Trabalho 2 de Cálculo 2

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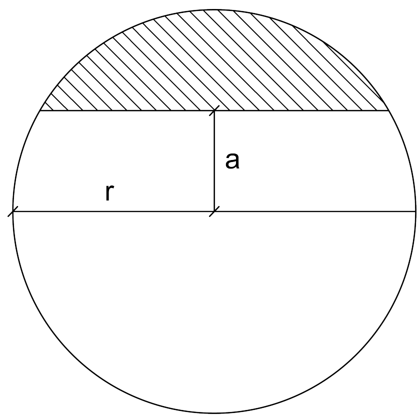
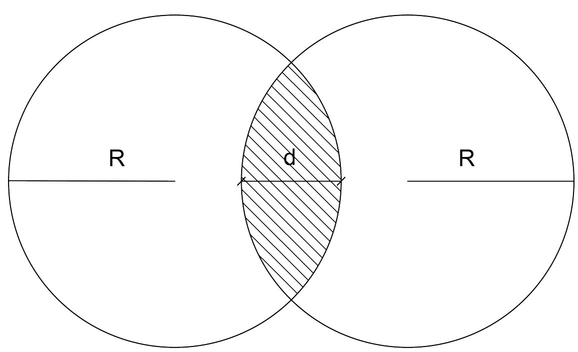
Base para os exercícios 1 e 2:



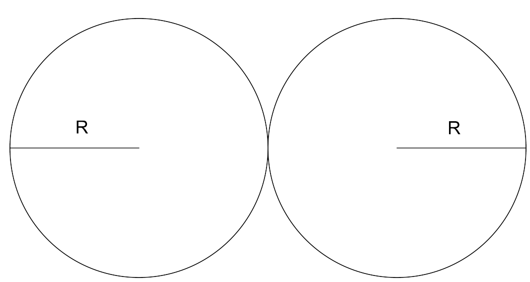
$$
\begin{aligned}
A(r,a) &= 2 \cdot \int_a^r \sqrt{r^2 - x^2}\, dx \\
&= \frac{\pi \cdot r^2}{2} - r^2 \left(\operatorname{arc\,sen} \frac{a}{r} + \frac{a}{r} \sqrt{1 - \left(\frac{a}{r}\right)^2} \right) \\
&= \frac{\pi \cdot r^2}{2} - r^2 \left(\operatorname{arc\,sen} \frac{a}{r} + \frac{a}{r^2} \sqrt{r^2 - a^2} \right).
\end{aligned}
$$

Exercício 1:



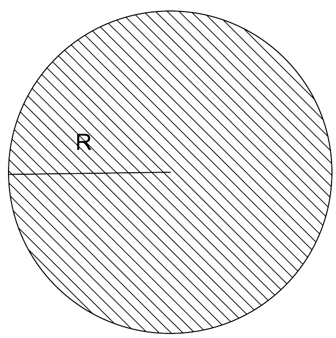
$$
\begin{aligned}
A\left(R, R-\frac{d}{2}\right) &= 2\cdot\left(2\cdot\int_{R-\frac{d}{2}}^{R}\sqrt{R^2-x^2}\;dx\right)\\[2ex]
&= 2\cdot\left(\frac{\pi\cdot R^2}{2}-R^2\left(\operatorname{arc\,sen}\left(\frac{R-\frac{d}{2}}{R}\right)+\frac{R-\frac{d}{2}}{R^2}\cdot\sqrt{R^2-\left(R-\frac{d}{2}\right)^2}\right)\right)\\[2ex]
&= \frac{2\pi R^2}{2}-2R^2\left(\operatorname{arc\,sen}\left(\frac{R-\frac{d}{2}}{R}\right)+\frac{R-\frac{d}{2}}{R^2}\cdot\sqrt{R^2-\left(R-\frac{d}{2}\right)^2}\right)\\[2ex]
&= \pi R^2-2R^2\left(\operatorname{arc\,sen}\left(\frac{R-\frac{d}{2}}{R}\right)+\frac{R-\frac{d}{2}}{R^2}\cdot\sqrt{R^2-\left(R^2-2\cdot R\frac{d}{2}+\frac{d^2}{4}\right)}\right)\\[2ex]
&= \pi R^2-2R^2\left(\operatorname{arc\,sen}\left(\frac{R-\frac{d}{2}}{R}\right)+\frac{R-\frac{d}{2}}{R^2}\cdot\sqrt{R^2-R^2+R\frac{d}{2}-\frac{d^2}{4}}\right)\\[2ex]
&= \pi R^2-2R^2\left(\operatorname{arc\,sen}\left(\frac{R-\frac{d}{2}}{R}\right)+\frac{R-\frac{d}{2}}{R^2}\cdot\sqrt{R^2-R^2+R\frac{d}{2}-\frac{d^2}{4}}\right)\\[2ex]
&= \pi R^2-2R^2\left(\operatorname{arc\,sen}\left(\frac{R-\frac{d}{2}}{R}\right)+\frac{R-\frac{d}{2}}{R^2}\cdot\frac{\sqrt{4Rd-d^2}}{2}\right)\\[2ex]
&= \pi R^2-2R^2\operatorname{arc\,sen}\left(\frac{R-\frac{d}{2}}{R}\right)+\left(R-\frac{d}{2}\right)\cdot\sqrt{4Rd-d^2}.
\end{aligned}
$$

