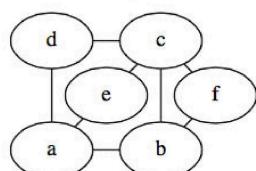
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**ANSWER:**

- $(V_2, e_1, V_3, e_3, V_4, e_4, V_3)$   
 $\therefore$  None of these
- $(V_4, e_7, V_5, e_6, V_1, e_1, V_2, e_2, V_3, e_3, V_4)$   
 $\therefore$  Simple circuit
- $(V_4, e_4, V_3, e_3, V_4, e_5, V_2, e_1, V_1, e_6, V_5, e_7, V_4)$   
 $\therefore$  Circuit

## Exercise 7

Q: Which of the following graphs has Euler circuit?  
Justify your answer.

**G<sub>1</sub>****G<sub>2</sub>****ANSWER:**

*(Ex 7)*

Which of the following graphs has Euler circuit?  
Justify your answer.

*G<sub>1</sub>*

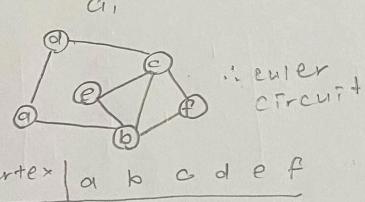


Diagram of graph  $G_1$  showing vertices a, b, c, d, e, f and edges connecting them. A path is highlighted in red starting from vertex a, passing through b, c, d, e, f, and returning to a.

$\therefore$  euler circuit

vertex	a	b	c	d	e	f
edge	2	2	2	2	2	2

*G<sub>2</sub>*

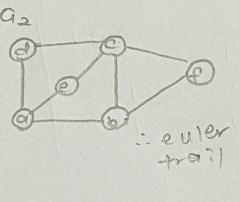


Diagram of graph  $G_2$  showing vertices a, b, c, d, e, f and edges connecting them. A path is highlighted in red starting from vertex a, passing through b, c, d, e, f, and returning to a.

$\therefore$  euler trail

vertex	a	b	c	d	e	f
edge	3	3	4	2	2	2

$\therefore$  because a and b have odd degree

*(Ex 7)*

because all vertex does not have odd degree

**Exercise 8**

**Question:** Is this graph has Hamiltonian cycle?

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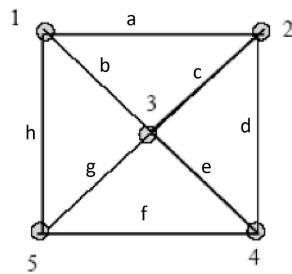
**ANSWER:**

yes, the graph has Hamiltonian cycle.  
The cycle is  $(1, 2, 6, 4, 3, 5, 7, 1)$



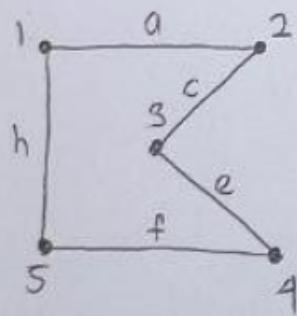
## Exercise 9

**Question:** Prove that this graph has Hamiltonian circuit.



**ANSWER:**

Exercise 9

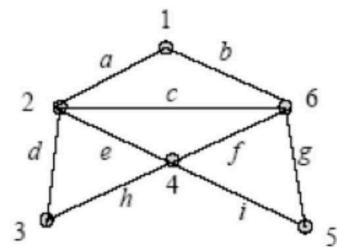


(1, a, 2, c, 3, e, 4, f, 5, h, 1)

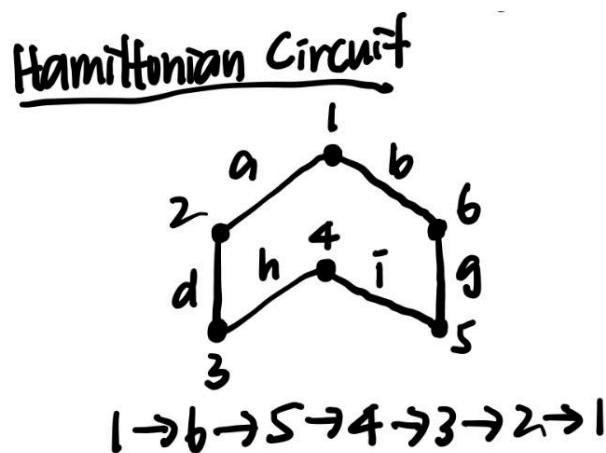
Proven it has Hamiltonian Circuit

## Exercise 10

Find a Hamiltonian circuit in this graph.



