

Project 4, SF2565

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Task 1: Redesigning the Domain class

The domain class was taken directly from project 3, with the following modifications.

- Functions `xsize()`, `ysize()` and `gridValid()` was added.
- The class uses `shared_ptr` to the boundary curves.
- The `writeFile()` function was changed to `writeFile(std::string fileName)` to be able to save the different grid functions with different filenames.
- Point `operator()(int i, int j) const` to access the x and y of the grid.

The `Curvebase` class and its derived classes are almost identical to those used in project 3. We have attempted to optimize the code slightly, for example by using inlining in the classes `xLine` and `yLine`.

Task 2: The Gfctn class

The class for gridfunctions `class Gfctn` has the following data members.

- A matrix `u` to store the grid function values.
- A `shared_ptr` to a `Domain` object called `grid`.

The class has a the following constructors.

- `Gfctn(shared_ptr<Domain> grid)` which initializes the grid function with the matrix `u` being the zero matrix.
- `Gfctn(const Gfctn& U)`, a copy constructor.

Overloaded operators are provided for adding and multiplying grid functions (+ and *). The following member functions are implemented.

- `void setFunction(fctnPtr f)` which sets the gridfunction values to those defined by the function `f`. This function needs to take `Point` objects as argument.
- `void print()` which prints the matrix `u`. This is useful for testing on small grids only.
- `void writeFile(std::string fileName) const` which saves the grid function values to a binary file, `fileName.bin`.

In addition to those functions listed above, the class has functions to compute the approximations to $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2}$. Finally the class has a function for computing the Laplacian of the grid function, $\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$. These functions are

- `Gfctn D0x() const`

- Gfctn D0y() const
- Gfctn DD0x() const
- Gfctn DD0y() const
- Gfctn laplace() const

Task 3: Results

We investigate the class using the function

$$u(x, y) = \sin(x^2/10^2) \cos(x/10) + y$$

Figure 1 shows the function on the domain from project 3.

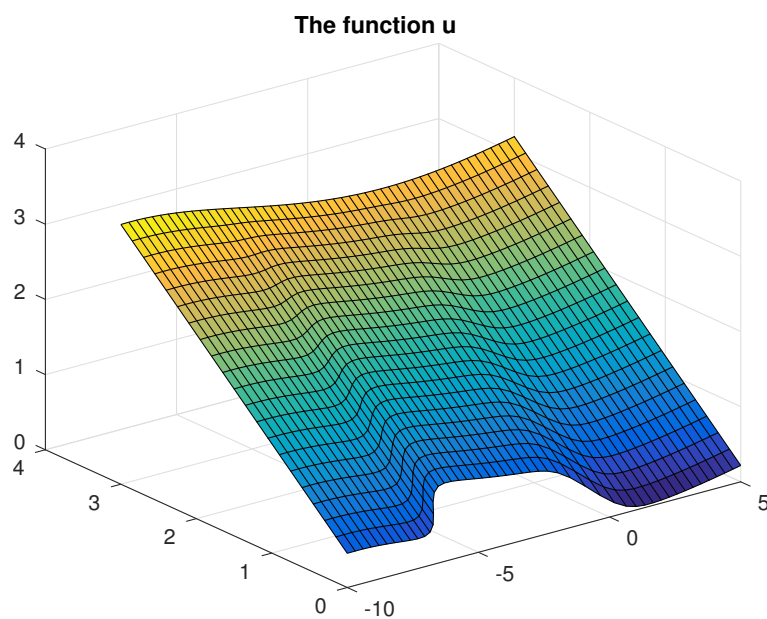


Figure 1: The function u defined on the domain from project 2.

Derivative w.r.t. x

Figures 2 and 3 shows the derivative w.r.t. x and the result from the implementation. Figure 4 show the difference between the true derivative w.r.t x and the implementation.

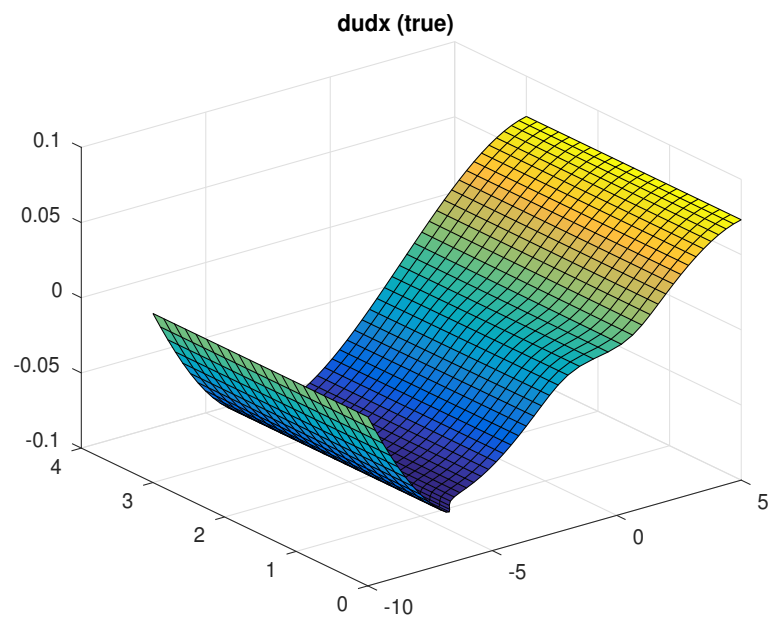


Figure 2: The true derivative $\frac{\partial u}{\partial x}$.

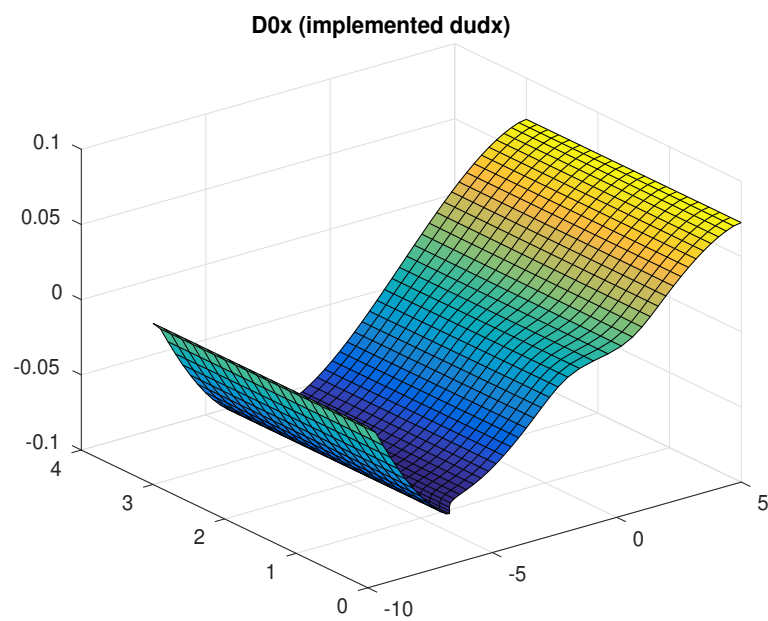


Figure 3: The result of the implementation of the derivative $\frac{\partial u}{\partial x}$.