Caso particular: para d = 0



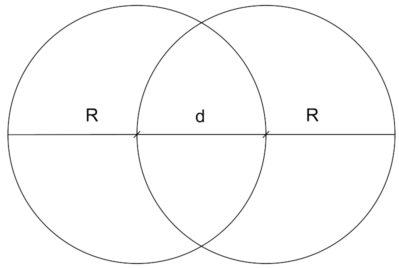
$$
\begin{aligned}
A &= \pi R^2 - 2R^2 \operatorname{arc\,sen}\left(\frac{R-\frac{0}{2}}{R}\right) + \left(R - \frac{0}{2}\cdot\sqrt{4R\cdot 0 - 0^2}\right) \\
&= \pi R^2 - 2R^2 \operatorname{arc\,sen}\left(\frac{R}{R}\right) \\
&= \pi R^2 - 2R^2 \operatorname{arc\,sen}(1) \\
&= \pi R^2 - 2R^2 \cdot \frac{\pi}{2} \\
&= \pi R^2 - \pi 2R^2 \\
&= 0.
\end{aligned}
$$

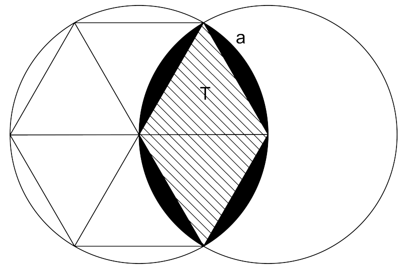
Caso particular: para d = 2R



$$
\begin{aligned}
A &= \pi R^2 - 2R^2 \operatorname{arc\,sen}\left(\frac{R-\frac{2R}{2}}{R}\right) + \left(R - \frac{2R}{2}\cdot\sqrt{4R\cdot 2R - (2R)^2}\right) \\
&= \pi R^2 - 2R^2 \operatorname{arc\,sen}(0) + 0\cdot\sqrt{8R - 4R^2} \\
&= \pi R^2 - 2R^2 \cdot 0 + 0 \\
&= \pi R^2.
\end{aligned}
$$

Caso particular: para d = R





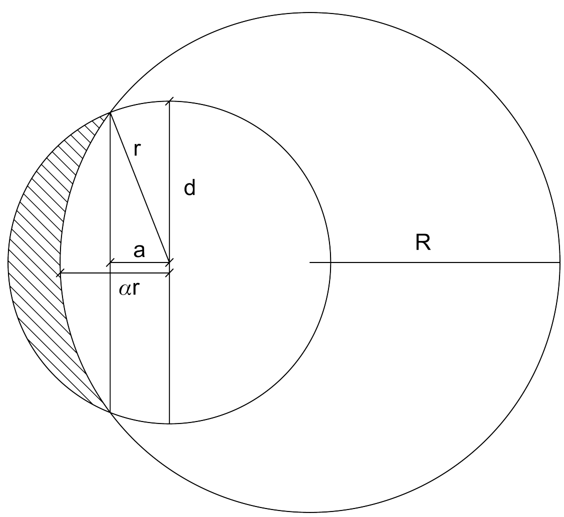
$$
\begin{aligned}
A &= \pi R^2 - 2R^2 \operatorname{arc\,sen}\left(\frac{R-\dfrac{R}{2}}{R}\right) + \left(R - \frac{R}{2}\cdot\sqrt{4R\cdot R - R^2}\right) \\
&= \pi R^2 - 2R^2 \operatorname{arc\,sen}\left(\frac{R}{2R}\right) + \frac{R}{2}\cdot\sqrt{4R^2 - R^2} \\
&= \pi R^2 - 2R^2 \operatorname{arc\,sen}\left(\frac{1}{2}\right) + \frac{R}{2}\cdot\sqrt{3}\cdot R \\
&= \pi R^2 - 2R^2 \frac{\pi}{6} + \frac{R^2}{2}\cdot\sqrt{3} \\
&= \pi R^2 - R^2 \frac{\pi}{3} + \frac{R^2}{2}\cdot\sqrt{3} \\
&= \frac{6\pi R^2 - 2\pi R^2 + 3R^2\cdot\sqrt{3}}{6} \\
&= \frac{4\pi R^2 + 3R^2\cdot\sqrt{3}}{6} \\
&= \frac{R^2}{6}\cdot(4\pi - 3\sqrt{3}).
\end{aligned}
$$