**ANSWER:**



## PART 2

**Exercise 1**

```

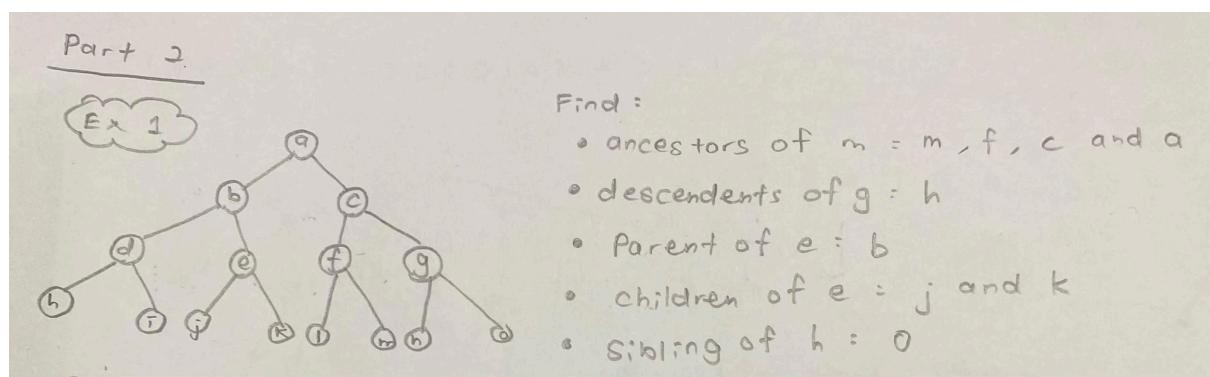
graph TD
    a((a)) --> b((b))
    a --> c((c))
    b --> d((d))
    b --> e((e))
    c --> f((f))
    c --> g((g))
    d --> h((h))
    d --> i((i))
    e --> j((j))
    e --> k((k))
    f --> l((l))
    f --> m((m))
    g --> n((n))
    n --> o((o))
  
```

Find:

- Ancestors of m
- Descendents of g
- Parent of e
- Children of e
- Sibling of h

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**ANSWER:**





## Exercise 2

- How many matches are played in a tennis tournament of 27 players?

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### ANSWER:

$$\begin{array}{ll} n = 27 & l = (n-1)/m \\ m = 2 & = (27-1)/2 \\ l = ? & = 13 \text{ matches} \end{array}$$

## Exercise 3

Suppose 1000 people enter a chess tournament. Use a rooted tree model of the tournament to determine how many games must be played to determine a champion, if a player is eliminated after one loss and games are played until only one entrant has not lost. (Assume there are no ties.)

**ANSWER:**

### Exercise 3

$$\begin{array}{ll} n=1000 & i=(n-1)/m \\ m=2 & = (1000-1)/2 \\ i=? & = 499 \end{array}$$

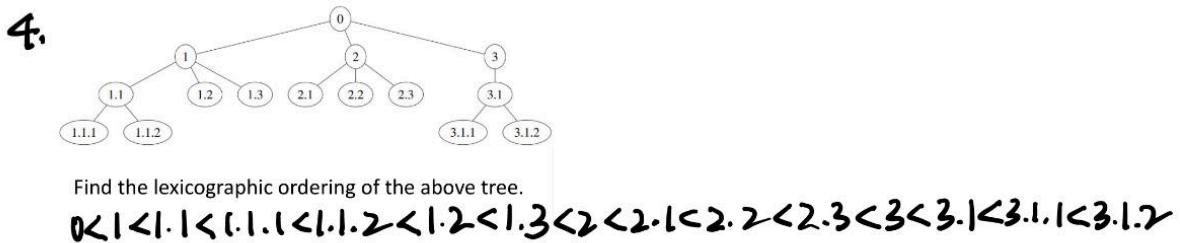
**Exercise 4**

```

graph TD
    0((0)) --- 1((1))
    0 --- 2((2))
    1 --- 1_1((1.1))
    1 --- 1_2((1.2))
    2 --- 2_1((2.1))
    2 --- 2_2((2.2))
    2 --- 2_3((2.3))
    3((3)) --- 3_1((3.1))
    3_1 --- 3_1_1((3.1.1))
    3_1 --- 3_1_2((3.1.2))
  