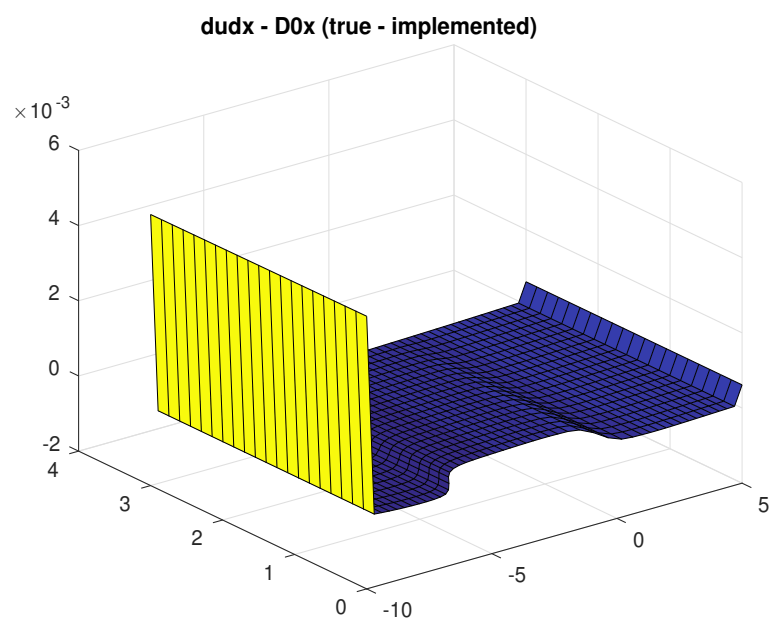


Figure 4: The difference of the true and implemented x -derivatives. Since we used one-sided differences on the boundary, the accuracy of the implementation is lower at the boundary.

Derivative w.r.t. y

Figure 5 shows the true derivative w.r.t. y while figures 6 and 7 show the implemented derivative and the difference of the true and implemented derivatives.

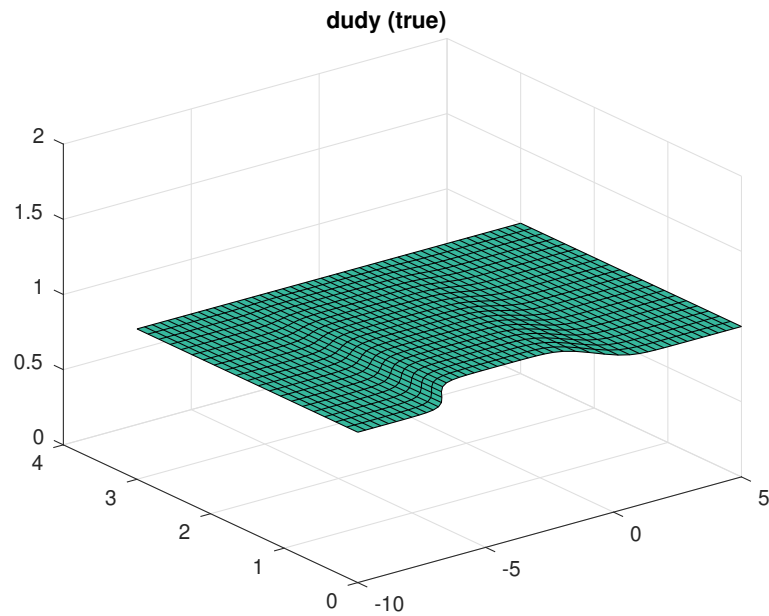


Figure 5: The true derivative $\frac{\partial u}{\partial y}$ is constant and equal to 1.

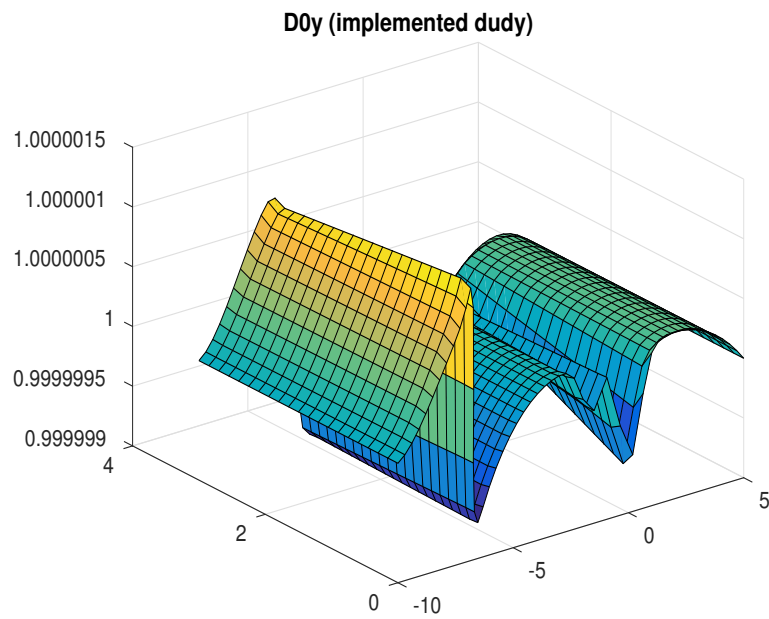


Figure 6: The implemented y -derivative. It is almost constantly equal to 1.

