IN SLIDE EXERCISE FOR CHAPTER 4

GROUP 5

SECTION 03 - SEM 1, 2024/2025

SECI1013 *(DISCRETE STRUCTURE)*

LECTURER : DR. MUHAMMAD ALIIF BIN AHMAD
DATE SUBMITTED : 5^{th} JANUARY 2025

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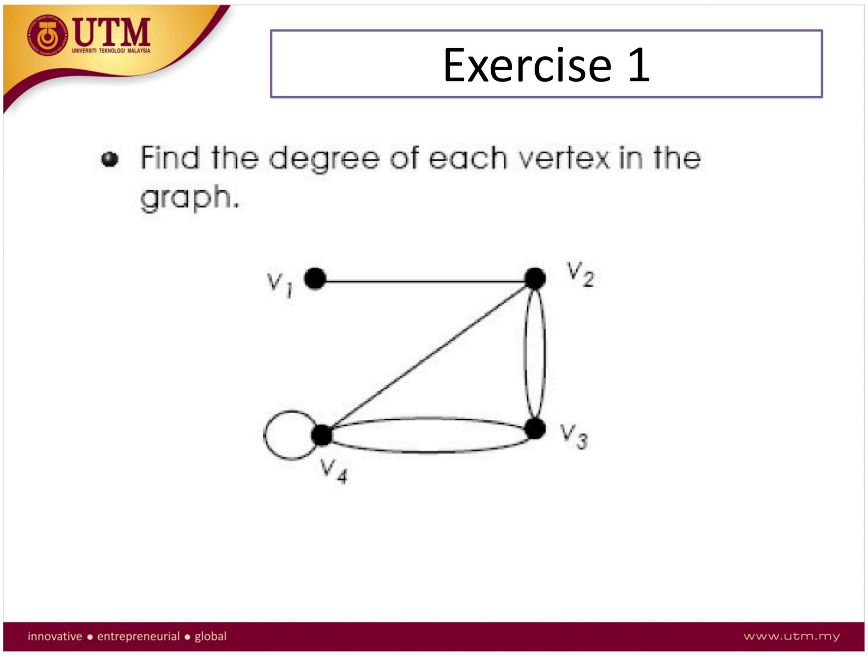
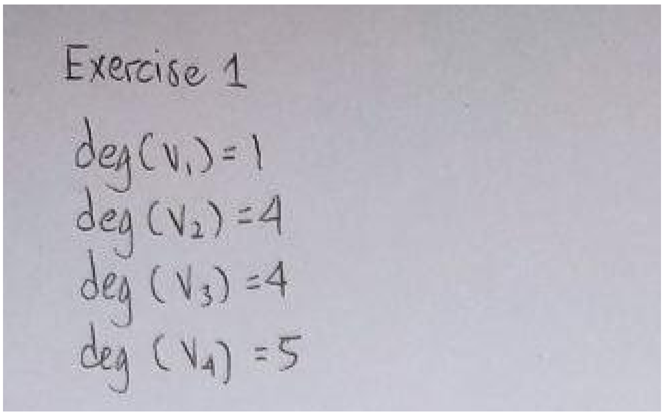
"Order between complexity"

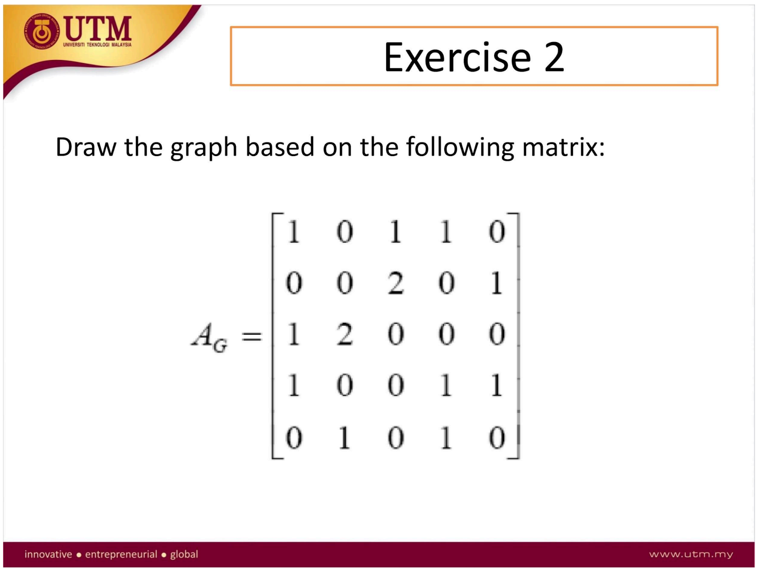
PART 1



ANSWER:

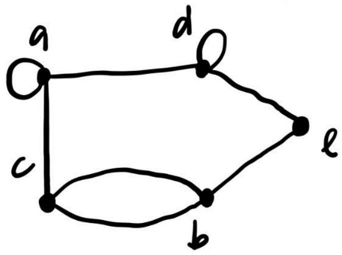

Exercise 1

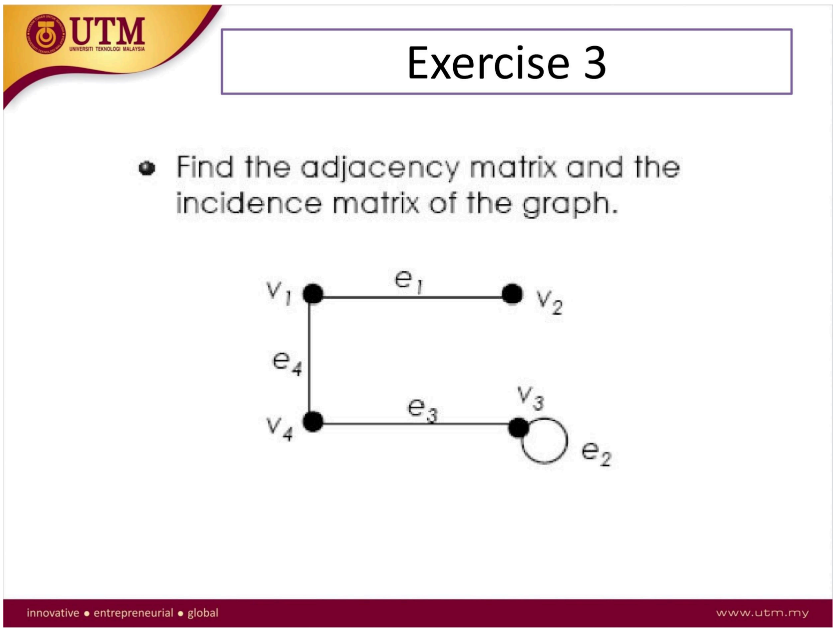
$deg(v_1) = 1$
$deg(v_2) = 4$
$deg(v_3) = 4$
$deg(v_4) = 5$



ANSWER:

Let $G = \{a, b, c, d, e\}$




UTM
Exercise 3
Find the adjacency matrix and the incidence matrix of the graph.
v1 e1 v2
e4
v4 e3 v3 e2
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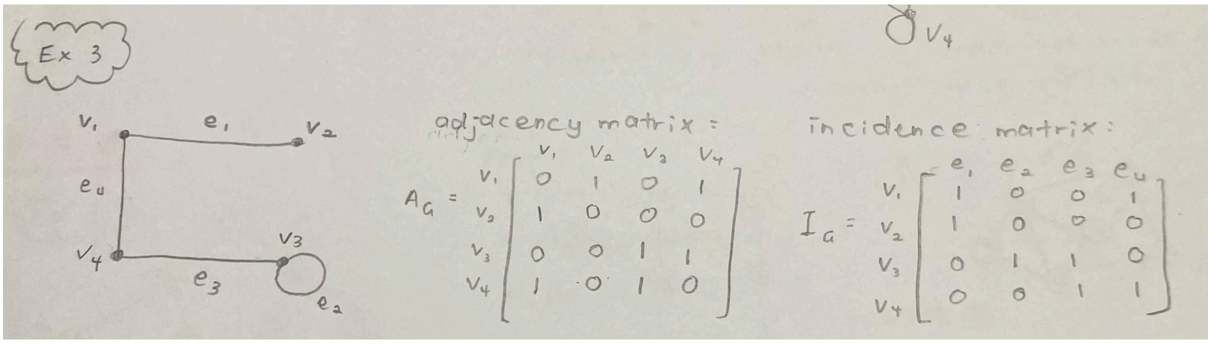
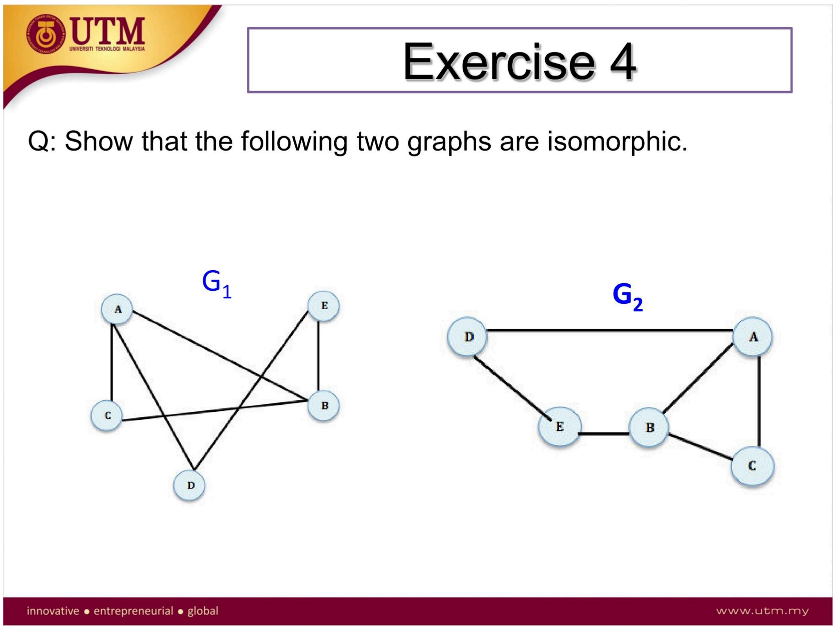
ANSWER:


Ex 3
v1 e1 v2
e4
v4 e3 v3 e2
v4
adjacency matrix :
$A_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$
incidence matrix :
$I_G = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$



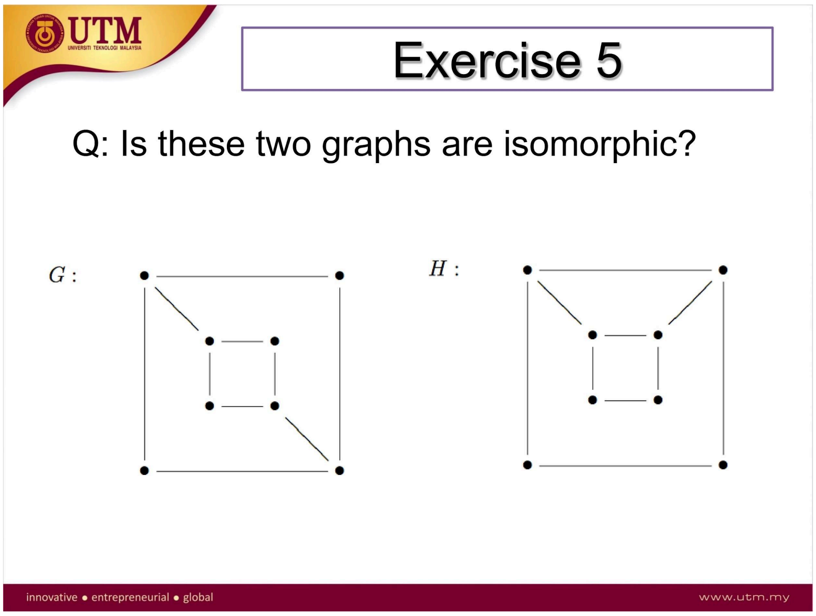
ANSWER:

Both graphs are simple and have the same number of vertices and the same number of edges.

Vertices of both graphs have equivalent number of degree.

$$A_{G_1} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad A_{G_2} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

A_{G_1} and A_{G_2} are the same, thus G_1 and G_2 are isomorphic.



ANSWER:

Exercise 5

~~$G = \{U_1, U_2$~~

$G = \{U_1, U_2, U_3, U_4, U_5, U_6, U_7, U_8, U_9\}$

$H = \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9\}$

$f(G) = H$

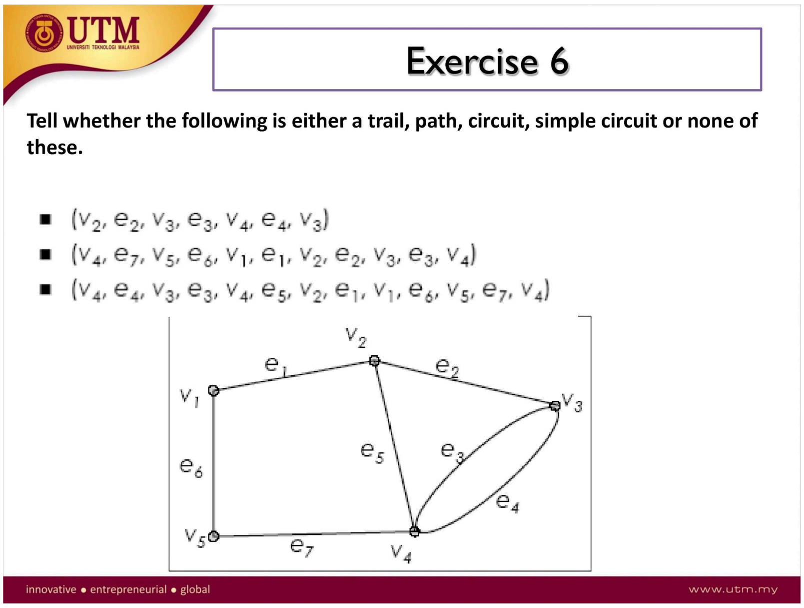
$f(U_1) = V_1$

$f(U_2) \neq V_2$

$f(U_3) = V_3$

$f(U_4) \neq V_4$

Since the degrees of corresponding vertices not equal for both graph, then it is not isomorphic



ANSWER:

