ASSIGNMENT 2

GROUP 5

SECTION 03 - SEM 1, 2024/2025

SECI1013 *(DISCRETE STRUCTURE)*

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DATE SUBMITTED : 26th NOVEMBER 2024

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"Logical order, defined limits"

ANSWER

CHAPTER 2 (2.3) : RECURRENCE RELATION

QUESTION 1

1. The price of a stock is initially RM50. Suppose that each morning the stock price increase by 2% and then in afternoon decreases by 2%. Let a_n be the price of the stock at the end of day n. [14 marks]

(i) Define recurrence relation.

$a_0 = 50$

Stock price in morning $= a_{n-1} \times 1.02$
$= 1.02a_{n-1}$

Stock price in afternoon $= 1.02a_{n-1} \times 0.98$
$= 0.9996a_{n-1}$

At the end of day, stock price $= 0.9996a_{n-1}$

Recurrence Relation: $a_n = 0.9996a_{n-1}, n \geq 1$ with initial condition $a_0 = 50$

(ii) Stock price at end of day 4, a_4

$a_0 = 50$

$a_1 = 0.9996a_0$
$= 0.9996(50)$
$= 49.98$

$a_2 = 0.9996a_1$
$= 0.9996(49.98)$
$= 49.96$

$a_3 = 0.9996a_2$
$= 0.9996(49.96)$
$= 49.94$

$a_4 = 0.9996a_3$
$= 0.9996(49.94)$
$= 49.92$

$\therefore$ Stock price at end of day 4, a_4 is RM49.92.

QUESTION 2

2. Given an arithmetic sequence 5, 37/7, 39/7, 41/7 ...

a. Find the sequence recursive relation

$5 \xrightarrow{+\frac{2}{7}} 37/7 \xrightarrow{+\frac{2}{7}} 39/7 \xrightarrow{+\frac{2}{7}} 41/7$

$a_0 = 5$

$a_1 = a_0 + \frac{2}{7}$

$\therefore a_n = a_{n-1} + \frac{2}{7}, \; n \geq 1, \; a_0 = 5$

b. Write a Pseudocode for function a(n)

Pseudo code :

```
a(n)
{
  if (n=0)
     return 5
  return a(n-1)+2/7
}
```

CHAPTER 3 (3.1): COUNTING METHODS

QUESTION 1

3.1

1. Iman on Faiha love to play snake game. They need to roll the dice to move forward. Then, two fair dice are rolled, one red and one blue. How mony ways that:

i) They can get the sum of dice rolled is 6 and 10.

Sum is 6 = {(1,5),(5,1),(2,4),(4,2),(3,3)}

T_1 (number of ways to get sum 6) = 5

Sum is 10 = {(5,5),(6,4),(4,6)}

T_2 (number of ways to get sum 10) = 3

$\therefore$ Sum of dice rolled is 6 and 10 $= T_1 + T_2$

$= 5 + 3$

$= 8$ ways

ii) They can get at least one dice shown the number of 3

At least one dice shows number 3 = {(1,3),(2,3),(3,3),(4,3),(5,3),(6,3),(3,1),(3,2),(3,4),(3,5),(3,6)}

$\therefore$ Total ways is 11

iii) They can get the red dice which shows the number of 3.

Red dice, number 3 = {(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)}

$\therefore$ Total ways is 6

QUESTION 2

2. An ambulance is given a route to reach patient's house (R3) from the hospital (R1) via route 2 (R2). To reach R2, there are possible 2 ways from R1 and to reach R3, there are possible 3 ways from R2.

i. How many ways it is possible to pass through R1 to R3 via R2. [2m]

$R1 \rightarrow R2 = 2$ ways

$R2 \rightarrow R3 = 3$ ways

$R1 \rightarrow R3 = 2 \times 3$

$= 6$ ways

ii. How many different round-trip routines are there from R1 to R3 and back to R1. [2m]

$R1 \rightarrow R3 = 6$ ways

$R3 \rightarrow R1 = 6$ ways

Total round-trip $= 6 \times 6$

$= 36$ ways

QUESTION 3

3. Nina would like to buy burger. She went to the restaurant and has been offered to buy a meal-deal. The meal-deal set can only have one main menu and one side or one beverage. [6 Marks]
Below shows the details of the menu:

