SF2822

Project assignment 2A1

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May 17, 2016

Contents

1 Background

In this problem a very simplified model of the traffic system of Stockholm was created and analyzed. The objective is to minimize the total traveling time for the commuters of Stockholm by finding the optimal distribution of cars on the edges between the nodes.

1.1 Introduction

A model of Stockholm with the city center and its suburbs was created and shown in Figure 1. The goal was to get a fairly realistic model to draw conclusions on the actual Stockholm region and on the non linear optimization approach analyzing the situation.

The traffic load from one region to another, in the morning of an average work day, was estimated and is shown in Table 1. Naturally, the amount of traffic to the city center and to industrial regions is higher than traffic to residential areas in the morning. Also, the average traveling time between two regions without any congestion, was estimated and is shown in Table 2. These times are approximate and based on experience from driving during low traffic conditions.

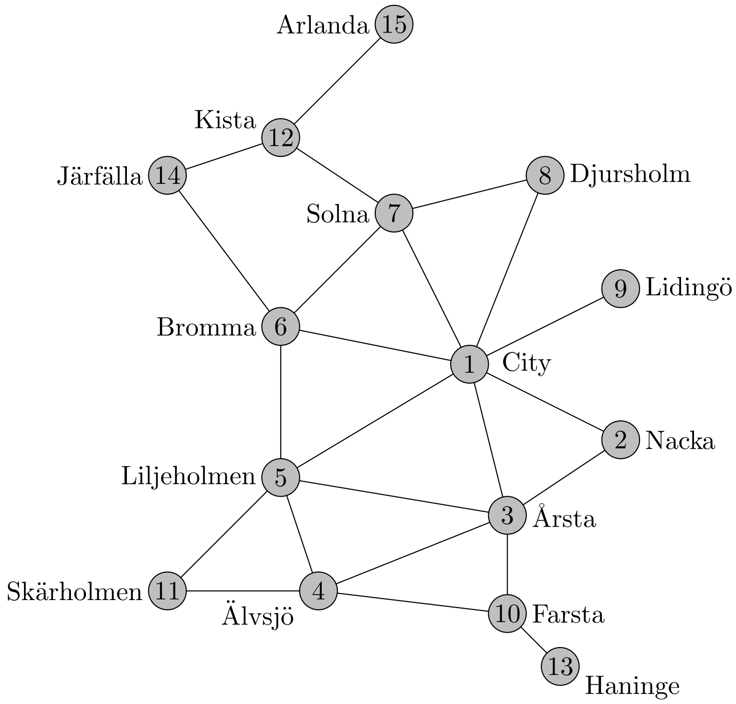

Figure 1: A simplified network model for the traffic flow in the greater Stockholm metropolitan area. The nodes represent suburbs and the city center while the edges connecting the nodes represent major roads used by commuters.

Table 1: The amount of cars (in thousands) traveling between different areas. For example, 20 thousand cars are traveling from Nacka to the city center, while only 5 thousand cars are traveling the opposite direction. In totat 745 thousand cars will be part of the traffic problem.

from \to	City 1	Nacka 2	Årsta 3	Älvsjö 4	Liljeholmen 5	Bromma 6	Solna 7	Djursholm 8	Lidingö 9	Farsta 10	Skärholmen 11	Kista 12	Haninge 13	Järfälla 14	Arlanda 15
City 1	-	5	0	5	0	5	10	5	0	0	0	0	5	0	10
Nacka 2	20	-	5	5	0	10	10	0	0	10	5	5	0	0	5
Årsta 3	10	5	-	15	10	5	5	5	0	5	0	0	0	0	0
Älv. 4	40	5	10	-	5	5	0	0	0	5	5	5	0	5	0
Liljeh.5	10	0	5	5	-	0	0	0	0	0	0	5	0	0	0
Brom. 6	30	5	0	5	0	-	5	10	5	0	0	5	0	5	5
Solna 7	40	5	5	0	5	10	-	10	0	5	0	10	0	0	5
Djursh. 8	30	0	0	0	5	5	10	-	5	0	0	5	0	0	0
Lid. 9	15	0	0	0	0	5	5	5	-	0	0	0	0	0	5
Farsta 10	15	5	5	5	5	0	5	5	0	-	0	0	0	0	0
Skärh. 11	5	5	0	10	5	5	5	0	0	0	-	0	0	0	0
Kista 12	10	0	0	0	5	5	5	0	0	0	0	-	0	0	5
Han. 13	10	5	5	0	0	0	0	5	0	5	0	0	-	0	0
Järf. 14	10	0	5	5	0	5	5	0	0	0	0	5	0	-	0
Arl. 15	10	0	0	0	0	0	0	0	0	0	0	5	0	0	-