```

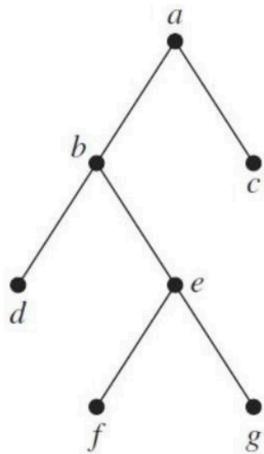
Find the lexicographic ordering of the above tree.

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**ANSWER:**



## Exercise 5

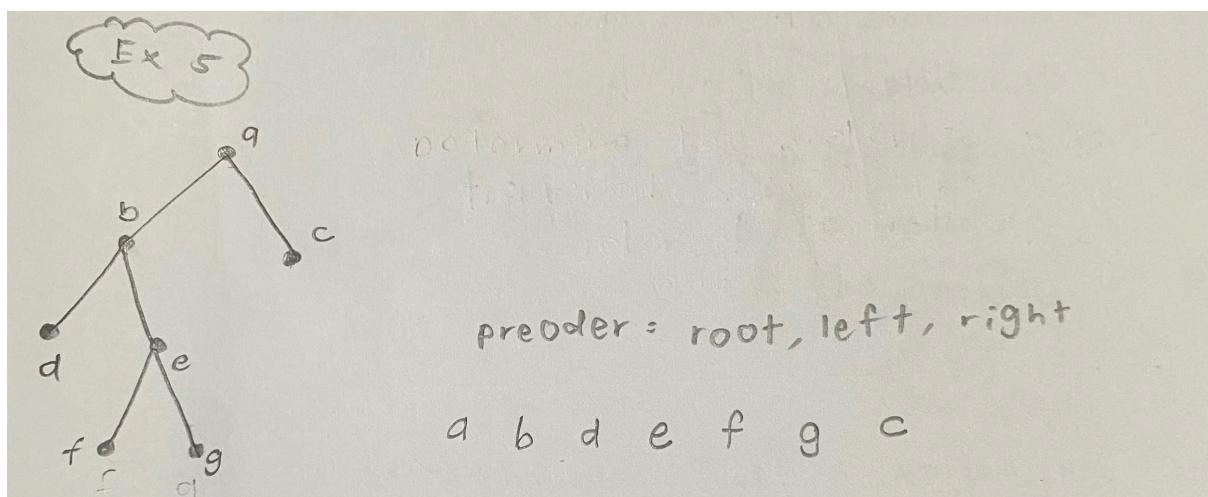


Determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.

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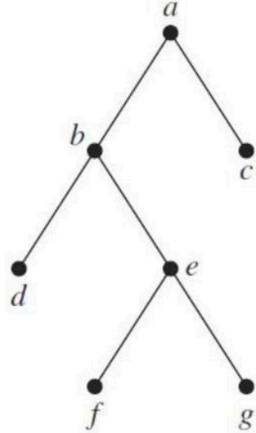
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### ANSWER:





## Exercise 6



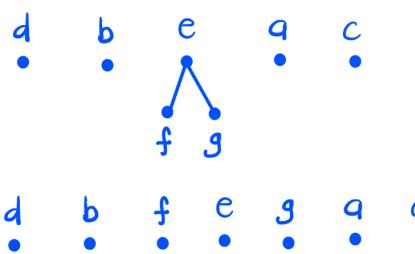
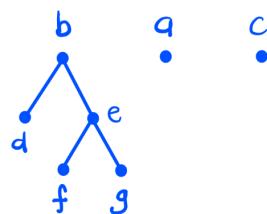
Determine the order in which a inorder traversal visits the vertices of the given ordered rooted tree.