Main Menu
- Burger – Meal mania
- Burger – Egg special
- Burger – Single cheese
- Burger – Double cheese

Side
- Crispy Fries
- Potato wedges
- Cheesy Pop
- Chicken Fries
- Onion Ring
- Sweet Potato Fries

Beverage
- Coca-cola
- 7 UP
- Lemonade
- Peach Tea
- Lemon Tea

(i) How many ways a meal-deal can be formed if Nina prefer a set that contain Burger? [2M]

Number of Ways = (Main Menu $\cap$ Side) + (Main Menu $\cap$ Beverage)
$= (4\times6)+(4\times5)$
$= 24 + 20$
$= 44$ ways

(ii) How many ways a meal-deal can be formed if Nina does not like peach tea or lemon tea? [2M]

X = meal-deal without peach tea & lemon tea
= Main Menu $\cap$ beverage without peach tea & lemon tea
$= 4\times3$
$= 12$

Y = Main Menu $\cap$ side
$= 4\times6$
$= 24$

Number of Ways = X + Y
$= 12 + 24$
$= 36$ ways

(iii) How many ways a meal-deal can be formed if Nina does prefer to choose main and side only? [2M]

Number of Ways = Main Menu $\cap$ Side
$= 4\times6$
$= 24$

QUESTION 4

4. I-syad's Bakery provides different types of cakes:
7 choc cakes, 2 chesecakes, 6 fruity cake and two layer cakes. How many options a customer can buy a cake from this shop?

7 - choc cakes
2 - cheese "
6 - fruity "
1 ~~2~~ - two layer "

Total ways to choose a cake:
$= 7 + 2 + 6 + 1$
$= 16$ ways #

CHAPTER 3 (3.2 & 3.3): PERMUTATION & COMBINATION

QUESTION 1

3.2 and 3.3

1. Suppose in a university, each subject offered to students has a subject code consisting of three letters followed by five digits

i) How many diffrent subject codes are possible?

26 26 26   10 10 10 10 10

$(26P_1)^3 \times (10P_1)^5 = 1\ 757\ 600\ 000$

$\therefore$ 1 757 600 000 possibilities of different subject codes.

ii) How many subject codes could begin with CS and end with digit 3 or 2?

C S _   _ _ _ _ 2/3

26   10 10 10 10 2

$26P_1 \times (10P_1)^4 \times 2P_1 = 520\ 000$

$\therefore$ 520 000 subject codes could begin with CS and end with digit 3 or 2.

iii) How many subject codes are possible in which all the letters and the digits are distinct?

26 25 24   10 9 8 7 6

$26P_3 \times 10P_5 = 15600 \times 30240$
$= 471\ 744\ 000$

$\therefore$ 471 744 000 possibilities of subject codes in which all the letters and the digits are distinct

QUESTION 2

2. Answer the following below: [6 marks]

i. In how many ways can 3 runners be selected from a group of 10 athletes?

$$C(10,3) = \frac{10!}{3!(10-3)!} = \frac{10!}{3!7!} = 120 \text{ ways}$$

ii. You have 15 chocolates in your hands. Unfortunately, you can only give 9 chocolates to your sister. In how many ways can could you give to your sisters?

$$C(15,9) = \frac{15!}{9!(15-9)!} = \frac{15!}{9!6!} = 5005 \text{ ways}$$

iii. How many strings can be formed in the length of 5 strings from the word DISCRETE?

