Project 1, SF2565

Hanna Hultin, TTMAM2
Mikael Persson, TTMAM2

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Task 1

Consider the $2N$ degree Taylor polynomial for $\cos x$

$$\begin{aligned}
\cos x \approx p(x) &= \sum_{n=0}^{N} (-1)^n \frac{x^{2n}}{(2n)!} \\
&= 1 + (-1)\frac{x^2}{2!} + (-1)^2\frac{x^4}{4!} + \cdots + (-1)^N \frac{x^{2N}}{(2N)!} \\
&= 1 - \frac{x\cdot x}{2\cdot 1}\Big(1 - \frac{x\cdot x}{4\cdot 3}\Big(1 - \ldots\Big(1 - \frac{x\cdot x}{(2N)(2N-1)}\Big)\ldots\Big).
\end{aligned}$$

Hence, the polynomial may be evaluated backwords using the following scheme

$$\begin{aligned}
b_N &= 1 - \frac{x\cdot x}{2N(2N-1)} \\
b_n &= 1 - \frac{x\cdot x}{2n(2n-1)} b_{n+1}, \quad n = N-1, N-2, \ldots, 2, 1 \\
b_1 &= p(x).
\end{aligned}$$

This is Horners' algorithm adjusted for the fact that each second term in the polynomial vanishes. Similarly for $\sin x$ the polynomial may be computed up to degree $2N+1$ by

$$\begin{aligned}
b_N &= 1 - \frac{x\cdot x}{2N(2N+1)} \\
b_n &= 1 - \frac{x\cdot x}{2n(2n+1)} b_{n+1}, \quad n = N-1, N-2, \ldots, 2, 1 \\
b_1 &= x\cdot p(x).
\end{aligned}$$