- $(v_2, e_1, v_3, e_3, v_4, e_4, v_3)$

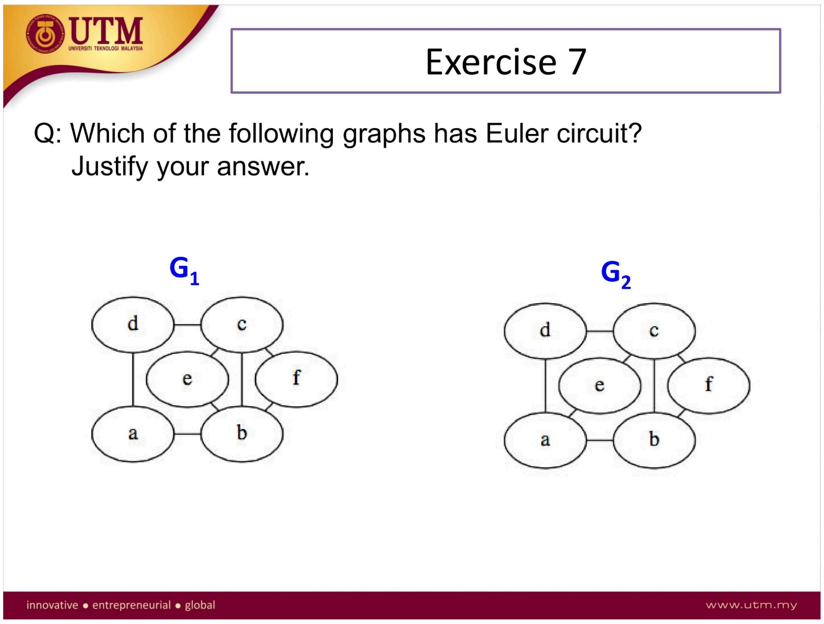
  $\therefore$ None of these

- $(v_4, e_7, v_5, e_6, v_1, e_1, v_2, e_2, v_3, e_3, v_4)$

  $\therefore$ Simple circuit

- $(v_4, e_4, v_3, e_3, v_4, e_5, v_2, e_1, v_1, e_6, v_5, e_7, v_4)$

  $\therefore$ Circuit


UTM
Exercise 7
Q: Which of the following graphs has Euler circuit?
Justify your answer.
G_1
G_2
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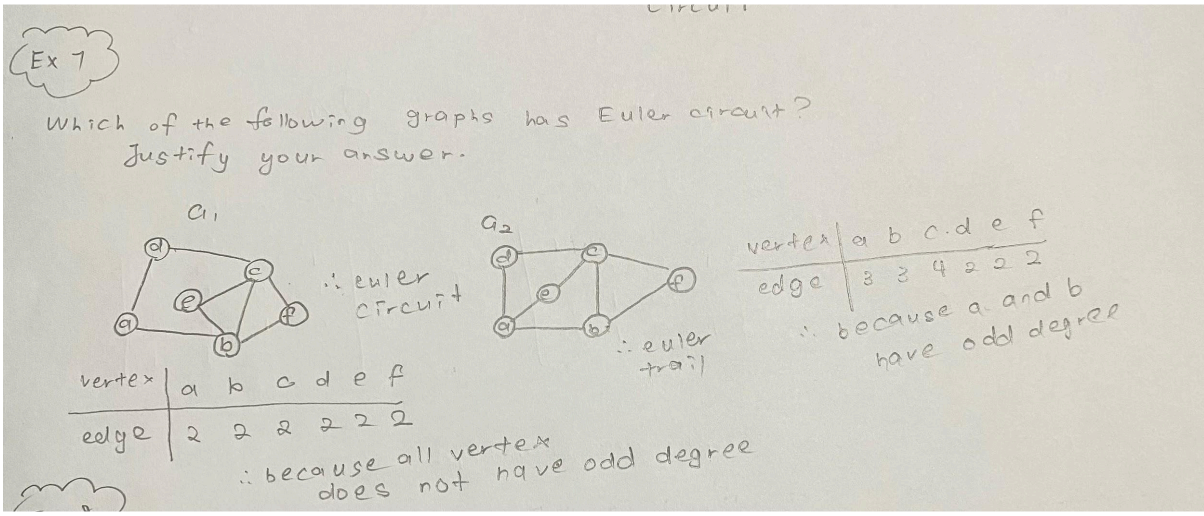
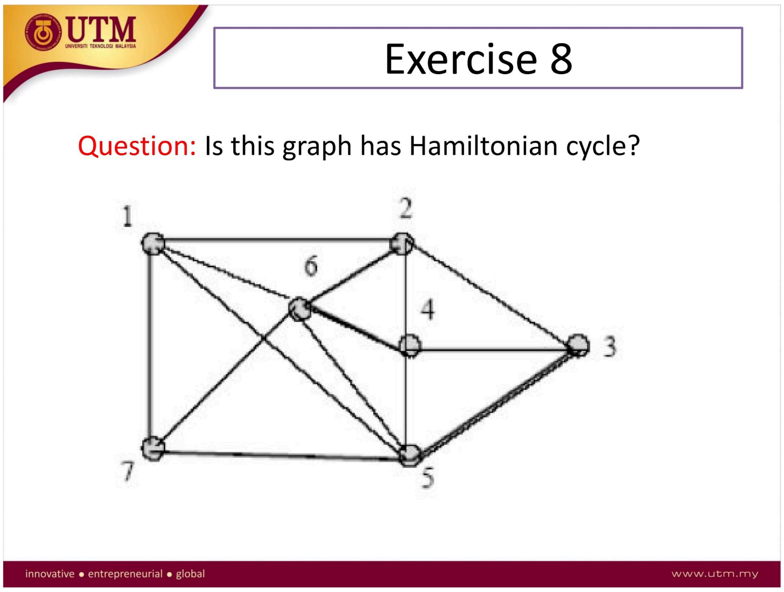
ANSWER:

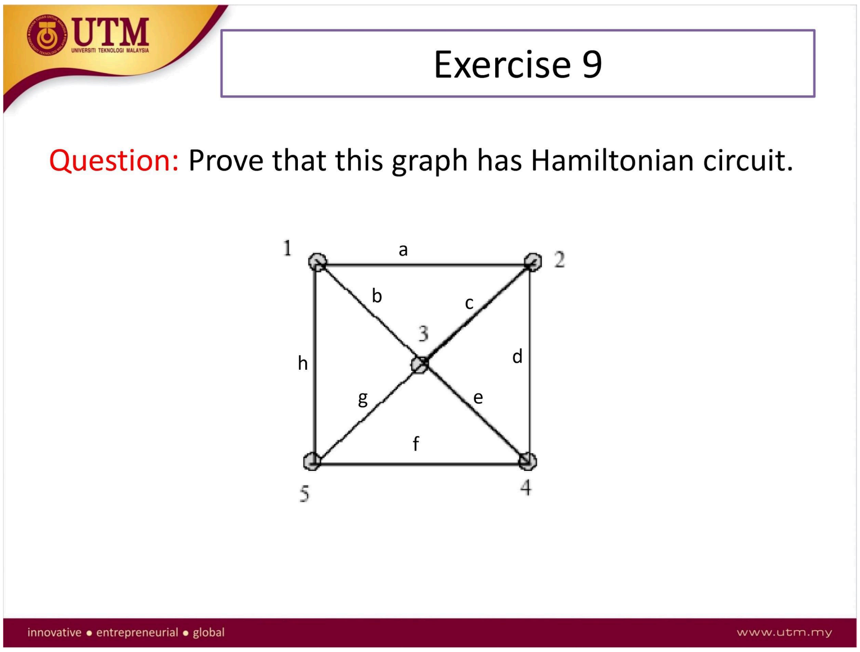

Ex 7
Which of the following graphs has Euler circuit?
Justify your answer.
G_1
∴ euler circuit
G_2
∴ euler trail
vertex | a b c d e f
edge | 3 3 4 2 2 2
∴ because a and b have odd degree
vertex | a b c d e f
edge | 2 2 2 2 2 2
∴ because all vertex does not have odd degree



ANSWER:

Yes, the graph has Hamiltonian cycle.

The cycle is (1, 2, 6, 4, 3, 5, 7, 1)


UTM
Exercise 9
Question: Prove that this graph has Hamiltonian circuit.
1
a
2
b
c
3
h
d
g
e
f
5
4
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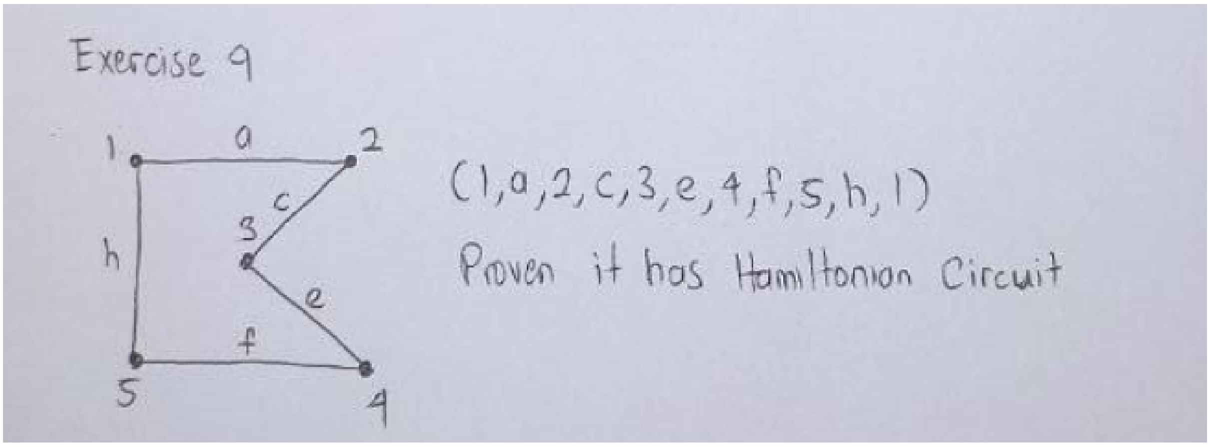
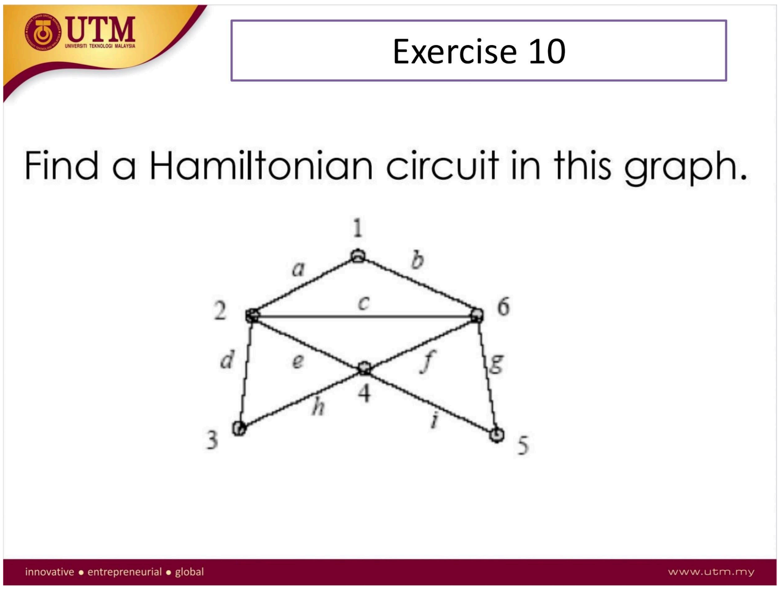
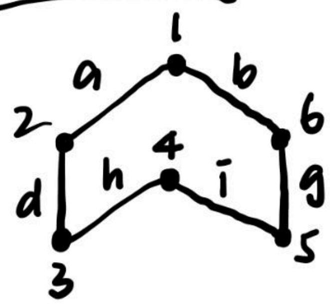
ANSWER:


Exercise 9
1
a
2
c
3
h
e
f
5
4
(1,a,2,c,3,e,4,f,5,h,1)
Proven it has Hamiltonian Circuit



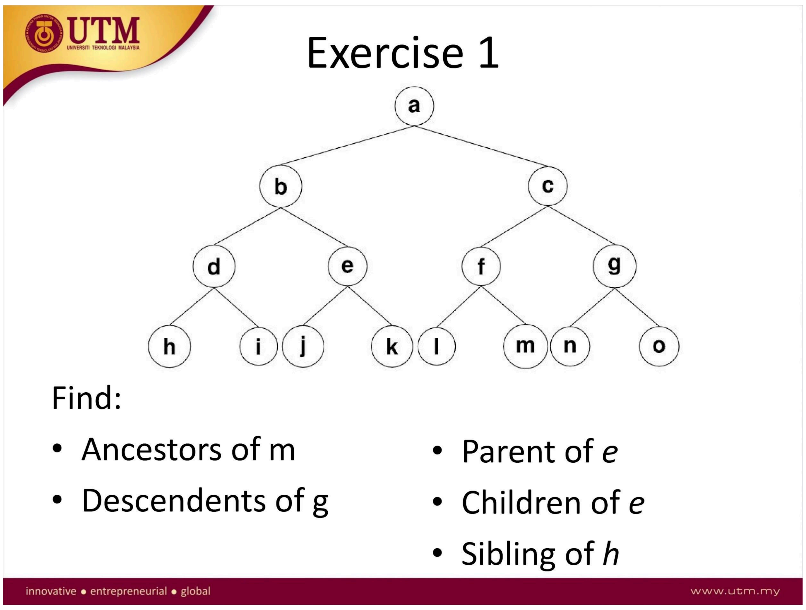
ANSWER:

Hamiltonian Circuit



1→6→5→4→3→2→1

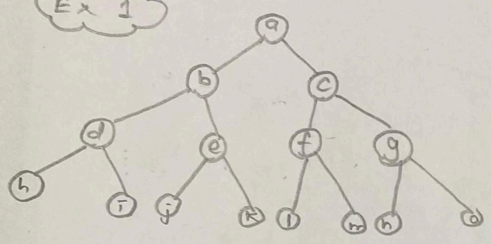
PART 2



ANSWER:

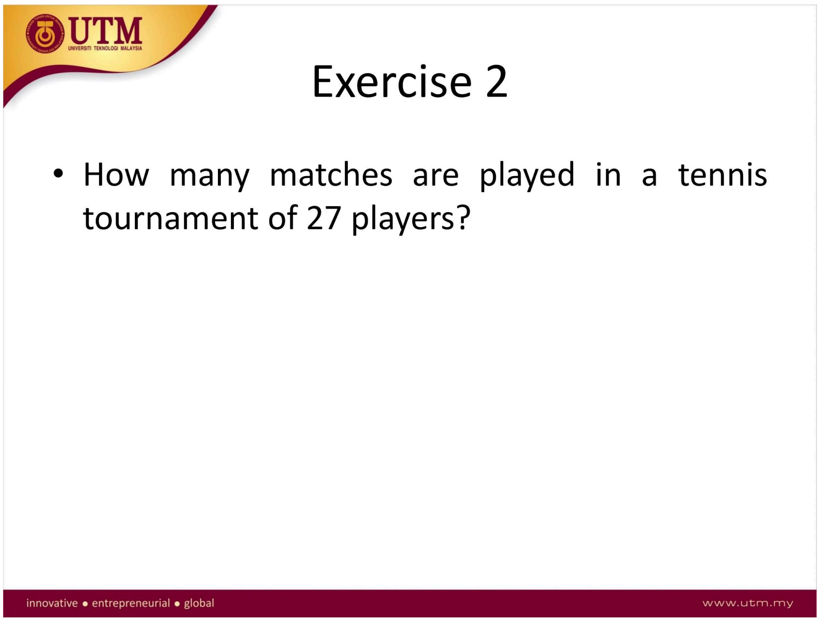
Part 2

Ex 1



Find :

- ancestors of m : m , f , c and a
- descendents of g : h
- Parent of e : b
- children of e : j and k
- Sibling of h : 0



ANSWER:

$n = 27$

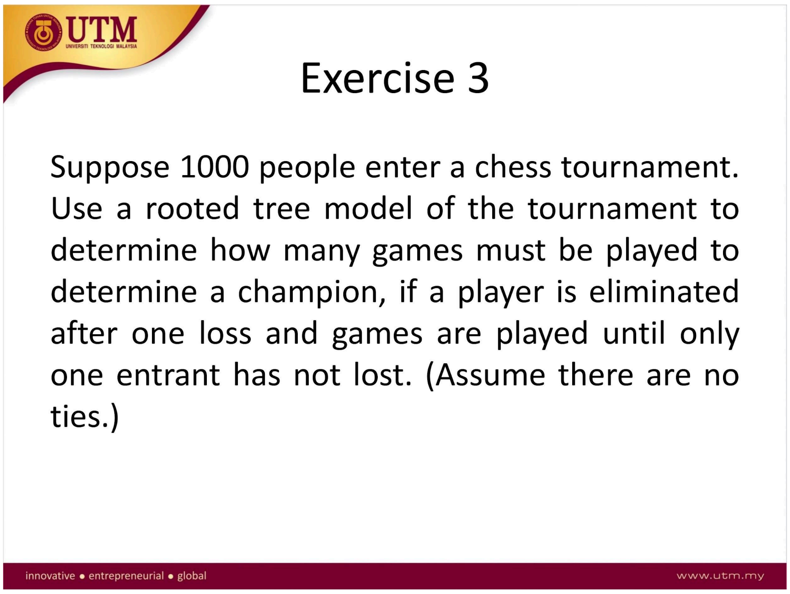
$m = 2$

$i = ?$

$i = (n-1)/m$

$= (27-1)/2$

$= 13$ matches



ANSWER:

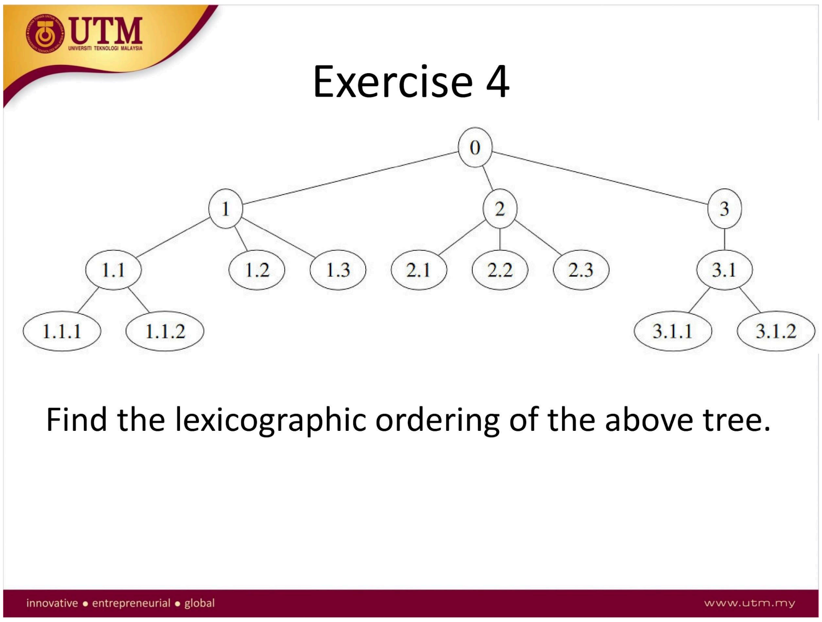
Exercise 3

$n = 1000$

$m = 2$

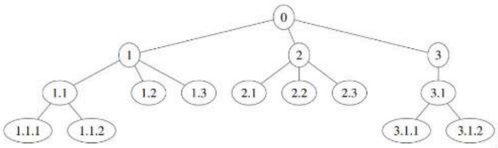
$i = ?$

$$\begin{aligned} i &= (n-1)/m \\ &= (1000-1)/2 \\ &= 499 \end{aligned}$$



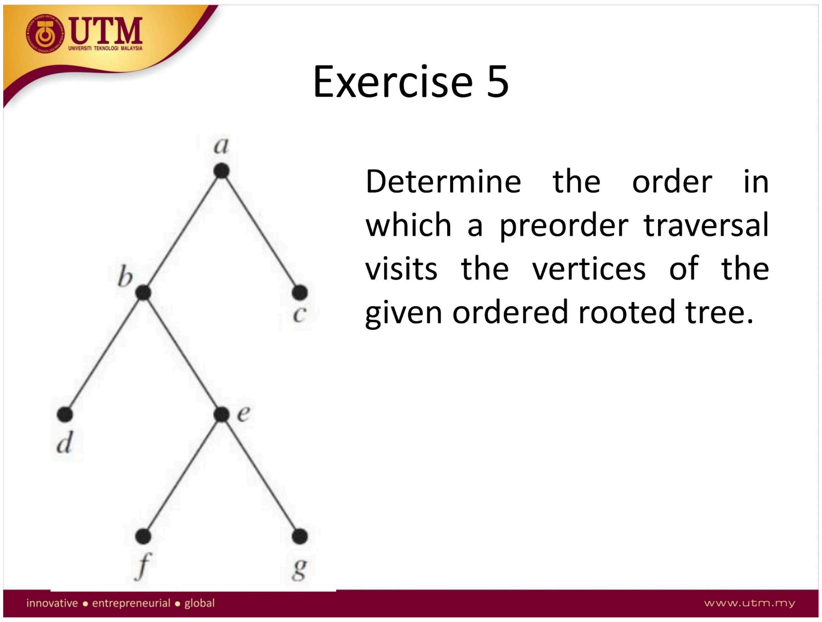
ANSWER:

4.



Find the lexicographic ordering of the above tree.

0<1<1.1<1.1.1<1.1.2<1.2<1.3<2<2.1<2.2<2.3<3<3.1<3.1.1<3.1.2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Exercise 5
Determine the order in which a preorder traversal visits the vertices of the given ordered rooted tree.
a
b
c
d
e
f
g
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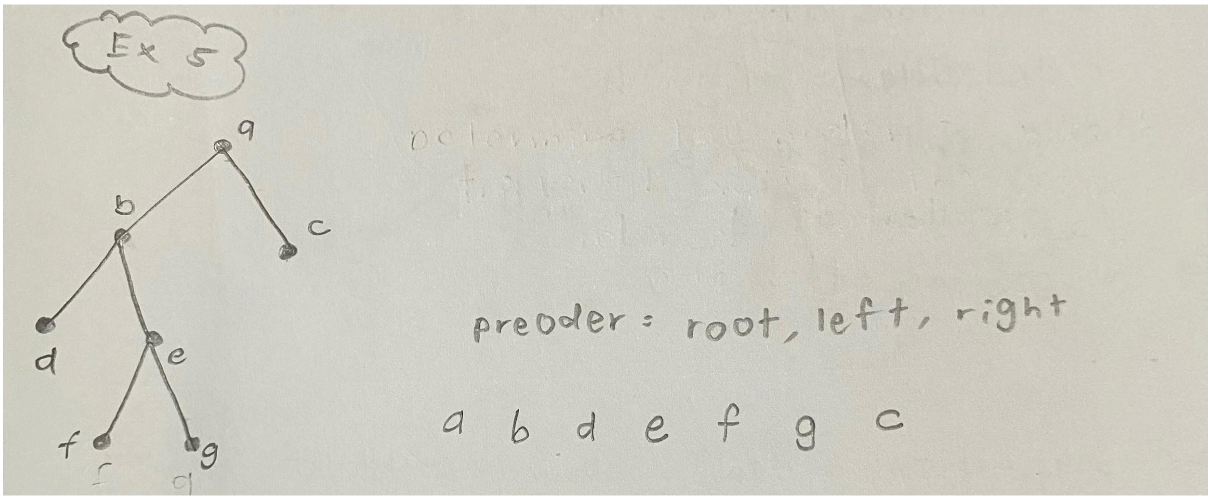
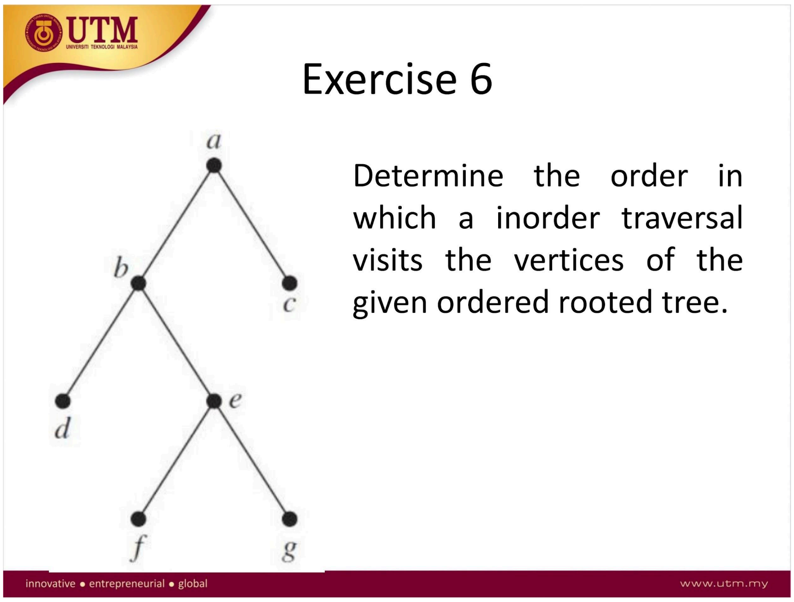
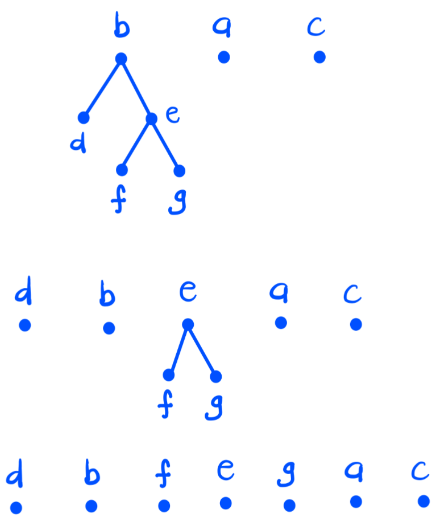
ANSWER:

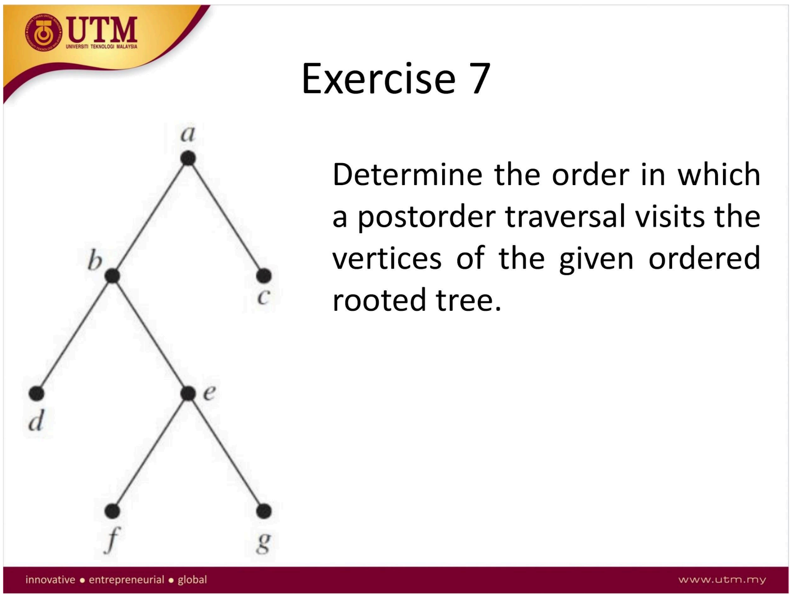

Ex 5
a
b
c
d
e
f
g
preoder : root, left, right
a b d e f g c



ANSWER:

Inorder traversal : Left-subtree, root, right-subtree

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Exercise 7
Determine the order in which a postorder traversal visits the vertices of the given ordered rooted tree.
a
b
c
d
e
f
g
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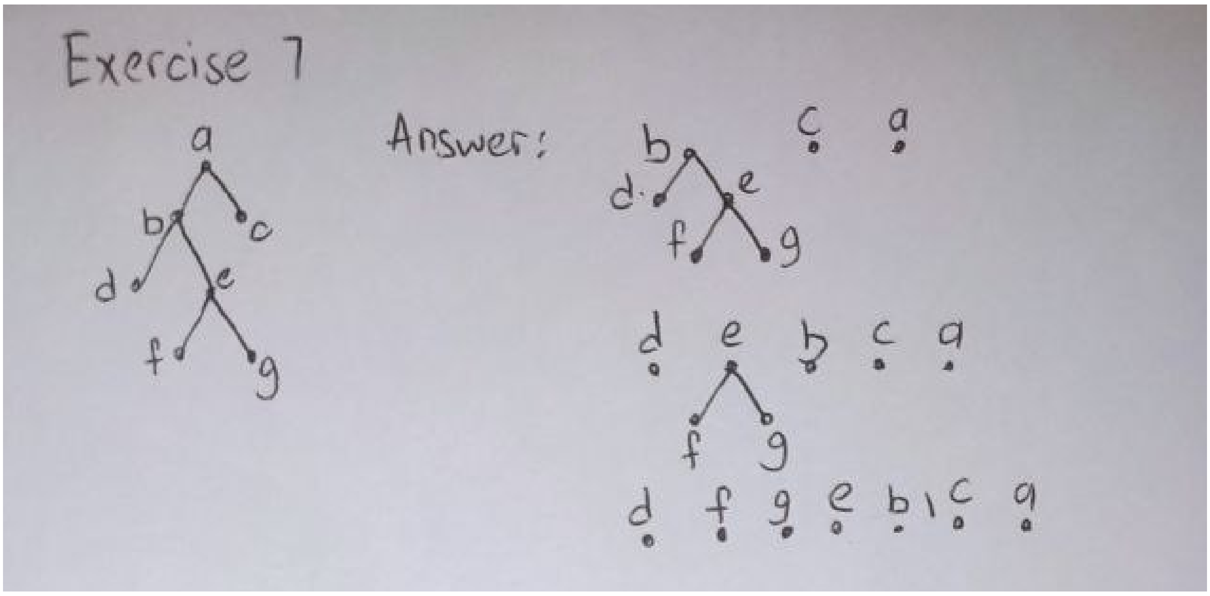
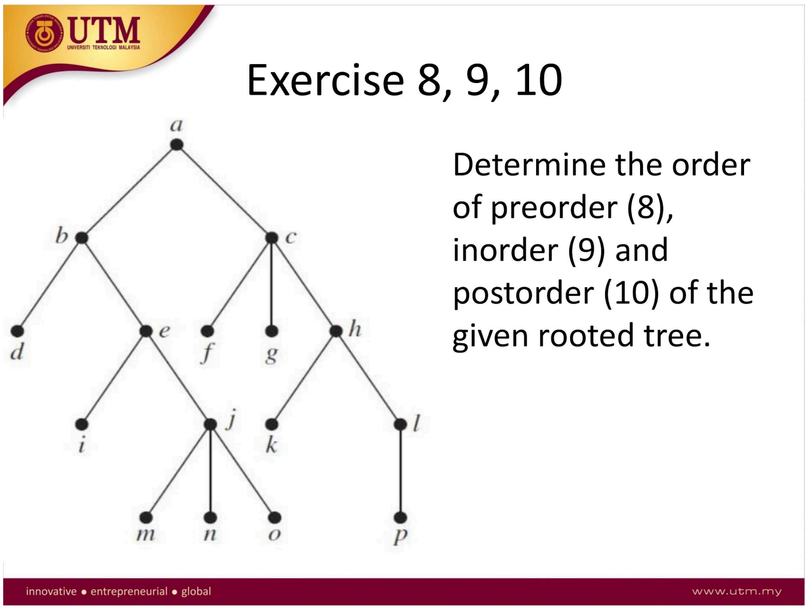
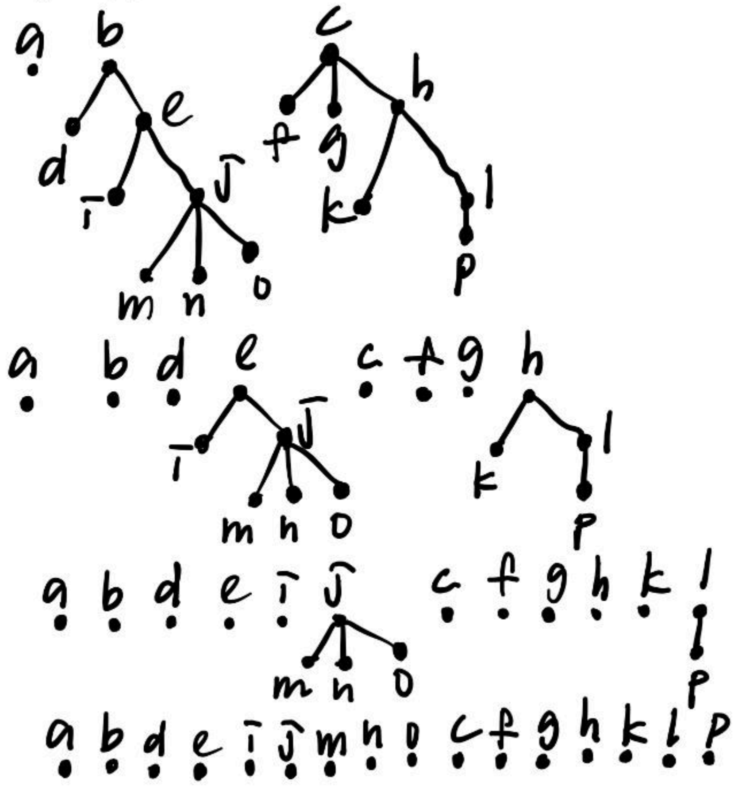
ANSWER:


Exercise 7
a
b
c
d
e
f
g
Answer:
d e b c a
d f g e b c a



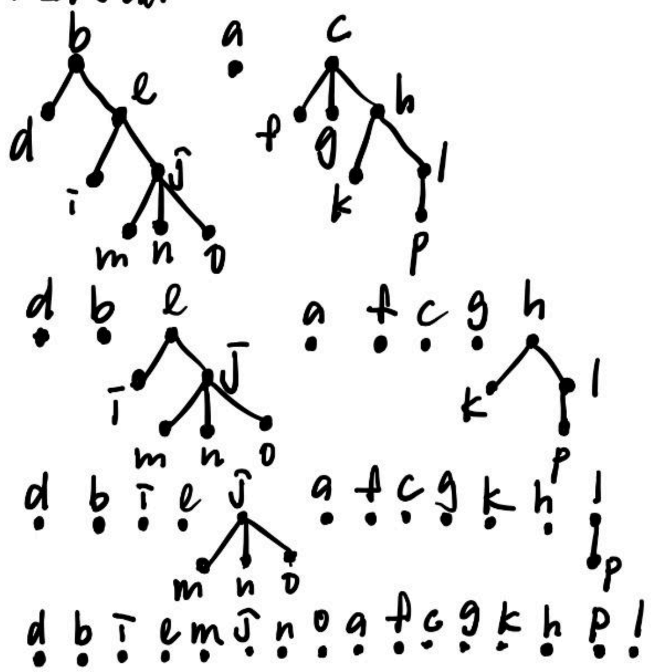
ANSWER:

8. Preorder



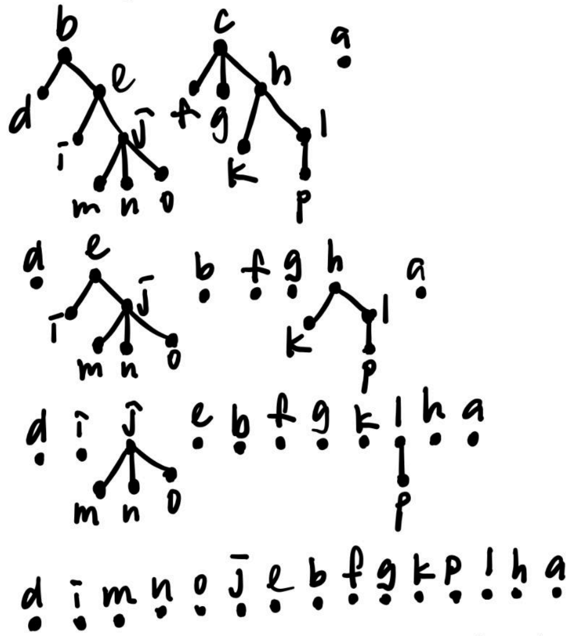
Preorder: a b d e i j m n o c f g h k l p