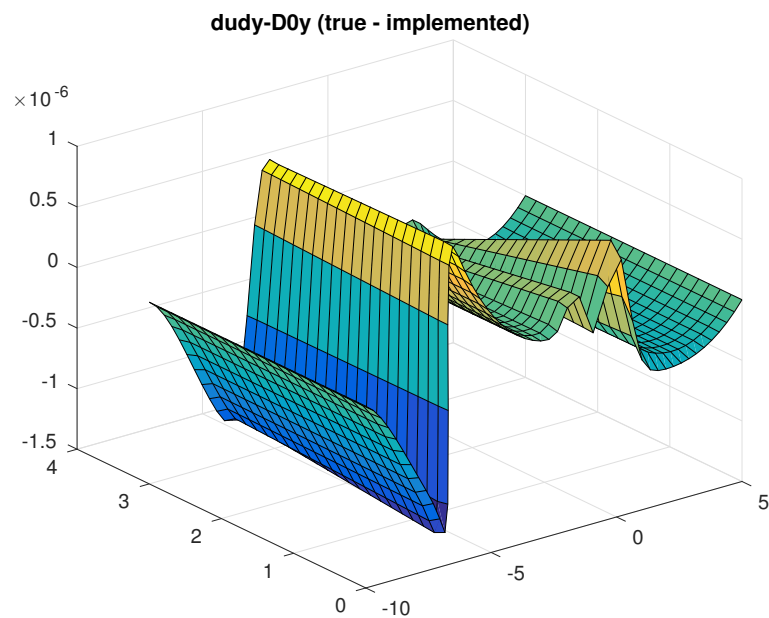


Figure 7: Difference between true and implemented derivatives.

Laplace operator

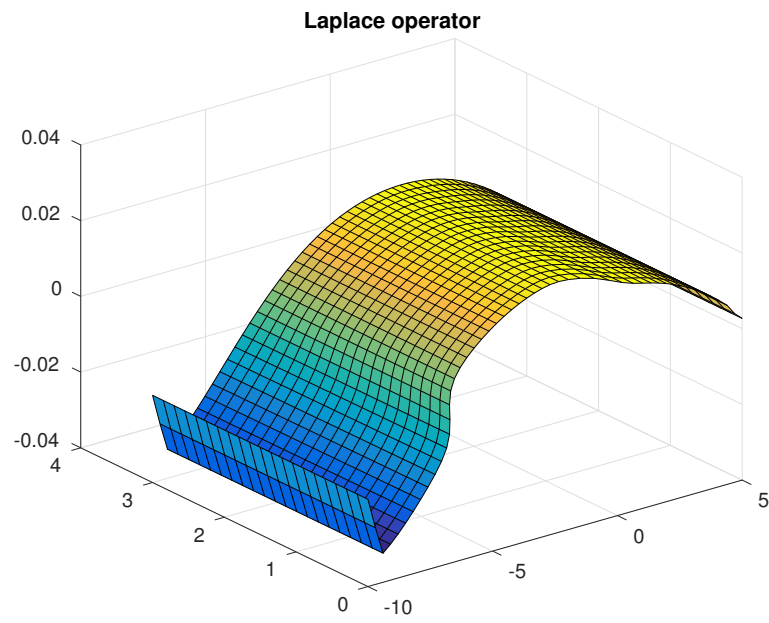


Figure 8: The Laplacian of the grid function u .

Code

Main

```
// file: testDomain.cpp

#include <iostream>
#include <memory>
#include <cmath>           // for sin and cos
#include "xline.hpp"
#include "yline.hpp"
#include "fxcurve.hpp"
#include "domain.hpp"
#include "gfctn.hpp"

using namespace std;

// function for testing the classes (as specified in lab instructions)
inline double f(Point p) {
    return sin((p.X()*p.X()*0.01))*cos(p.X()*0.1) + p.Y();
}

int main(int argc, char *argv[])
{
    shared_ptr<fxCurve> a = make_shared<fxCurve>(-10.0,5.0);
    shared_ptr<yLine> b = make_shared<yLine>(0.0,3.0,5.0);
    shared_ptr<xLine> c = make_shared<xLine>(-10.0,5.0,3.0);
    shared_ptr<yLine> d = make_shared<yLine>(0.0,3.0,-10.0);

    shared_ptr<Domain> grid = make_shared<Domain>(a,b,c,d);
    grid->grid_generation(50,20);
    grid->writeFile("gridOut.bin");
    Gfctn U = Gfctn(grid);
    U.setFunction(&f);
    //U.print();

    Gfctn DxU = U.D0x();
    cout << "derivative_x" << endl;
    DxU.writeFile("DxOut.bin");

    Gfctn DyU = U.D0y();
    cout << "derivative_y" << endl;
    DyU.writeFile("DyOut.bin");

    Gfctn Laplace = U.laplace();
    cout << "Laplace" << endl;
    Laplace.writeFile("laplaceOut.bin");

    return 0;
}
```

The Curvebase Class

```
// file: curvebase.hpp

#ifndef CURVEBASE_HPP
#define CURVEBASE_HPP

#include <cmath>
#include <iostream>

class Curvebase {
private:
    double newtonsolve(double p0, double s) const;
    double i2Simpson(double a, double b) const;
    double iSimpson(double a, double b) const;
    double dL(double t) const;           // integrand for arc length

protected:
    double a;
    double b;
    double length;

    // Pure virtual, ("= 0"), derived classes must implement:
    virtual double xp(double p) const = 0; //parametrized by user
    virtual double yp(double p) const = 0; //parametrized by user
}
```



```

virtual double dxp(double p) const = 0; //dx(p)/dp for arc length
virtual double dyp(double p) const = 0; //dy(p)/dp for arc length

double integrate(double a, double b) const;

public:
    Curvebase(); //default constructor
    virtual double x(double s) const; //parametrized by normalized arc length
    virtual double y(double s) const; //parametrized by normalized arc length

    // TODO from slides 6 F-Inheritance.pdf page 30:
    // the destructor of abstract base class should always be virtual
    // virtual ~Curvebase();
};

#endif // CURVEBASE_HPP

// file: Curvebase.cpp

#include <cmath>
#include <iostream>
#include "curvebase.hpp"

Curvebase::Curvebase() {}; // Default constructor

/* Integrate , i2Simpson, iSimpson all taken
 * directly from project 1.
 */
inline double Curvebase::i2Simpson(double a, double b) const {
    return iSimpson(a,0.5*(a+b)) + iSimpson(0.5*(a+b),b);
}

inline double Curvebase::iSimpson(double a, double b) const {
    return ((b-a)/6.0)*(dL(a)+4.0*dL(0.5*(a+b)) + dL(b));
}

inline double Curvebase::dL(double p) const {
    return sqrt(dxp(p)*dxp(p) + dyp(p)*dyp(p));
}

double Curvebase::integrate(double a, double b) const{

    double tolI = 1e-8;
    double I = 0, I1, I2, errest;
    int node = 1;

    while (true) {
        I1 = iSimpson(a,b);
        I2 = i2Simpson(a,b);
        errest = std::abs(I1-I2);
        if (errest < 15*tolI) { //if leaf
            I += I2;
            while (node % 2 != 0) { // while uneven node

                if (node == 1) {
                    return I; // return if we are back at root again
                }

                node = 0.5*node;
                a = 2*a-b;
                tolI *= 2;
            }
            // First even node: go one node up - go to right child
            b = 2*b-a;
            node = node+1;
            a = 0.5*(a+b);
        } else { //if not a leaf: go to left child
            node *= 2;
            b = 0.5*(a+b);
            tolI *= 0.5;
        }
    }
}

/* Newton solver for equation f(p) = l(p) - s*l(b)
 * input: p0 is initial guess for Newtons method.