Table 2: Traveling times in minutes assuming zero traffic. The table is symmetric meaning the assumption is made that it takes equal time to travel in both directions along any edge.

from \to	City 1	Nacka 2	Årsta 3	Älvsjö 4	Liljeholmen 5	Bromma 6	Solna 7	Djursholm 8	Lidingö 9	Farsta 10	Skärholmen 11	Kista 12	Haninge 13	Järfälla 14	Arlanda 15
City 1	-														
Nacka 2	20	-													
Årsta 3	10	15	-												
Älv. 4	-	-	10	-											
Liljeh. 5	10	-	10	10	-										
Brom. 6	15	-	-	-	15	-									
Solna 7	15	-	-	-	-	15	-								
Djursh. 8	20	-	-	-	-	-	15	-							
Lid. 9	15	-	-	-	-	-	-	-	-						
Farsta 10	-	-	15	20	-	-	-	-	-	-					
Skärh. 11	-	-	-	15	15	-	-	-	-	-	-				
Kista 12	-	-	-	-	-	-	15	-	-	-	-	-			
Han. 13	-	-	-	-	-	-	-	-	-	15	-	-	-		
Järf. 14	-	-	-	-	-	25	-	-	-	-	-	25	-	-	
Arl. 15	-	-	-	-	-	-	-	-	-	-	-	20	-	-	-

Table 3: The maximum capacity in thousands of cars on the edges. Again the assumption is made that the capacity is equal in both directions. The existence of an edge is indicated by a positive value in this table.

from \to	City 1	Nacka 2	Årsta 3	Älvsjö 4	Liljeholmen 5	Bromma 6	Solna 7	Djursholm 8	Lidingö 9	Farsta 10	Skärholmen 11	Kista 12	Haninge 13	Järfälla 14	Arlanda 15
City 1	-														
Nacka 2	60	-													
Årsta 3	60	80	-												
Älv. 4	-	-	60	-											
Liljeh. 5	60	-	90	60	-										
Brom. 6	50	-	-	-	80	-									
Solna 7	50	-	-	-	-	60	-								
Djursh. 8	70	-	-	-	-	-	50	-							
Lid. 9	80	-	-	-	-	-	-	-	-						
Farsta 10	-	-	80	40	-	-	-	-	-	-					
Skärh. 11	-	-	-	60	120	-	-	-	-	-	-				
Kista 12	-	-	-	-	-	-	90	-	-	-	-	-			
Han. 13	-	-	-	-	-	-	-	-	-	70	-	-	-		
Järf. 14	-	-	-	-	-	80	-	-	-	-	-	60	-	-	
Arl. 15	-	-	-	-	-	-	-	-	-	-	-	80	-	-	-

Table 4: The sensitivity to traffic jam, a_{ij}, on edge e_{ij}. The table is symmetric as above and a "-" indicates there is no edge between the nodes in question.