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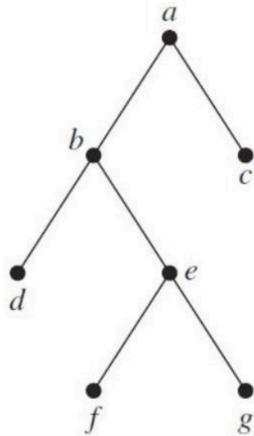
### ANSWER:

Inorder traversal : Left-subtree, root, right-subtree





## Exercise 7



Determine the order in which a postorder traversal visits the vertices of the given ordered rooted tree.

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**ANSWER:**

Exercise 7

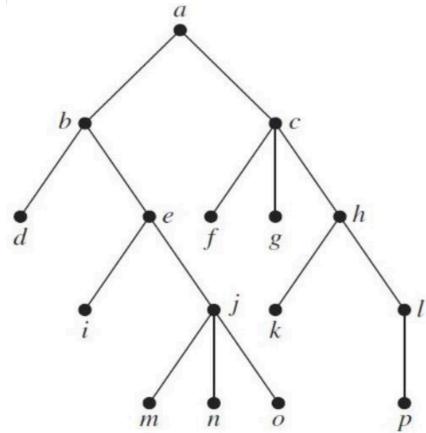
Answer:

Handwritten notes show two traversals of the tree:

- Top traversal: d → f → g → e → b → c → a
- Bottom traversal: d → f → g → e → b → c → a