```

```

*/
double Curvebase::newtonsolve(double p0, double s) const{

    int iter = 0, maxiter = 150;
    double tolN = 1e-6;
    double err = 1.0;
    double p1,p;
    p = p0;
    while (err > tolN && iter < maxiter) {

        p1 = p - (integrate(a,p)-s*length)/dL(p);    // Newtons method
        err = fabs(p1 - p);                          // Check error
        p = p1; iter++;                             // Update
    }

    if (iter == maxiter) {                          // maxiter reached
        std::cout << "No_convergence_in_Newton_solver" << std::endl;
    }

    return p;
}

// Curve parametrized by grid coordinate
double Curvebase::x(double s) const{
    double p, p0;
    p0 = a + s*length;
    p = newtonsolve(p0,s);                        // Initial guess for Newtons meth.
    return xp(p);
}

// Curve parametrized by grid coordinate
double Curvebase::y(double s) const{
    double p, p0;
    p0 = a + s*length;
    p = newtonsolve(p0,s);                        // Initial guess for Newtons meth.
    return yp(p);
}

```

The derived classes from the Curvebase Class

ylene).

```

// file: xline.hpp

#ifndef XLINE_HPP
#define XLINE_HPP

/* xLine: curves for lines with constant y.
 * Derived class from base class Curvebase.
 * Constructor: y0 constant y,
 *             x0, x1 interval in x: [x0, x1].
 * Overwrite integrate, xp, yp, dxp, dyp, x(s) and y(s).
 */

#include "curvebase.hpp"

class xLine: public Curvebase{
public:
    xLine(double x0, double x1, double y0)    // Constructor
    {
        a = x0;
        b = x1;
        yConst = y0;
        length = x1 - x0;
    }
    ~xLine() {}                                // Destructor

    // Overwrite x(s) and y(s) in normalized coordinates
    double x(double s) const { return a+s*length; }
    double y(double s) const { return yConst; }

protected:
    double yConst;

    // user parametrizations
    double xp(double p) const { return p; }
    double yp(double p) const { return yConst; }
    double dxp(double p) const { return 1.0; }

```

```

    double dyp(double p) const { return 0.0; }

    // Arc length
    double integrate(double a, double b) const { return b-a; }
};

#endif // XLINE_HPP

#ifndef YLINE_HPP
#define YLINE_HPP

/* yLine: curves for lines with constant x.
 * Derived class from base class Curvebase.
 * Constructor: x0 is constant x,
 *             y0, y1 interval in y: [y0,y1].
 * Overwrite integrate, xp, yp, dxp, dyp, x(s) and y(s)
 */

#include "curvebase.hpp"

class yLine: public Curvebase{
public:
    yLine(double y0, double y1, double x0) // Constructor
    {
        a = y0;
        b = y1;
        xC = x0;
        length = y1 - y0;
    }
    ~yLine() {} // Destructor

    // Overwrite x(s) and y(s) in normalized coordinates
    double x(double s) const { return xC; }
    double y(double s) const { return a+s*length; }

protected:
    double xC;

    // user parametrizations
    double xp(double p) const { return xC; }
    double yp(double p) const { return p; }
    double dxp(double p) const { return 0.0; }
    double dyp(double p) const { return 1.0; }

    // Arc length
    double integrate(double a, double b) const { return b-a; }
};

#endif // YLINE_HPP

#ifndef FXCURVE_HPP
#define FXCURVE_HPP

/* fxCurve: Derived class from base class Curvebase.
 * Constructor: interval length in x: [x0,x1].
 */

class fxCurve: public Curvebase{
public:
    fxCurve(double xx0, double xx1); // Constructor
    ~fxCurve(); // Destructor

protected:
    double xp(double p) const;
    double yp(double p) const;
    double dxp(double p) const;
    double dyp(double p) const;
};

#endif // FXCURVE_HPP

#include <cmath> // for exp in xp, yp, dxp, dyp

#include "curvebase.hpp"
#include "fxcurve.hpp"

```

```

// Constructor
fxCurve::fxCurve(double xx0, double xx1) {
    a = xx0;
    b = xx1;
    length = integrate(a,b);
}

// Destructor
fxCurve::~fxCurve() {}

// Curve parametrized in user parameter p
double fxCurve::xp(double p) const { return p; }
double fxCurve::yp(double p) const {
    if (p < -3.0) {
        return 0.5/(1.0 + exp(-3.0*(p + 6.0)));
    } else {
        return 0.5/(1.0 + exp(3.0*p));
    }
}

// Derivatives w.r.t. the user parameter p
double fxCurve::dyp(double p) const { return 1.0; }
double fxCurve::dyp(double p) const {
    if (p < -3.0) {
        //return 6.0*exp(-3.0*(p+6))*yp(p)*yp(p);
        return 1.5*exp(3.0*(p+6))/(1.0 + 2.0*exp(3.0*(p + 6.0)) + exp(6.0*(p+6.0)));
    } else {
        //return -6.0*exp(3.0*p)*yp(p)*yp(p);
        return -1.5*exp(3.0*p)/(1.0 + 2.0*exp(3.0*p) + exp(6.0*p));
    }
}