from \to	City 1	Nacka 2	Årsta 3	Älvsjö 4	Liljeholmen 5	Bromma 6	Solna 7	Djursholm 8	Lidingö 9	Farsta 10	Skärholmen 11	Kista 12	Haninge 13	Järfälla 14	Arlanda 15
City 1	-														
Nacka 2	2	-													
Årsta 3	2	2	-												
Älv. 4	-	-	2	-											
Liljeh. 5	2	-	1	2	-										
Brom. 6	2	-	-	-	1	-									
Solna 7	2	-	-	-	-	1	-								
Djursh. 8	2	-	-	-	-	-	2	-							
Lid. 9	2	-	-	-	-	-	-	-	-						
Farsta 10	-	-	1	2	-	-	-	-	-	-					
Skärh. 11	-	-	-	2	1	-	-	-	-	-	-				
Kista 12	-	-	-	-	-	-	1	-	-	-	-	-			
Han. 13	-	-	-	-	-	-	-	-	-	1	-	-	-		
Järf. 14	-	-	-	-	-	2	-	-	-	-	-	2	-	-	
Arl. 15	-	-	-	-	-	-	-	-	-	-	-	1	-	-	-

Each road was estimated to have a maximum capacity of cars which is shown in Table 3. This is based on the kind of major roads connecting the regions. For example, a six lane highway has higher max capacity than a two lane road. Finally the sensitivity to traffic jam was estimated, given by Table 4. The sensitivity was deemed higher on smaller roads and vice versa. The time, T, taken to drive on one edge of the network is given by the following model taking into account the congestion,

$$T = t + \frac{ax}{u - x}. \tag{1}$$

Here, t is the time taken during zero traffic conditions, u is the maximum capacity, a is the sensitivity to traffic jam and x is the amount of cars traveling on the particular road.

2 Mathematical formulation

The mathematical model defines constraints and the objective function to formulate the problem as a non linear programming problem.

2.1 Nomenclature

Sets

Let i denote the nodes in the network, that is the suburbs and city center,

$$i = 1, 2, \ldots, 15.$$

Also define three similar indices which will be used in the sequel, j, k, l. For the corresponding suburb to each index, please refer to Table 1. The edges in the network in Figure 1 will be referred to as e_{ij}.

Decision variables

Denote the total number of cars traveling from node k to node l via edge e_{ij} by

$$x_{ijkl} \qquad [\# \text{ cars} \times 10^3].$$

Variables for the total number of cars on edge e_{ij} irrespective of departure or destination will be denoted by

$$\xi_{ij} \qquad [\# \text{ cars} \times 10^3].$$

Constants

The total number of cars traveling from node k to node l is given by Table 1 and is denoted by

$$y_{kl} \qquad [\# \text{ cars} \times 10^3],$$

and is the entry on row k, column l in the table. Similarly, the traveling time, max capacity and sensitivity for the edges are given by Tables 2, 3 and 4, denoted by

$$t_{ij} \ \text{[minutes]} \ , u_{ij} \ [\# \text{ cars} \times 10^3] \ \text{ and } \ a_{ij}$$

respectively. Note that a_{ij} are dimensionless constants.

2.2 Constraints

The problem at hand is not surprisingly a version of the network flow problem. Recall that for such a problem, flow balance at node i is given by

$$\sum_{j:e_{ij}\in\mathcal{E}} x_{ij} - \sum_{k:e_{ki}\in\mathcal{E}} x_{ki} - s_i = 0, \quad \forall i, \tag{2}$$

where s_i represents a source or sink if present at node i. Here $\mathcal{E}$ is the set of edges in the network. The natural interpretation is that the flow into the node minus the flow out of the node equals the amount entering or leaving the network at node i, given by a positive or negative s_i respectively. Now, these flow balances presumes there is just one kind of flow in the network.