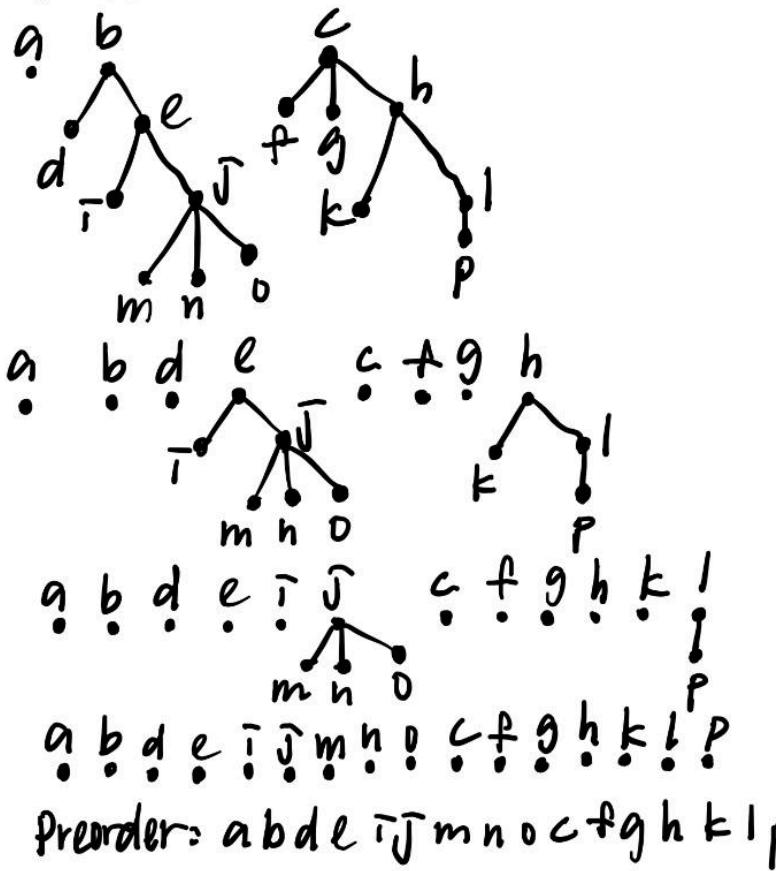
## Exercise 8, 9, 10



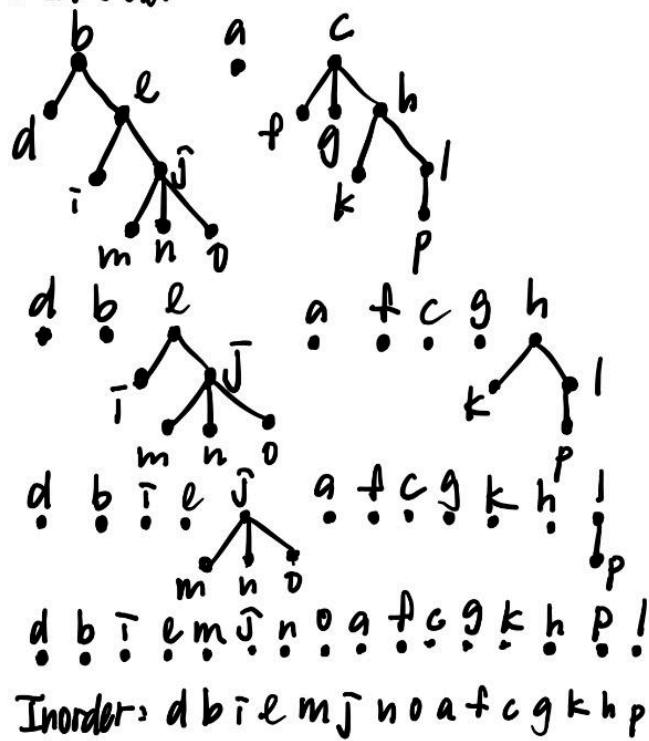
Determine the order of preorder (8), inorder (9) and postorder (10) of the given rooted tree.