```

The Domain Class

```

// file: domain.hpp

#ifndef DOMAIN_HPP
#define DOMAIN_HPP

#include <memory> // for shared_ptr (use -std=c++11)
#include "curvebase.hpp"
#include "point.hpp"

using namespace std;

class Domain {
private:
    shared_ptr<Curvebase> sides[4]; // Pointers to curves of the 4 sides
    int n_, m_; // # of grid points in x and y
    double *x_, *y_; // Arrays for coordinates in grid
    bool cornersOk; // Corners connected = ok

    inline double phi1(double t) const {return t;}; // Linear interpolation functions
    inline double phi2(double t) const {return 1.0-t;};

public:
    // CONSTRUCTOR

    Domain(shared_ptr<Curvebase> s1,
           shared_ptr<Curvebase> s2,
           shared_ptr<Curvebase> s3,
           shared_ptr<Curvebase> s4);

    // DESTRUCTOR
    ~Domain();

    Point operator()(int i, int j) const; // Coordinates at i,j

    // FUNCTIONS
    void grid_generation(int n, int m); // Generates the grid (x_ and y_)
    void print() const; // Print points of grid to console
    void writeFile(std::string fileName) const; // Write points to .bin-file
    bool checkCorners() const; // Check if corners are connected

    // new functions for pro4:
    inline int xsize() const {return n_;};

```

```

    inline int ysize() const {return m-;}
    bool gridValid() const;
};

#endif //DOMAIN_HPP

// file: domain.cpp

#include <cstdio>           // for writeFile()
#include <iostream>
#include <cmath>           // for fabs

#include "domain.hpp"
// #include "curvebase.hpp"
// #include "point.hpp"

/*
 * .cpp-file for class domain. See also domain.hpp.
 */

using namespace std;

// CONSTRUCTOR
Domain::Domain(shared_ptr<Curvebase> s1,
               shared_ptr<Curvebase> s2,
               shared_ptr<Curvebase> s3,
               shared_ptr<Curvebase> s4): n_(0), m_(0), x_(nullptr), y_(nullptr) {

    sides[0] = s1;
    sides[1] = s2;
    sides[2] = s3;
    sides[3] = s4;

    cornersOk = checkCorners();           // Indicator for corners connected
    if (!cornersOk) {
        sides[0] = sides[1] = sides[2] = sides[3] = nullptr;
    }
}

// DESTRUCTOR
Domain::~Domain() {
    if (m_ > 0) {
        delete [] x_;           // Could as well check if n_>0, since both
        delete [] y_;           // need to be positive to generate the grid
    }
}

Point Domain::operator()(int i, int j) const
{
    if (i < 0 || i > n_ || j < 0 || j > m_) {
        cout << "invalid_index_ij" << endl;
        exit(1);
    }
    int ind = j+i*(m_+1);
    return Point(x_[ind], y_[ind]);
}

// MEMBER FUNCTIONS
// Generates the grid and sets it to
void Domain::grid_generation(int n, int m) {
    if ((n < 1) || (m < 1)) {
        // Need n and m > 0 to generate any grid. Else:
        std::cout << "Warning: _Non_positive_grid_size." << std::endl;
        std::cout << "No_grid_generated" << std::endl;
        return;           // No grid is generated
    } else if (!cornersOk) {
        // Dont generate grid if corners are disconnected
        std::cout << "No_grid_generated_(corner_disconnected)" << std::endl;
        return;           // No grid is generated
    }
}

```

```

if (n != 0) {                                     // Reset the arrays
    delete[] x_;
    delete[] y_;
}

n_ = n;
m_ = m;

/* The sides' coordinates are computed once only, i.e. there is
 * 4*(n+1)+4*(m+1) calls to x(s) and y(s). If instead, one would
 * call x(s) and y(s) for each of the grid points there would be
 * 16*(n+1)*(m+1) calls.
 * Consider MEMORY if n,m are large.
 */

double *xLo,*xRi,*xTo,*xLe,*yLo,*yRi,*yTo,*yLe;

xLo = new double[n_+1];           // Lower boundary x-coords
xRi = new double[m_+1];           // Right boundary
xTo = new double[n_+1];           // Top boundary
xLe = new double[m_+1];           // Left boundary

yLo = new double[n_+1];           // same for the y-coords
yRi = new double[m_+1];
yTo = new double[n_+1];
yLe = new double[m_+1];

double h1= 1.0/n; double h2 = 1.0/m; // Step sizes

for (int i=0; i <= n_; i++) {       // Loop the normalized coordinate for x
    xLo[i] = sides[0]->x(i*h1);
    xTo[i] = sides[2]->x(i*h1);

    yLo[i] = sides[0]->y(i*h1);
    yTo[i] = sides[2]->y(i*h1);
}

for (int j=0; j <= m_; j++) {       // Loop the normalized coordinate for y
    xRi[j] = sides[1]->x(j*h2);
    xLe[j] = sides[3]->x(j*h2);

    yRi[j] = sides[1]->y(j*h2);
    yLe[j] = sides[3]->y(j*h2);
}

x_ = new double[(n_+1)*(m_+1)];     // x-coordinates for the entire grid
y_ = new double[(n_+1)*(m_+1)];     // y-coordinates for the same

for (int i = 0; i <= n_; i++) {
    for (int j = 0; j <= m_; j++) {
        x_[j+i*(m_+1)] =
            phi2(i*h1)*xLe[j]           // left side
            + phi1(i*h1)*xRi[j]         // right side
            + phi2(j*h2)*xLo[i]         // bottom side
            + phi1(j*h2)*xTo[i]         // top side
            - phi2(i*h1)*phi2(j*h2)*xLo[0] // lower left
            - phi1(i*h1)*phi2(j*h2)*xLo[n_] // lower right
            - phi2(i*h1)*phi1(j*h2)*xTo[0] // top left
            - phi1(i*h1)*phi1(j*h2)*xTo[n_]; // top right

        y_[j+i*(m_+1)] =
            phi2(i*h1)*yLe[j]           // equivalent to x above
            + phi1(i*h1)*yRi[j]
            + phi2(j*h2)*yLo[i]
            + phi1(j*h2)*yTo[i]
            - phi2(i*h1)*phi2(j*h2)*yLo[0]
            - phi1(i*h1)*phi2(j*h2)*yLo[n_]
            - phi2(i*h1)*phi1(j*h2)*yTo[0]
            - phi1(i*h1)*phi1(j*h2)*yTo[n_];
    }
}

delete[] xLo; // Delete temporary
delete[] xRi; // boundary values
delete[] xTo;
delete[] xLe;