* If all letters are different:

$$^{7}C_5 \times {}^{5}P_5 = \frac{7!}{5!(7-5)!} \times \frac{5!}{(5-5)!} = 21 \times 120 = 2520 \text{ strings}$$

* If letter E is used twice:

$$^{2}C_2 \times {}^{6}C_3 \times {}^{5}P_3 = \frac{2!}{2!(2-2)!} \times \frac{6!}{3!(6-3)!} \times \frac{5!}{(5-3)!} = 1 \times 20 \times 60 = 1200 \text{ strings}$$

∴ Total number of strings can be formed = 2520 + 1200

= 3720 strings

iv. There are 10 girls and 7 boys in a class and need to select 4 students as a school representative. In how many ways can we select of 2 girls and 2 boys?

$$C(10,2) \times C(7,2) = \frac{10!}{2!(10-2!)} \times \frac{7!}{2!(7-2!)} = 45 \times 21$$

$$= 945 \text{ ways}$$

QUESTION 3

3. A teacher has prepared 20 questions of permutation and 15 questions of combination from the topic learned. How many ways are there to setup the quiz which consists of 3 permutation and 2 combination questions. [4 Marks]

P = Permutation C = Combination

$\underline{P}\ \underline{P}\ \underline{P}\ \underline{C}\ \underline{C}$

$$\text{Ways to choose } 3P = {}^{20}C_3 = \frac{20!}{3!(20-3)!} = \frac{20!}{3!\,17!} = 1140$$

$$\text{Ways to choose } 2C = {}^{15}C_2 = \frac{15!}{2!(15-2)!} = \frac{15!}{2!\,13!} = 105$$

$$\text{Number of Ways} = \text{ways to choose } 3P \times \text{ways to choose } 2C = 1140 \times 105 = 119700$$

CHAPTER 3 (3.4): PIGEONHOLE PRINCIPLE

QUESTION 1

1. In a group of 40 people, at least how many must have been born in the same month?

n = 40 people
k = 12 month

$m = \lceil \frac{n}{k} \rceil = \lceil \frac{40}{12} \rceil$

$m = \lceil 3.33 \rceil$

$m = 4$

$\therefore$ at least 4 people with same month

QUESTION 2

3.4

2. In a quiz taken by 35 students, the sore ranges of grade A+ is from 90 to 100. At least how many students must have the same sore?

Pigeons (students), $n = 50$, $k = ?$
Pigeonholes (range A+), $m = 11$
$n > m$

Using 3^{rd} Form,

$$k = \left\lceil \frac{n}{m} \right\rceil$$

$$= \left\lceil \frac{35}{11} \right\rceil$$

$$= \lceil 3.18 \rceil$$ (since student must be real number, round to 4)

$$= 4$$

$\therefore$ 4 students must have the same score.

QUESTION 3

3. Given $X = \{1,2,3,4,5,6,7,8,9,10\}$. Show that if you pick six numbers from set X, the sum of two of them is exactly 11.

Let $P = \{p_1, p_2, p_3, p_4, p_5, p_6\}$ be any subset of six distinct elements of X.

Let $Q = \{\{1,10\},\{2,9\},\{3,8\},\{4,7\},\{5,6\}\}$, a set of 5 distinct elements and a part of X.

Define $f: P \to Q$ by $f(x) = p_i$ if $x \in p_i$. For example, if $x = 3 \in P$, then $f(3) = \{3,8\}$.

$|P| = 6$, $|Q| = 5$. According to 2^{nd} form of pigeonhole principle, at least two distinct elements of P must be mapped to the same element of Q.

Hence, if we choose any six numbers of X, then for at least one pair of these six numbers, their sum is 11.

QUESTION 4

4. There are 53 different time periods during each class of a university can be scheduled. How many different rooms will be needed if there are 115 different classes are available?

$n = 115$

$k = 53$

$m = \left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{115}{53} \right\rceil$

$m = \lceil 2.17 \rceil$

$m = 3$

Hence, each period requires at least 3 classrooms.

QUESTION 5

5. There are 25 computers in a lab. Each computer is directly connected to at least one of the computers. Show there are at least two computers in the network that are directly connected to the same number of other computers.

According to Pigeonhole Principle, if there are 25 computers and only 24 possible connection counts, at least two computers must share the same connection. This is because there are more computer than possible connection, so two or more computers will necessarily overlap in the number of connections.

$n = 25$, $k = 1$, $m = n - 1 = 24$

Possible connection : $m - k + 1 = 24 - 1 + 1 = 24$

$\therefore$ $n = 25 > 24$, at least two computers will share the same connection.