**ANSWER:**

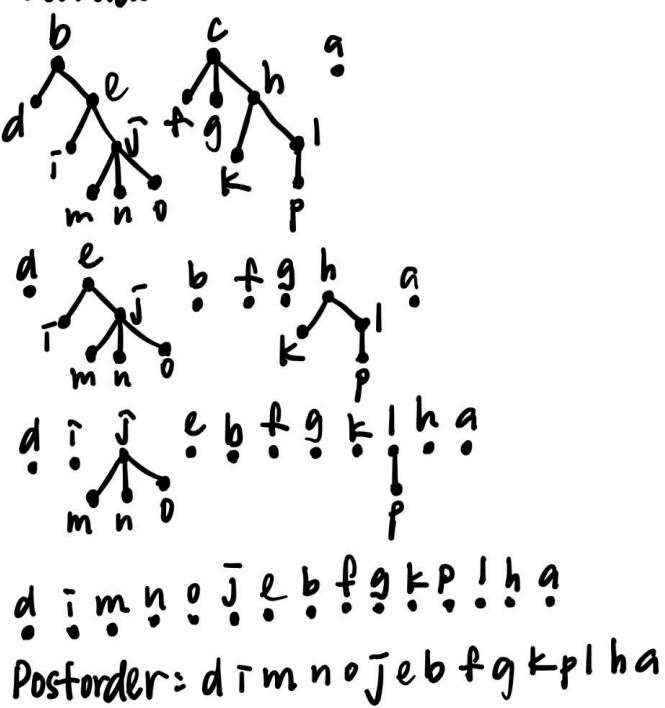
### 8. Preorder



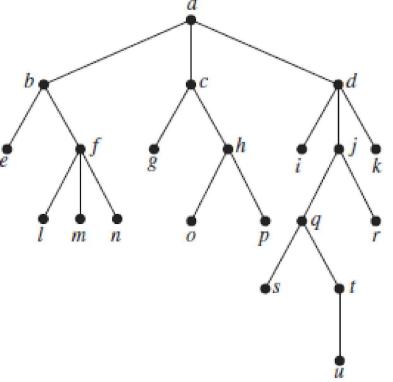
## 9. Inorder



## 10. Postorder

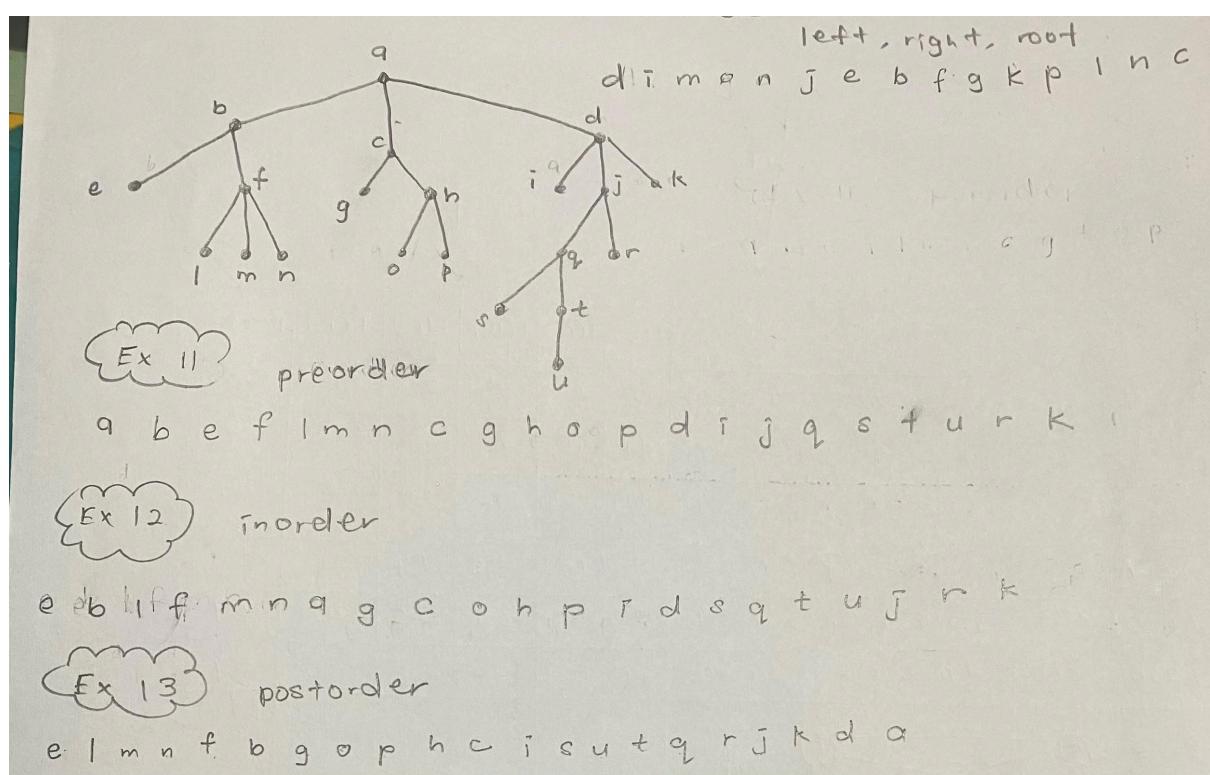


 **Exercise 11, 12, 13**



Determine the order of preorder (11), inorder (12) and postorder (13) of the given rooted tree.

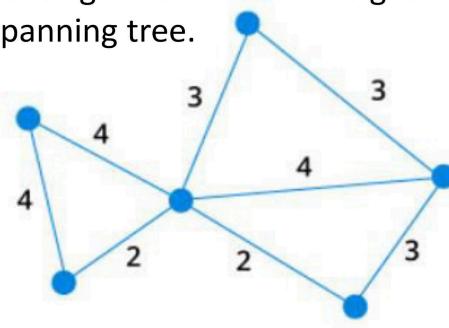
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**ANSWER:**



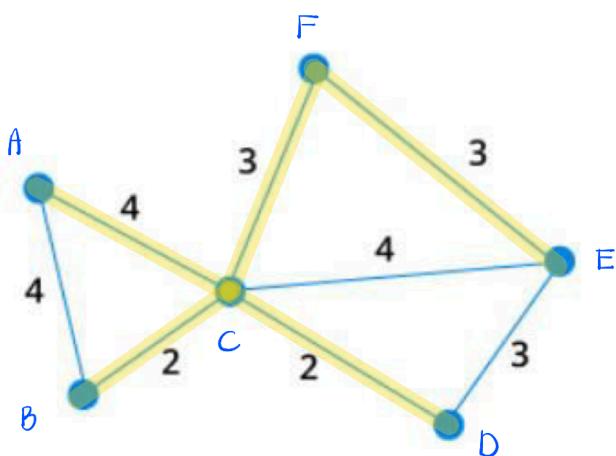
## Exercise 14

Find the minimum spanning tree using Kruskal's Algorithm and give the total weight for the minimum spanning tree.