```

```

delete[] yLo;
delete[] yRi;
delete[] yTo;
delete[] yLe;
}

// Print (for testing) the grid coordinates: Careful if n,m are large.
void Domain::print() const {
    if (n_ < 1 || m_ < 1) {
        std::cout << "No_grid_to_print" << std::endl;
        return;
    }
    for (int i = 0; i < (n_+1)*(m_+1); i++) {
        std::cout << "[" << x_[i] << "," << y_[i] << "]" << std::endl;
    }
}

// Write the grid to an external file to enable visualization in e.g. matlab.
void Domain::writeFile(std::string fileName) const {
    if (n_ < 1 || m_ < 1) {
        std::cout << "No_grid_available_for_writeFile()" << std::endl;
        return;
    }
    FILE *fp;
    fp = fopen(fileName.c_str(), "wb");
    if (fp == nullptr) {
        std::cout << "Error_opening_file_to_write_to" << std::endl;
    }
    fwrite(&n_, sizeof(int), 1, fp);
    fwrite(&m_, sizeof(int), 1, fp);
    fwrite(x_, sizeof(double), (n_+1)*(m_+1), fp);
    fwrite(y_, sizeof(double), (n_+1)*(m_+1), fp);
    fclose(fp);
}

// Function to check if the boundaries are connected (corners)
bool Domain::checkCorners() const {
    if (fabs(sides[0]->x(1) - sides[1]->x(0)) > 1e-4 ||
        fabs(sides[0]->y(1) - sides[1]->y(0)) > 1e-4) {
        std::cout << "Low-Right_corner_disconnected" << std::endl;
        return false;
    }
    if (fabs(sides[1]->x(1) - sides[2]->x(1)) > 1e-4 ||
        fabs(sides[1]->y(1) - sides[2]->y(1)) > 1e-4) {
        std::cout << "Top-Right_corner_disconnected" << std::endl;
        return false;
    }
    if (fabs(sides[2]->x(0) - sides[3]->x(1)) > 1e-4 ||
        fabs(sides[2]->y(0) - sides[3]->y(1)) > 1e-4) {
        std::cout << "Top-Left_corner_disconnected" << std::endl;
        return false;
    }
    if (fabs(sides[3]->x(0) - sides[0]->x(0)) > 1e-4 ||
        fabs(sides[3]->y(0) - sides[0]->y(0)) > 1e-4) {
        std::cout << "Low-Left_corner_disconnected" << std::endl;
        return false;
    }
    return true;
}

// new functions for pro4:
bool Domain::gridValid() const {
    if (m_ != 0 && checkCorners()) {
        //std::cout << "grid valid!" << std::endl;
        return true;
    } else {
        std::cout << "grid_NOT_valid!" << std::endl;
        return false;
    }
}

```

The Gfctn Class

```

// file: gfctn.hpp

#ifndef GFCTN_HPP
#define GFCTN_HPP

```

```

#include <memory>           // for shared_ptr (use -std=c++11)
#include "matrix.hpp"
#include "domain.hpp"

// from slides "Implementation of Grid Functions"

typedef double (*fctnPtr)(Point);

class Gfctn
{
private:
    Matrix u;
    shared_ptr<Domain> grid;

public:
    // CONSTRUCTORS
    Gfctn(shared_ptr<Domain> grid_);
    Gfctn(const Gfctn& U);

    // OPERATORS
    Gfctn& operator=(const Gfctn& U); // copy assignment
    Gfctn& operator=(Gfctn&& U) noexcept; // move assignment

    Gfctn operator+(const Gfctn& U) const;
    Gfctn operator*(const Gfctn& U) const;

    // MEMBER FUNCTIONS
    void setFunction(const fctnPtr f); // set grid function values
    inline void writeFile(std::string fileName) const {u.writeFile(fileName);} //write to binary file
    Gfctn D0x() const; // du/dx
    Gfctn D0y() const; // du/dy
    Gfctn laplace() const; // d2u/dx2 + d2u/dy2

    // etc
};

#endif // GFCTN_HPP

```

```

// file: gfctn.cpp

#include <iostream>
#include "gfctn.hpp"

/* Source file for Gfctn class.
 * See gfctn.hpp for declarations.
 *
 * TODO more comments
 */

// Constructors -----
Gfctn::Gfctn(shared_ptr<Domain> grid_)
    : u(grid_>xsize() + 1, grid_>ysize() + 1), grid(grid_) {}

Gfctn::Gfctn(const Gfctn &U)
    : u(U.u), grid(U.grid) {}

// Destructor -----

/*
Gfctn::~~Gfctn()
{
    // TODO implement destructor
}
*/

// Operator overloadings -----

// Copy assignment
Gfctn &Gfctn::operator=(const Gfctn &U) {
    u = U.u;
    grid = U.grid;
    return *this;
}

// Move assignment
Gfctn &Gfctn::operator=(Gfctn &&U) noexcept {
    u = U.u;
    grid = U.grid;
}

```



```

    U.u = Matrix();
    U.grid = nullptr;
    return *this;
}

Gfctn Gfctn::operator+(const Gfctn &U) const {
    if (grid == U.grid) { // Defined on same grid?
        Gfctn tmp = Gfctn(grid);
        tmp.u = u + U.u; // Matrix operator +()
        return tmp;
    } else {
        std::cout << "error:_different_grids" << std::endl;
        exit(1);
    }
}

Gfctn Gfctn::operator*(const Gfctn &U) const {
    if (grid == U.grid) {
        Gfctn tmp = Gfctn(grid);
        for (int j = 0; j < grid->ysize(); j++) {
            for (int i = 0; i < grid->xsize(); i++) {
                tmp.u(i, j) = u.get(i, j) * U.u.get(i, j);
            }
        }
        return tmp;
    } else {
        std::cout << "error:_different_grids_(*)" << std::endl;
        exit(1);
    }
}

// Member functions -----

/* setFunction
 * Computes the value of the function f in all grid points and puts in the matrix u
 */
void Gfctn::setFunction(const fctnPtr f)
{
    for (int j = 0; j <= grid->ysize(); j++) {
        for (int i = 0; i <= grid->xsize(); i++) {
            u(i, j) = f((*grid)(i, j));
            //cout << (*grid)(i,j) << endl;
        }
    }
}

/* du/dx of grid function u
 * usage: Gfctn DxU = U.D0x();
 * Implementation of derivative from slide F.PDEs
 */
Gfctn Gfctn::D0x() const {
    Gfctn tmp(grid);
    if (grid->gridValid()) {
        double xi, xj, yi, yj, ui, uj;
        double h1 = 1.0 / grid->xsize();
        double h2 = 1.0 / grid->ysize();

        for (int j = 0; j <= grid->ysize(); j++) {
            for (int i = 0; i <= grid->xsize(); i++) { //start at i=1, end at i=n-1
                if (i == 0) {
                    xi = ((*grid)(i + 1, j).X() - (*grid)(i, j).X()) / h1;
                    yi = ((*grid)(i + 1, j).Y() - (*grid)(i, j).Y()) / h1;
                    ui = (u.get(i + 1, j) - u.get(i, j)) / h1;
                } else if (i == grid->xsize()) {
                    xi = ((*grid)(i, j).X() - (*grid)(i - 1, j).X()) / h1;
                    yi = ((*grid)(i, j).Y() - (*grid)(i - 1, j).Y()) / h1;
                    ui = (u.get(i, j) - u.get(i - 1, j)) / h1;
                } else {
                    xi = ((*grid)(i + 1, j).X() - (*grid)(i - 1, j).X()) / (2.0 * h1);
                    yi = ((*grid)(i + 1, j).Y() - (*grid)(i - 1, j).Y()) / (2.0 * h1);
                    ui = (u.get(i + 1, j) - u.get(i - 1, j)) / (2.0 * h1);
                }
                if (j == 0) {
                    xj = ((*grid)(i, j + 1).X() - (*grid)(i, j).X()) / h2;
                    yj = ((*grid)(i, j + 1).Y() - (*grid)(i, j).Y()) / h2;
                    uj = (u.get(i, j + 1) - u.get(i, j)) / h2;
                } else if (j == grid->ysize()) {