The traffic problem requires one set of such constraints for each node pair for which there are travellers wanting to depart from and arrive at, hereafter referred to as departure-destination pairs. The flow balance equations in node i for the traffic problem becomes

$$\sum_j (x_{ijkl} - x_{jikl}) = \begin{cases} y_{kl}, & \text{if } k = i, \\ -y_{kl}, & \text{if } l = i, \\ 0, & \text{else}, \end{cases} \qquad \forall i, k, l. \tag{3}$$

For each departure-destination pair k, l this gives a set of flow balance equations of the form (2). One may think of each pair k, l as one particular type of flow for which the flow balance has to be fulfilled. Now this equation says nothing about the set of edges in the network. To assure now flow is allowed where there is no edges a capacity constraint is used according to, where u_{ij} is the capacity on edge e_{ij},

$$\xi_{ij} \le u_{ij}, \quad \forall i, j, \tag{4}$$

and ξ_{ij} is the total flow on the edge for all departure-destination pairs k, l,

$$\xi_{ij} = \sum_{kl} x_{ijkl}, \quad \forall i, j. \tag{5}$$

Finally, since the number of cars traveling an edge in the network is a positive number, a non negativity constraint is used,

$$x_{ijkl} \ge 0, \quad \forall i, j, k, l. \tag{6}$$

Note that in equation (3), if $k = i$ but there are no cars wanting to travel from node k to node l the flow balance is preserved for this particular departure-destination pair since in that case $y_{kl} = 0$ by construction. Similarly for l and if $k = l$. The flow balance equation may be written more compactly using Dirac-delta notation as

$$\sum_j (x_{ijkl} - x_{jikl}) = y_{kl}(\delta_{ki} - \delta_{li}), \quad \forall i, k, l$$

2.3 Objective function

The objective is to minimize the total time spent by the citizens commuting. Each car that belongs ξ_{ij} has to spend the time it takes to travel e_{ij} so the grand total time is given by

$$\sum_{ij} \xi_{ij} T_{ij}.$$

where T_{ij} is the actual time taken to travel the edge e_{ij}. If not taking into account the congestion given by equation (1), simply set

$$T_{ij} = t_{ij}. \tag{7}$$

This gives a linear objective function. The non linear model for the actual travel time is given by

$$T_{ij} = t_{ij} + \frac{a_{ij}\xi_{ij}}{u_{ij} - \xi_{ij}}.$$

The first term is the time it takes to travel the edge in zero traffic conditions. The time increases without bound as the traffic approaches the maximum capacity u_{ij}. The constant a_{ij} may be seen as a representation as to how sensitive the particular edge is to traffic jam.

2.4 Full model formulation

At this point, the mathematical formulation is summarized into the following nonlinear optimization problem

$$\boxed{\begin{array}{lll} & \min & \sum_{ij} \xi_{ij}(t_{ij} + \frac{a_{ij}\xi_{ij}}{u_{ij}-\xi_{ij}}) & \\ & \text{s.t.} & \xi_{ij} = \sum_{kl} x_{ijkl}, & \forall i,j, \\ (\text{NLP}) & & \xi_{ij} \le u_{ij}, & \forall i,j, \\ & & \sum_j (x_{ijkl} - x_{jikl}) = y_{kl}(\delta_{ki} - \delta_{li}), & \forall i,k,l, \\ & & x_{ijkl} \ge 0, & \forall i,j,k,l, \\ & & i,j,k,l \in \{1,2,\dots,15\} & \end{array}}$$

If disregarding the non linear model for the increased travel time due to congestion, the objective function is changed to

$$\Sigma_{ij}\xi_{ij}t_{ij}$$

to obtain a linear programming problem (since the constraints are all linear), hereafter referred to as (LP). Note that in this problem the maximum capacities on the roads will still be considered while in the non linear problem, the minimizer should keep the solution far from these capacities implicitly.

2.5 On convexity

The feasible set is given by linear equalities and inequalities in the variables x and ξ implying the feasible set is convex. It follows that the full problem (P) is convex if the objective function is convex. This is the case if the Hessian matrix of the objective function is positive semidefinite. An analysis of the second derivatives of the traveling times on the edges of the network gives