Resolução sem o uso de integral para d = R

$$
\begin{aligned}
\pi R^2 &= 6\ T + 6\ a \\
\frac{\pi R^2}{6} &= T + a \\
a &= \frac{\pi R^2}{6} - T
\end{aligned}
$$

$$
\begin{aligned}
A &= 2\ T + 4\ a \\
&= 2T + \frac{4\pi R^2}{6} - 4T \\
&= \frac{4\pi R^2}{6} - 2T \\
&= \frac{4\pi R^2}{6} - 2\left(\frac{R^2\cdot\sqrt{3}}{4}\right) \\
&= 4\pi\frac{R^2}{6} - \frac{R}{2}\cdot\sqrt{3} \\
&= 4\pi\frac{R^2}{6} - \frac{3\cdot R}{6}\cdot\sqrt{3} \\
&= \frac{R^2}{6}\cdot(4\pi - 3\sqrt{3}).
\end{aligned}
$$

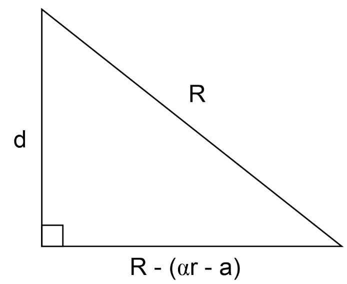
Exercício 2:



$$
\begin{aligned}
r^2 &= a^2 + d^2 \\
d^2 &= r^2 - a^2
\end{aligned}
$$

e também

$$
\begin{aligned}
R^2 &= b^2 + d^2 \\
d^2 &= R^2 - b^2
\end{aligned}
$$



R para $a > 0$

$$R^2 = (R - (\alpha r - a))^2 - (r^2 - a^2)$$

$$R^2 = R^2 - 2R \cdot (\alpha r - a) + (\alpha r - a)^2 + r^2 - a^2$$

$$2R \cdot (\alpha r - a) = (\alpha r - a)^2 + r^2 - a^2$$

$$\begin{aligned} R &= \frac{(\alpha r - a)^2 + r^2 - a^2}{2(\alpha r - a)} \\ &= \frac{(\alpha \cdot r)^2 - 2 \cdot \alpha \cdot r \cdot a + a^2 + r^2 - a^2}{2 \cdot (\alpha r - a)} \\ &= \frac{r^2(\alpha^2 + 1) - 2 \cdot \alpha \cdot r \cdot a}{2 \cdot (\alpha r - a)}. \end{aligned}$$

R para $a = 0$

$$\begin{aligned} R &= \frac{r^2(\alpha^2 + 1) - 2 \cdot \alpha \cdot r \cdot a}{2 \cdot (\alpha r - a)} \\ &= \frac{r^2(\alpha^2 + 1) - 2 \cdot \alpha \cdot r \cdot 0}{2 \cdot (\alpha r - 0)} \\ &= (\frac{r^2(\alpha^2 + 1)}{2 \cdot (\alpha r)}. \end{aligned}$$