```

```

        xj = ((*grid)(i, j).X() - (*grid)(i, j - 1).X()) / h2;
        yj = ((*grid)(i, j).Y() - (*grid)(i, j - 1).Y()) / h2;
        uj = (u.get(i, j) - u.get(i, j - 1)) / h2;
    } else {
        xj = ((*grid)(i, j + 1).X() - (*grid)(i, j - 1).X()) / (2.0 * h2);
        yj = ((*grid)(i, j + 1).Y() - (*grid)(i, j - 1).Y()) / (2.0 * h2);
        uj = (u.get(i, j + 1) - u.get(i, j - 1)) / (2.0 * h2);
    }
    tmp.u(i, j) = (ui * yj - uj * yi) / (xi * yj - yi * xj);
}
}
} else {
    cout << "grid_invalid_in_D0x" << endl;
}
}
return tmp;
}

/* du/dy of grid function u
 * Analogous to above
 */
Gfctn Gfctn::D0y() const {
    Gfctn tmp(grid);
    if (grid->gridValid()) {
        double xi, xj, yi, yj, ui, uj;
        double h1 = 1.0 / grid->xsize();
        double h2 = 1.0 / grid->ysize();

        for (int j = 0; j <= grid->ysize(); j++) {
            for (int i = 0; i <= grid->xsize(); i++) { //start at i=1, end at i=n-1
                if (i == 0) {
                    xi = ((*grid)(i + 1, j).X() - (*grid)(i, j).X()) / h1;
                    yi = ((*grid)(i + 1, j).Y() - (*grid)(i, j).Y()) / h1;
                    ui = (u.get(i + 1, j) - u.get(i, j)) / h1;
                } else if (i == grid->xsize()) {
                    xi = ((*grid)(i, j).X() - (*grid)(i - 1, j).X()) / h1;
                    yi = ((*grid)(i, j).Y() - (*grid)(i - 1, j).Y()) / h1;
                    ui = (u.get(i, j) - u.get(i - 1, j)) / h1;
                } else {
                    xi = ((*grid)(i + 1, j).X() - (*grid)(i - 1, j).X()) / (2.0 * h1);
                    yi = ((*grid)(i + 1, j).Y() - (*grid)(i - 1, j).Y()) / (2.0 * h1);
                    ui = (u.get(i + 1, j) - u.get(i - 1, j)) / (2.0 * h1);
                }
                if (j == 0) {
                    xj = ((*grid)(i, j + 1).X() - (*grid)(i, j).X()) / h2;
                    yj = ((*grid)(i, j + 1).Y() - (*grid)(i, j).Y()) / h2;
                    uj = (u.get(i, j + 1) - u.get(i, j)) / h2;
                } else if (j == grid->ysize()) {
                    xj = ((*grid)(i, j).X() - (*grid)(i, j - 1).X()) / h2;
                    yj = ((*grid)(i, j).Y() - (*grid)(i, j - 1).Y()) / h2;
                    uj = (u.get(i, j) - u.get(i, j - 1)) / h2;
                } else {
                    xj = ((*grid)(i, j + 1).X() - (*grid)(i, j - 1).X()) / (2.0 * h2);
                    yj = ((*grid)(i, j + 1).Y() - (*grid)(i, j - 1).Y()) / (2.0 * h2);
                    uj = (u.get(i, j + 1) - u.get(i, j - 1)) / (2.0 * h2);
                }
                tmp.u(i, j) = (-ui * xj + uj * xi) / (xi * yj - yi * xj);
            }
        }
    } else {
        cout << "grid_invalid_in_D0y" << endl;
    }
    return tmp;
}

/* Laplacian of grid function
 */
Gfctn Gfctn::laplace() const {
    Gfctn laplace = D0x().D0x() + D0y().D0y();
    return laplace;
}

```

```
// file: gfctn.cpp
```

The Matrix Class

```
// file: matrix.hpp
```

```

#ifndef MATRIX_HPP
#define MATRIX_HPP

#include <iostream>

class Matrix
{
private:
    int m, n;    // matrix dim.
    double *a;   // matrix elements

public:
    // Constructors and destructors
    Matrix(int n_ = 0, int m_ = 0);
    Matrix(const Matrix &M);
    ~Matrix();

    // Functions
    void fillMatrix(double b[]);
    void identity();
    void print() const;
    inline int rowSizeMatrix() const {return n;}
    inline int colSizeMatrix() const {return m;}
    void randomize();
    void writeFile(std::string fileName) const;
    inline double get(int i, int j) const {
        return a[i*m+j];           // get element from matrix
    }

    // Operator overloadings
    Matrix &operator=(const Matrix &M);
    Matrix &operator=(Matrix &&M) noexcept;
    const Matrix &operator*=(const double d);
    const Matrix &operator+=(const Matrix &M);
    const Matrix operator+(const Matrix& M) const;
    double& operator()(int i, int j) const;
    friend std::ostream& operator<<(std::ostream& os, const Matrix& M);
};

#endif // MATRIX_HPP