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\Big(x\big(t+\frac{ax}{u-x}\big)\Big) &= \frac{\mathrm{d}}{\mathrm{d}x}\big(xt+\frac{ax^2}{u-x}\big) = t + a\frac{\mathrm{d}}{\mathrm{d}x}\big(\frac{x^2}{u-x}\big) \\
&= t + a\frac{2x(u-x)-(-1)x^2}{(u-x)^2} = t + a\frac{2xu-2x^2+x^2}{(u-x)^2} \\
&= t + a\frac{2xu-x^2}{(u-x)^2} \Rightarrow \\
\frac{\mathrm{d}^2}{\mathrm{d}x^2}\Big(x\big(t+\frac{ax}{u-x}\big)\Big) &= \frac{\mathrm{d}}{\mathrm{d}x}\big(t+a\frac{2xu-x^2}{(u-x)^2}\big) = 0 + a\frac{\mathrm{d}}{\mathrm{d}x}\big(\frac{2xu-x^2}{(u-x)^2}\big) = \\
&= a\frac{(2u-2x)(u-x)^2-2(-1)(2xu-x^2)(u-x)}{(u-x)^4} \\
&= a\frac{2u^3-2u^2x}{(u-x)^4} = \frac{2au^2}{(u-x)^3} > 0, \quad x<u.
\end{aligned}$$

Since each term in the objective function contains only one instance of $\xi_{ij}T_{ij} = (t_{ij}\xi_{ij} + (a_{ij}\xi_{ij}^2/(u_{ij}-\xi_{ij}))$ and this term contains only one variable, it follows that the Hessian matrix is diagonal. A diagonal matrix with non negative elements is clearly positive semidefinite so the promlem is convex. Hence any local minima found will also be a global minima.

2.6 Recommendations to increase capacity

When the optimal solution is found, the natural question is where increased capacity is needed. Since the optimal solution will be an interior point of the problem (as the objective function tends to infinity when approaching the capacity constraints), marginal analysis will not answer the question. Recall that for a function of several variables $f(x_1,\ldots,x_n)$, the change of moving in the direction given by the vector $v=e_i$ from a point $\hat{x}$ is given by the directional derivative

$$f'_v = \frac{\nabla f(\hat{x})\cdot v}{v\cdot v} = (\nabla f(\hat{x}))_i.$$

The question of where to increase the capacity can thus be answered by viewing the objective function at the optimal solution as a function of the parameters u_{ij} and investigating where the gradient with respect to u_{ij} is smallest (increase in capacity should mean lower traveling time). The components of this gradient is given by

$$\begin{aligned}
\big[\nabla_u f(\hat{\xi},u)\big]_{ij} &= \frac{\partial}{\partial u_{ij}}\Big(\sum_{kl}\hat{\xi}_{kl}(t_{kl}+\frac{a_{kl}\hat{\xi}_{kl}}{u_{kl}-\hat{\xi}_{kl}})\Big) \\
&= \frac{\partial}{\partial u_{ij}}\frac{a_{ij}\hat{\xi}_{ij}^2}{u_{ij}-\hat{\xi}_{ij}} = -\frac{a_{ij}\hat{\xi}_{ij}^2}{(u_{ij}-\hat{\xi}_{ij})^2},
\end{aligned}$$

where $\hat{\xi}$ means this expression is evaluated at the optimal solution to (NLP). The value of this expression is to be interpreted as the change of the objective function value when increasing the capacity of the edge by 1 i.e. 1000 cars. Note however, due to the high non linearity that this will only be true at the optimal solution $\hat{\xi}$.

3 Result

The models stated above is implemented using the GAMS modeling language and solved using suitable solvers. First, the result of the linear model (LP) is given.

3.1 Linear model without traffic congestion

With z^*_{LP} denoting the objective value for the optimal solution found

$$z^*_{\text{LP}} = 19650 \times 10^3 \quad \text{[minutes]}.$$

With a total of 745 thousand cars this means the average commuter would spend some 26 minutes on the roads. In Table 5 the number of cars on each edge of the network is given for the optimal solution to (LP). Several edges in the network is on max capacity, which is not surprising considering that this is a linear programming problem.