Área para $a > 0$

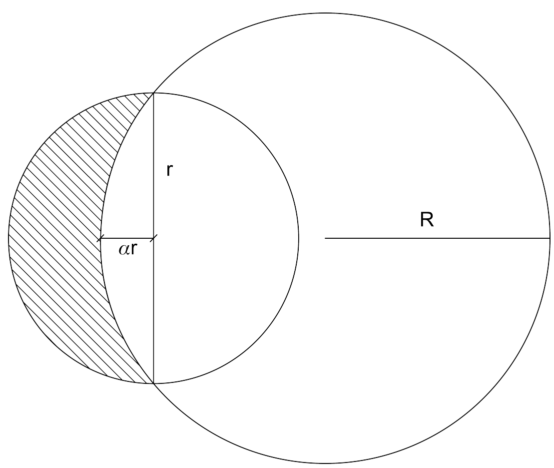
$$\begin{aligned} A &= A(r,a) - A(R, R - (\alpha r - a)) \\ &= A(r,a) - A(R,b) \end{aligned}$$

$$A(r,a) = \frac{\pi \cdot r^2}{2} - r^2 \left(\operatorname{arc\,sen} \frac{a}{r} + \frac{a}{r^2} \sqrt{r^2 - a^2} \right)$$

$$
\begin{aligned}
A(R, R-(\alpha r - a)) &= \frac{\pi \cdot R^2}{2} - R^2 \left(\operatorname{arc\,sen} \frac{R-(\alpha r - a)}{R} + \frac{R-(\alpha r - a)}{R^2} \sqrt{R^2 - (R-(\alpha r - a))^2} \right) \\
&= \frac{\pi \cdot R^2}{2} - R^2 \left(\operatorname{arc\,sen} \frac{R-(\alpha r - a)}{R} + \frac{R-(\alpha r - a)}{R^2} \sqrt{R^2 - R^2 + (\alpha r - a)^2} \right) \\
&= \frac{\pi \cdot R^2}{2} - R^2 \left(\operatorname{arc\,sen} \frac{R-(\alpha r - a)}{R} + \frac{R-(\alpha r - a)}{R^2} \sqrt{(\alpha r - a)^2} \right) \\
&= \frac{\pi \cdot R^2}{2} - R^2 \left(\operatorname{arc\,sen} \frac{R-(\alpha r - a)}{R} + \frac{R-(\alpha r - a)}{R^2} \cdot (\alpha r - a) \right) \\
&= \frac{\pi \cdot R^2}{2} - R^2 \left(\operatorname{arc\,sen} \frac{R-(\alpha r - a)}{R} + \frac{R-(\alpha r - a)^2}{R^2} \right) \\
&= R^2 \left(\frac{\pi}{2} - \operatorname{arc\,sen} \left(1 - \frac{(\alpha r - a)}{R} \right) - \frac{1}{R} + \frac{(\alpha r - a)^2}{R^2} \right)
\end{aligned}
$$

$$
\begin{aligned}
A &= A(r, a) - A(R, R-(\alpha r - a)) \\
&= \frac{\pi \cdot r^2}{2} - r^2 \left(\operatorname{arc\,sen} \frac{a}{r} + \frac{a}{r^2} \sqrt{r^2 - a^2} \right) - R^2 \left(\frac{\pi}{2} - \operatorname{arc\,sen} \left(1 - \frac{(\alpha r - a)}{R} \right) - \frac{1}{R} + \frac{(\alpha r - a)^2}{R^2} \right).
\end{aligned}
$$

Área para $a = 0$



$$
A = A(r, 0) - A(R, R - \alpha r)
$$

$$
A(r, 0) = \frac{\pi r^2}{2}
$$

$$A(R, R-\alpha r) = \left(\frac{r^2(\alpha^2+1)}{2\cdot(\alpha r)}\right)^2 \left(\frac{\pi}{2} - \operatorname{arc\,sen}\left(1 - \frac{(\alpha r)}{\left(\frac{r^2(\alpha^2+1)}{2\cdot(\alpha r)}\right)}\right) - \frac{1}{\left(\frac{r^2(\alpha^2+1)}{2\cdot(\alpha r)}\right)} + \frac{\alpha r^2}{\left(\frac{r^2(\alpha^2+1)}{2\cdot(\alpha r)}\right)^2}\right)$$

$$= \left(\frac{r^4(\alpha^2+1)^2}{4\cdot(\alpha r)^2}\right)\left(\frac{\pi}{2} - \operatorname{arc\,sen}\left(1 - \frac{2(\alpha r)^2}{r^2(\alpha^2+1)}\right) - \frac{2\alpha r}{r^2(\alpha^2+1)} + \frac{4(\alpha r)^4}{r^4(\alpha^2+1)^2}\right)$$