```

```

// file: matrix.cpp

#include <iostream>
#include <iomanip>           // for setprecision in operator<<
#include "matrix.hpp"

/* Source file for Matrix class.
 * See matrix.hpp for declarations
 * This class implements matrix using C-style array
 * a[i+j*n] is the element on row i, col j i.e. A[i,j].
 */

using namespace std;

// Constructors -----

Matrix::Matrix(int n_, int m_): m(m_), n(n_), a(nullptr)
{
    if (m*n > 0) {
        a = new double[m*n];
        fill(a, a+m*n, 0.0);
    }
}

Matrix::Matrix(const Matrix &M)
{
    n = M.n;
    m = M.m;
    a = new double [m*n];
    for (int i = 0; i < n*m; i++) {
        a[i] = M.a[i];
    }
    //cout << "matrix copy-constructor:" << this << endl;
}

// Destructor -----

Matrix::~Matrix()

```

```

{
    delete [] a;
}

// Member functions
void Matrix::fillMatrix(double b[])
{
    for (int i = 0; i < m*n; i++) {
        a[i] = b[i];
    }
}

void Matrix::identity()
{
    if (n!=m) {
        cout << "A_non-square_matrix_can_not_be_the_identity_matrix" << endl;
        return;
    }
    for (int i = 0; i < n*n; i++) {
        (i%n == i/n)? a[i] = 1: a[i] = 0;
    }
}

void Matrix::print() const
{
    cout << endl;
    if (n == 0 || m == 0) {
        cout << "[]" << endl;
        return;
    }
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++) {
            cout << a[j + i*m] << " ";
        }
        cout << endl;
    }
}

void Matrix::randomize()
{
    if (n == 0 || m == 0) {
        cout << "empty_matrix, _no_randomizing_done" << endl;
        return;
    }
    //srand(time(0)); gives the same random number every time
    for (int i = 0; i < n*m; i++) {
        a[i] = rand()%10;
    }
}

// Write the grid to an external file to enable visualization in e.g. matlab.
void Matrix::writeFile(string fileName) const{
    if (n < 1 || m < 1) {
        cout << "No_matrix_available_for_writeFile()" << endl;
        return;
    }
    FILE *fp;
    fp = fopen(fileName.c_str(), "wb");
    if (fp == nullptr) {
        cout << "Error_opening_file_to_write_to" << endl;
        return;
    }
    fwrite(&n, sizeof(int), 1, fp);
    fwrite(&m, sizeof(int), 1, fp);
    fwrite(a, sizeof(double), n*m, fp);
    fclose(fp);
}

// Operator overloadings
/* Equality operator
 * Usage: M1 = M2; where M1 and M2 are Matrix-obj.
 */
Matrix &Matrix::operator=(const Matrix &M)
{
    if (this == &M) {
        return *this;
    }

```

```

    }
    if (n == M.n && m == M.m) {
        for (int i = 0; i < n*m; i++) {
            a[i] = M.a[i];
        }
    } else {
        if (a) { // if initialized, delete a
            delete a;
        }
        n = M.n;
        m = M.m;
        a = new double[n*m];
        for (int i = 0; i < n*m; i++) {
            a[i] = M.a[i];
        }
    }
    return *this;
}

Matrix &Matrix::operator=(Matrix &&M) noexcept{
    if (this == &M) {
        return *this;
    }
    m = M.m;
    n = M.n;
    a = M.a;
    M.m = 0;
    M.n = 0;
    M.a = nullptr;

    return *this;
}

/* Matrix-scalar multiplication operator
 * Usage: M *= d; where M is Matrix-obj and d is double
 */
const Matrix &Matrix::operator*=(const double d)
{
    for (int i = 0; i < n*m; i++) {
        a[i] *= d;
    }
    return *this;
}

/* Matrix addition operator
 * Usage: M1 += M2
 */
const Matrix &Matrix::operator+=(const Matrix &M)
{
    if (n != M.n || m != M.m) {
        cerr << "Dimensions_mismatch_in_sum..Exiting." << endl;
        exit(1);
    }
    for (int i = 0; i < n*m; i++) {
        a[i] += M.a[i];
    }

    return *this;
}

/* Matrix addition operator
 * Usage: A = B+C;
 */
const Matrix Matrix:: operator+(const Matrix &M) const
{
    if (n != M.n || m != M.m) {
        cerr << "Dimensions_mismatch_in_sum..Exiting" << endl;
        exit(1);
    }
    Matrix A(n,m);
    for (int i = 0; i < n*m; i++) {
        A.a[i] = a[i]+M.a[i];
    }
    return A;
}

/* Matrix element access operator
 * Usage: e = M(i,j)

```

```

*/
double& Matrix:: operator()(int i, int j) const
{
    if (i < 0 || i >= n || j < 0 || j >= m) {
        cerr << "Bad_index_in_matrix" << endl;
        exit(1);
    }
    return a[j+i*m];
}

/* Stream insertion operator
 * Usage: cout << M << endl;
 */
ostream& operator<<(ostream& os, const Matrix& M)
{
    int n = M.n;
    int m = M.m;
    os << endl;
    os << fixed << setprecision(4);
    if (n <= 0 || m <= 0) {
        os << "[]" << endl;
        return os;
    }
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < m; j++) {
            if (M.a[j + i*m] >= 0) {
                os << "_";
            }
            os << M.a[j + i*m] << " _";
        }
        os << endl;
    }
    return os;
}

// matrix.cpp

```

The Point Class

```

// file: point.hpp

#ifndef POINT_HPP
#define POINT_HPP

#include <iostream>

using namespace std;

class Point
{
private:
    double x;
    double y;

public:
    // Constructors and destructor
    Point(double xx = 0.0, double yy = 0.0);    // constructor
    Point(const Point& Q);                      // copy constructor
    ~Point();                                   // destructor

    // Member functions
    double X() const { return x; }
    double Y() const { return y; }
    friend ostream& operator<<(ostream& os, const Point& P);
};

#endif // POINT_HPP

// file: point.cpp

/* Source file for class Point.
 * See also point.hpp for declarations

```

```
*/  
  
#include "point.hpp"  
  
using namespace std;  
  
// CONSTRUCTORS AND DESTRUCTORS      ----      ----      ----      ----  
  
// constructor using initializer list  
Point::Point(double xx, double yy) :  
    x(xx),  
    y(yy)  
{  
  
// Copy constructor  
Point::Point(const Point& Q) :  
    x(Q.x), y(Q.y)  
{  
  
// destructor  
Point::~~Point()  
{  
  
// output operator for ostream  
ostream& operator<<(ostream& os, const Point& P)  
{  
    os << "[" << P.x << ", " << P.y << "]" ; //friend function  
    return os;  
}
```