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**ANSWER:**

Let all vertices labelled with A,B,C,D,E,F  
List out all edges in order of its size:

$BC = 2$  ①  
 $CD = 2$  ②  
 $CF = 3$  ③  
 $EF = 3$  ④  
 $DE = 3$   
 $AB = 4$   
 $AC = 4$  ⑤  
 $CE = 4$

The solution is:  
 $BC = 2$   
 $CD = 2$   
 $CF = 3$   
 $EF = 3$   
 $AC = 4$   

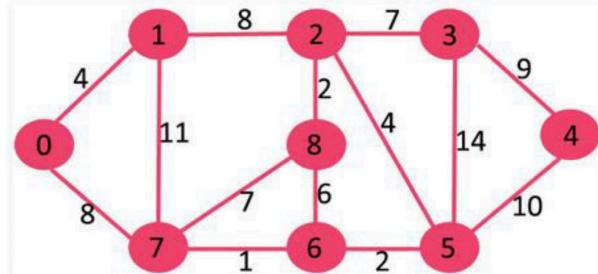

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 Total : 14 weight



## Exercise 15

Find the minimum spanning tree using Kruskal's Algorithm and give the total weight for the minimum spanning tree.

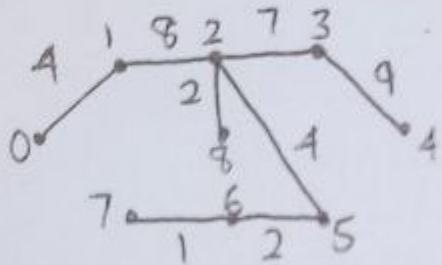


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**ANSWER:**

Exercise 15



Total spanning = 37

## PART 3

**Exercise 1**

Q: Given a weighted digraph, find the shortest path from **S** to **T**, using Dijkstra Algorithm.

Note: Weights are arbitrary numbers (i.e., not necessarily distances).

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### ANSWER:

Iteration	S	N	L(S)	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(T)
0	{S}	{S, A, B, C, D, E, F, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{S}	{A, B, C, D, E, F, T}	0	9	14	15	∞	∞	∞	∞
2	{S, A}	{B, C, D, E, F, T}	0	9	14	15	23	∞	∞	∞
3	{S, A, B}	{C, D, E, F, T}	0	9	14	15	32	44	∞	∞
4	{S, A, B, C}	{D, E, F, T}	0	9	14	15	32	35	∞	59
5	{S, A, B, C, D}	{E, F, T}	0	9	14	15	32	34	00	51
6	{S, A, B, C, D, E}	{F, T}	0	9	14	15	32	34	45	40
7	{S, A, B, C, D, E, T}	{F}	0	9	14	15	32	34	45	00

Shortest path:  $S \rightarrow B \rightarrow D \rightarrow E \rightarrow T$

Total weight = 40

**Exercise 2**

**Figure 5**

Based on Dijkstra's algorithm, complete Table 1 to find the shortest path from city A to city H. (Note: Copy Table 1 into your answer booklet).

(8 marks)

**ANSWER:**

*(Ex 23)*

$A \rightarrow C \rightarrow F \rightarrow H$

Iteration	S	N	$L(A)$	$L(B)$	$L(C)$	$L(D)$	$L(E)$	$L(F)$	$L(G)$	$L(H)$
0	{ } {A}	{A,B,C,D,E,F,G,H}	0	$\infty$						
1	{A}	{B,C,D,E,F,G,H}	0	1	2↑	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$
2	{A,B}	{C,D,E,F,G,H}	0	1	2	6	4	$\infty$	$\infty$	$\infty$
3	{A,B,C}	{D,E,F,G,H}	0	1	2	5	4	6↑	$\infty$	$\infty$
4	{A,B,C,E}	{D,F,G,H}	0	1	2	5	4	6	11	$\infty$
5	{A,B,C,E,F}	{D,G,H}	0	1	2	5	4	6	11	8↑
6	{A,B,C,E,F,H}	{D,G}	0	1	2	5	4	6	11	8