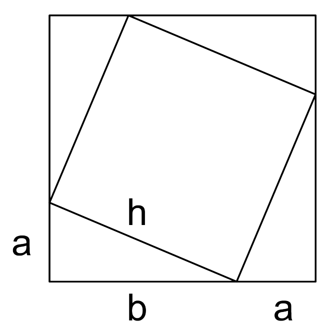
$$= \frac{r^2(\alpha^2+1)^2}{4\cdot\alpha^2}\left(\frac{\pi}{2} - \operatorname{arc\,sen}\left(1 - \frac{2\alpha^2}{\alpha^2+1}\right) - \frac{2\alpha}{r(\alpha^2+1)} + \frac{4\alpha^4}{(\alpha^2+1)^2}\right)$$

$$A = A(r,0) - A(R, R-\alpha r)$$

$$= \frac{\pi r^2}{2} - \frac{r^2(\alpha^2+1)^2}{4\cdot\alpha^2}\left(\frac{\pi}{2} - \operatorname{arc\,sen}\left(1 - \frac{2\alpha^2}{\alpha^2+1}\right) - \frac{2\alpha}{r(\alpha^2+1)} + \frac{4\alpha^4}{(\alpha^2+1)^2}\right).$$

Base para o exercício 3

Teorema de Pitágoras

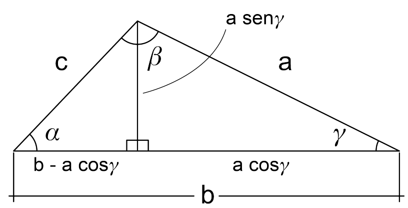


$$(a+b)^2 = h^2 + 4\cdot\frac{a\cdot b}{2}$$

$$a^2 + 2ab + b^2 = h^2 + 2ab$$

$$a^2 + b^2 = h^2$$

Teorema do Cosseno

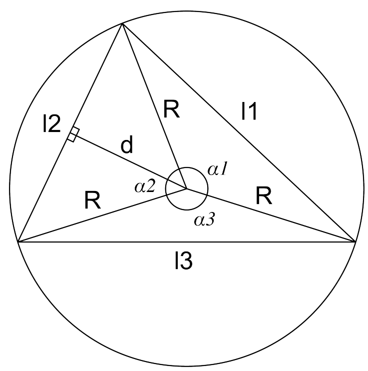


$$c^2 = (a \cdot \operatorname{sen}\gamma)^2 + (b - a \cdot \cos\gamma)^2$$

$$= a^2 \operatorname{sen}^2\gamma + b^2 - 2ab\cos\gamma + a^2\cos^2\gamma$$

$$= a^2 + b^2 - 2ab \cdot \cos\gamma$$

Exercício 3:



$$\begin{aligned}
l_i^2 &= R^2 + R^2 - 2R \cdot R \cdot \cos\alpha_i \\
&= 2R^2 - 2R^2 \cdot \cos\alpha_i \\
&= 2R^2 \cdot (1 - \cos\alpha_i) \\
A_i^2 &= \frac{l_i^2 \cdot d_i^2}{2^2} \\
&= \frac{2R^2(1-\cos\alpha_i)}{4} \cdot R^2 \cdot \cos^2\alpha_i \\
&= R^2 \cdot R^2 \cdot \frac{(1-\cos\alpha_i)}{2} \cdot \cos^2\alpha_i \\
&= R^4 \cdot \operatorname{sen}^2\alpha_i \cdot \cos^2\alpha_i
\end{aligned}$$

$$
\begin{aligned}
A_i &= \frac{R^2}{2} \cdot 2 \operatorname{sen} \alpha_i \cdot \cos \alpha_i \\
&= \frac{R^2}{2} \cdot \operatorname{sen} 2\alpha_i
\end{aligned}
$$

$$
\begin{aligned}
A_T &= A_1 + A_2 + A_3 \\
&= \frac{R^2}{2} \cdot \operatorname{sen} 2\alpha_1 + \frac{R^2}{2} \cdot \operatorname{sen} 2\alpha_2 + \frac{R^2}{2} \cdot \operatorname{sen} 2\alpha_3 \\
&= \frac{R^2}{2} \left(\operatorname{sen} 2\alpha_1 + \operatorname{sen} 2\alpha_2 + \operatorname{sen} 2\alpha_3 \right).
\end{aligned}
$$