9. Inorder

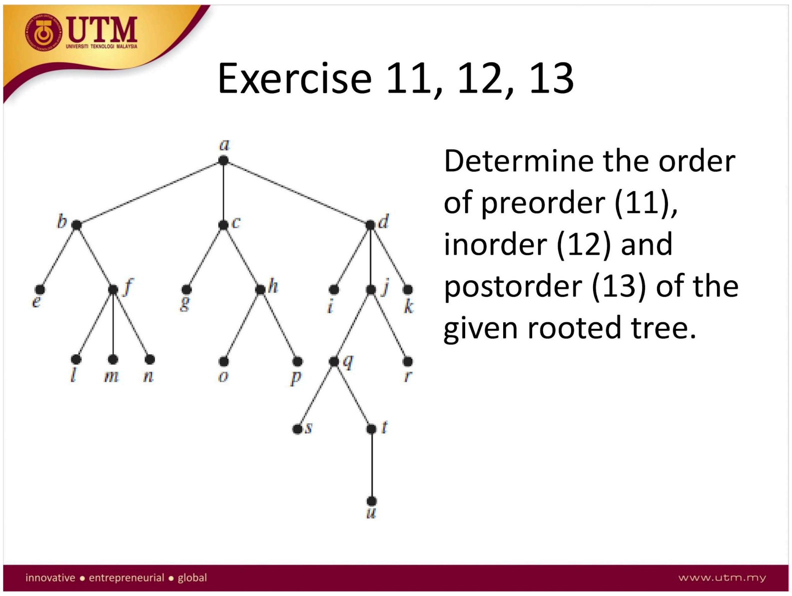


Inorder: d b i e m j n o a f c g k h p l

10. Posforder



Posforder: d i m n o j e b f g k p l h a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Exercise 11, 12, 13
Determine the order of preorder (11), inorder (12) and postorder (13) of the given rooted tree.
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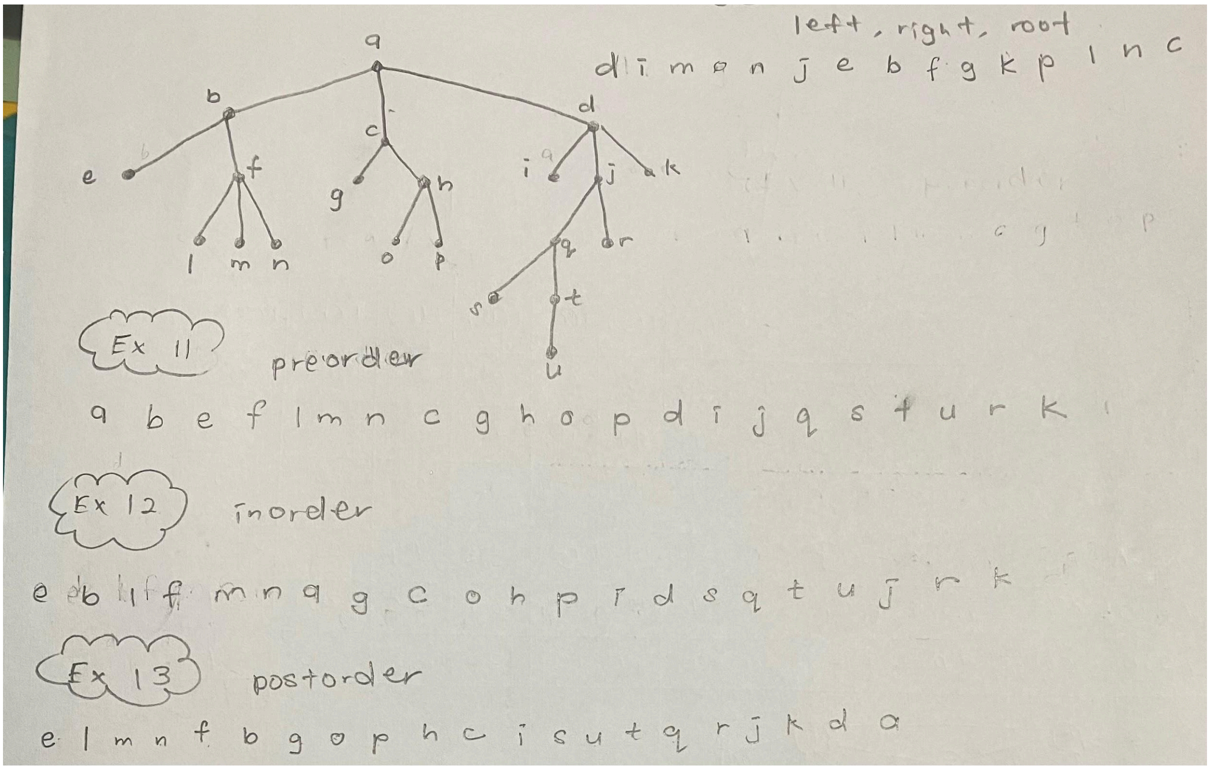
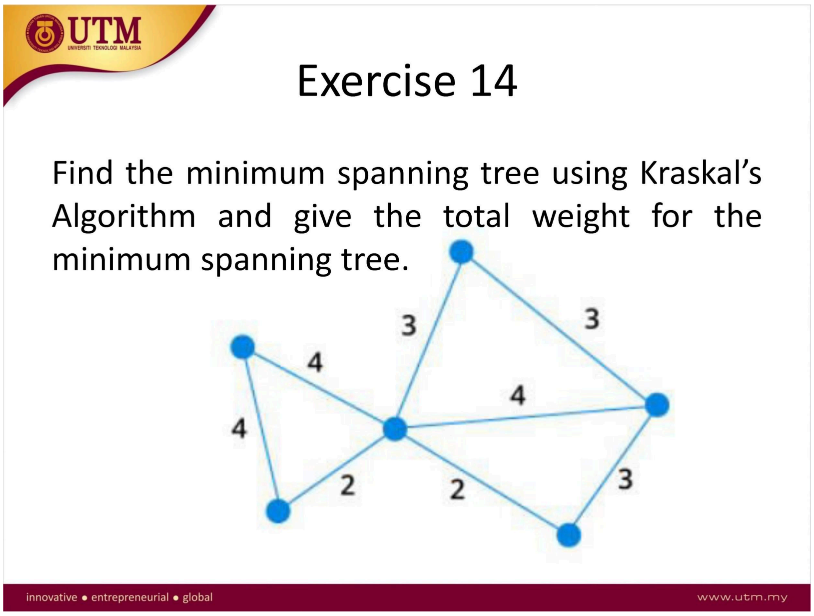
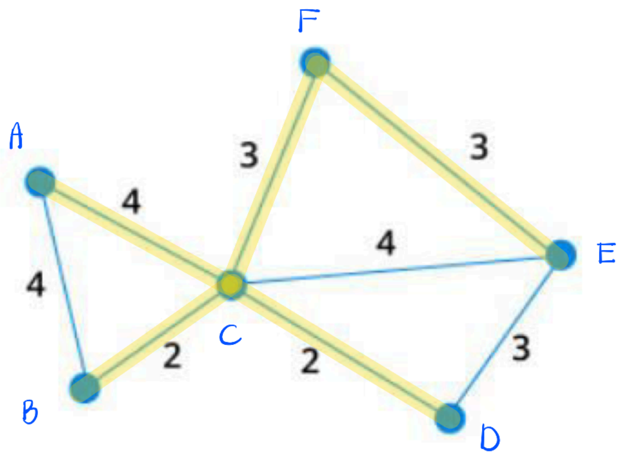
ANSWER:


left, right, root
Ex 11
preorder
a b e f l m n c g h o p d i j q s t u r k
Ex 12
inorder
e b l f m n a g c o h p r d s q t u j r k
Ex 13
postorder
e l m n f b g o p h c i s u t q r j k d a



ANSWER:



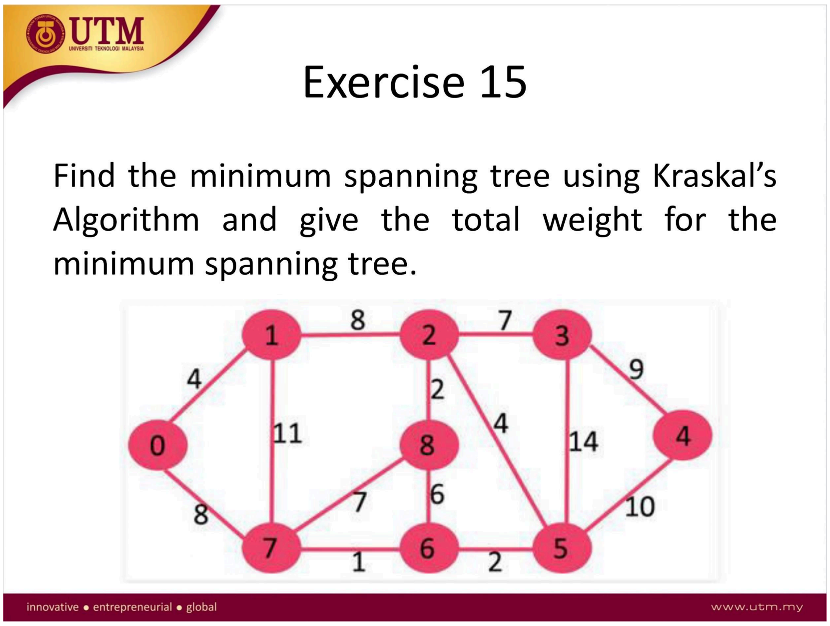
Let all vertices labelled with A,B,C,D,E,F.
List out all edges in order of its size:

BC = 2 ①
CD = 2 ②
CF = 3 ③
EF = 3 ④
DE = 3
AB = 4
AC = 4 ⑤
CE = 4

The solution is:

BC = 2
CD = 2
CF = 3
EF = 3
AC = 4

Total weight : 14


UTM
Exercise 15
Find the minimum spanning tree using Kraskal's Algorithm and give the total weight for the minimum spanning tree.
0 1 2 3 4 5 6 7 8
4 8 7 9 11 2 4 14 10 8 7 6 1 2
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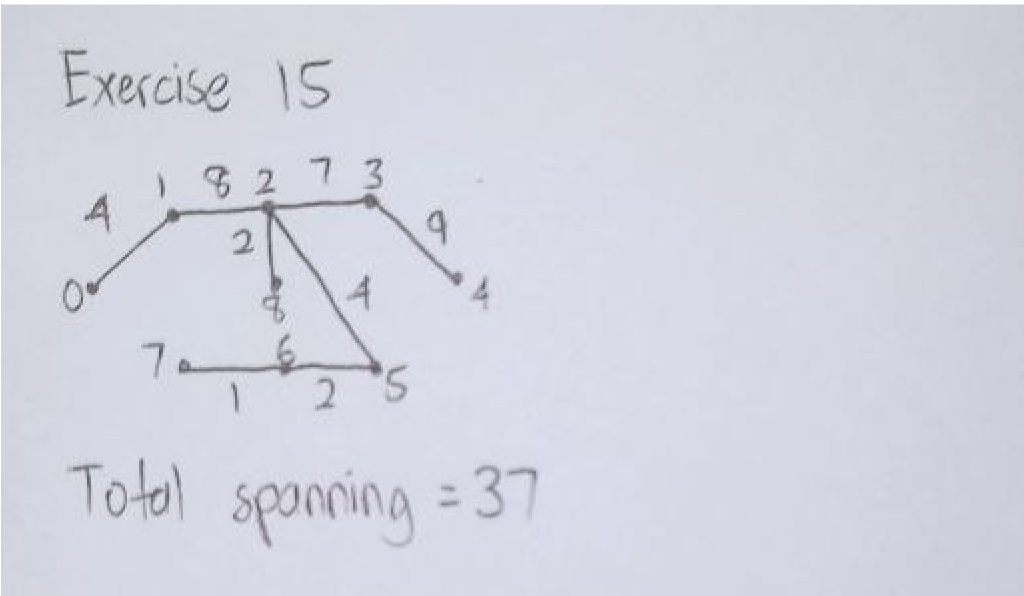
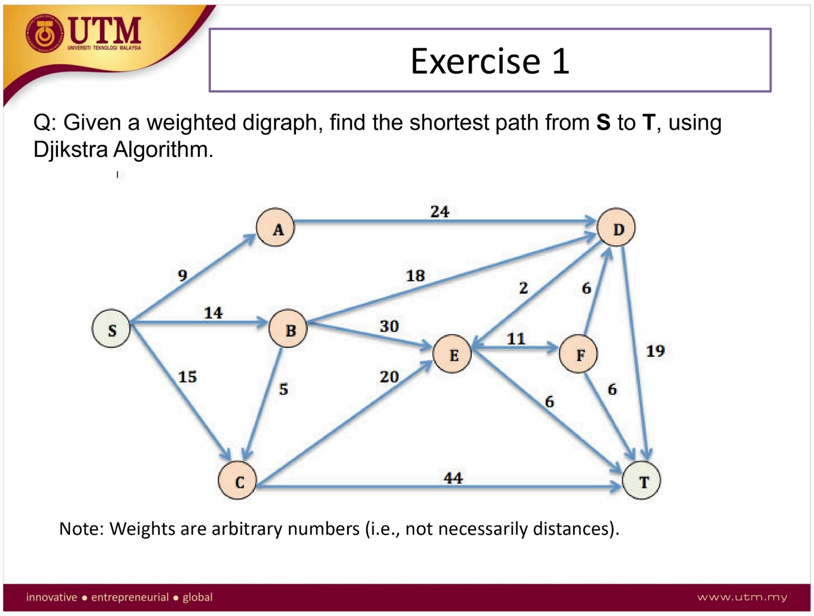
ANSWER:


Exercise 15
0 1 2 3 4 5 6 7 8
4 8 7 9 2 4 1 2
Total spanning = 37

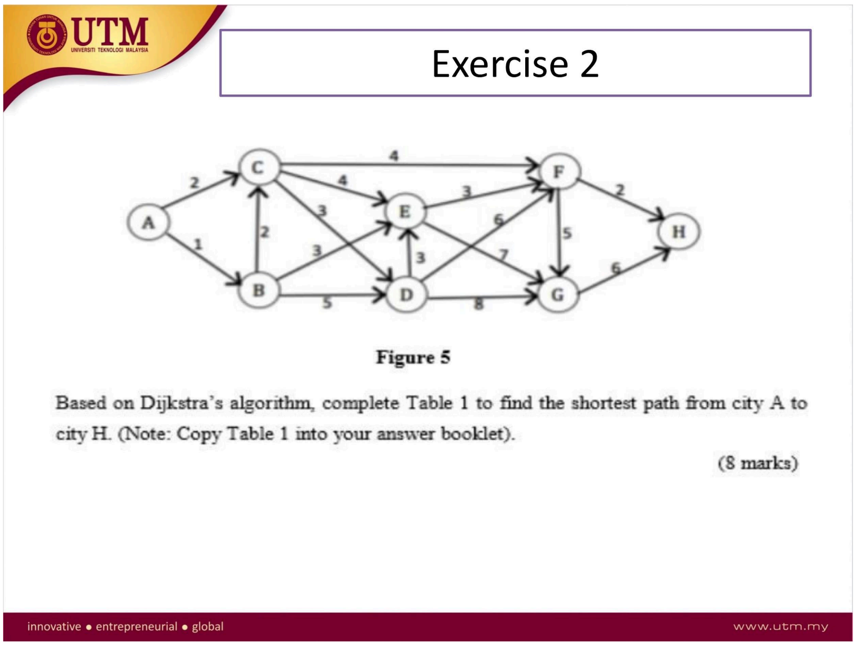
PART 3



ANSWER:

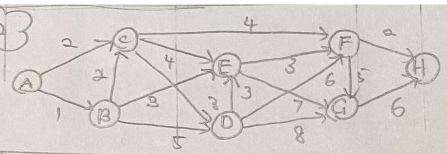
Iteration	S	N	L(S)	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(T)
0	{ }	{S,A,B,C,D,E,F,T}	0	∞	∞	∞	∞	∞	∞	∞
1	{S}	{A,B,C,D,E,F,T}	0	9	14	15	∞	∞	∞	∞
2	{S,A}	{B,C,D,E,F,T}	0	9	14	15	33	∞	∞	∞
3	{S,A,B}	{C,D,E,F,T}	0	9	14	15	32	44	∞	∞
4	{S,A,B,C}	{D,E,F,T}	0	9	14	15	32	35	∞	59
5	{S,A,B,C,D}	{E,F,T}	0	9	14	15	32	34	∞	51
6	{S,A,B,C,D,E}	{F,T}	0	9	14	15	32	34	45	40
7	{S,A,B,C,D,E,T}	{F}	0	9	14	15	32	34	45	40

Shortest Path: S → B → D → E → T
Total weights = 40



ANSWER:

Ex 2



A → C → F → H

Iteration	S	N	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(G)	L(H)
0	{ }	{A,B,C,D,E,F,G,H}	0	∞	∞	∞	∞	∞	∞	∞
1	{A}	{B,C,D,E,F,G,H}	0	1	2	∞	∞	∞	∞	∞
2	{A,B}	{C,D,E,F,G,H}	0	1	2	6	4	∞	∞	∞
3	{A,B,C}	{D,E,F,G,H}	0	1	2	5	4	6	∞	∞
4	{A,B,C,E}	{D,F,G,H}	0	1	2	5	4	6	11	∞
5	{A,B,C,E,F}	{D,G,H}	0	1	2	5	4	6	11	8
6	{A,B,C,E,F,H}	{D,G}	0	1	2	5	4	6	11	8