Table 5: The total amount of cars (in thousands) traveling the roads of the model as given by the optimal solution to (LP), i.e. without considering congestion. An entry "-" indicates there is no edge between the nodes. A graphical description of the result is given in Figure 2.

from \to	City 1	Nacka 2	Årsta 3	Älvsjö 4	Liljeholmen 5	Bromma 6	Solna 7	Djursholm 8	Lidingö 9	Farsta 10	Skärholmen 11	Kista 12	Haninge 13	Järfälla 14	Arlanda 15
City 1	-	15	10	-	5	30	50	25	10	-	-	-	-	-	-
Nacka 2	50	-	25	-	-	-	-	-	-	-	-	-	-	-	-
Årsta 3	60	25	-	25	25	-	-	-	-	25	-	-	-	-	-
Älv. 4	-	-	20	-	60	-	-	-	-	5	5	-	-	-	-
Liljeh.5	55	-	20	15	-	35	-	-	-	-	5	-	-	-	-
Brom. 6	50	-	-	-	35	-	50	-	-	-	-	-	-	10	-
Solna 7	50	-	-	-	-	40	-	35	-	-	-	65	-	-	-
Djursh. 8	55	-	-	-	-	-	20	-	-	-	-	-	-	-	-
Lid. 9	35	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Farsta 10	-	-	65	5	-	-	-	-	-	-	-	-	5	-	-
Skärh. 11	-	-	-	15	20	-	-	-	-	-	-	-	-	-	-
Kista 12	-	-	-	-	-	-	40	-	-	-	-	-	-	0	35
Han. 13	-	-	-	-	-	-	-	-	-	30	-	-	-	-	-
Järf. 14	-	-	-	-	-	25	-	-	-	-	-	10	-	-	-
Arl. 15	-	-	-	-	-	-	-	-	-	-	-	15	-	-	-

3.2 Non linear model with traffic congestion

The non linear problem (NLP) has optimal objective value

$$z^*_{\text{NLP}} = 25592 \times 10^3 \quad \text{[minutes]}.$$

As argued above, this is also the global optimal value. Average time is 34 minutes per car for this model. The total number of cars on each edge is given in Table 6 and a graphical description of the result is given in Figure 3. Comparing with the solution to (LP), it is interesting to note that more traffic is directed around the city centre. As predicted, the capacity constraints are not active at the optimal solution.

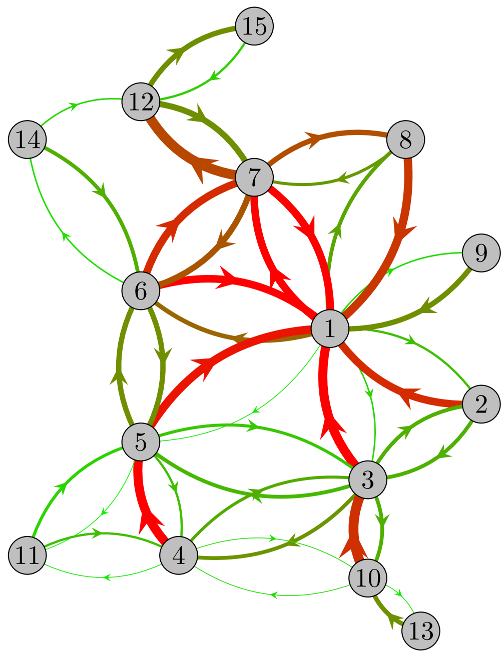

Figure 2: The result of the linear model (LP). The thickness of the directed edges is directly proportional to the flow. The color is an indication of the relative load on the edge where green means the load is low and full red means $\xi_{ij}/u_{ij} = 1$.

Table 6: The result of (NLP). Also for this solution, a graphical description is given in Figure 3.

from \to	City 1	Nacka 2	Årsta 3	Älvsjö 4	Liljeholmen 5	Bromma 6	Solna 7	Djursholm 8	Lidingö 9	Farsta 10	Skärholmen 11	Kista 12	Haninge 13	Järfälla 14	Arlanda 15
City 1	-	5	5	-	8.7	10	38	31.6	10	-	-	-	-	-	-
Nacka 2	45.3	-	40.4	-	-	-	-	-	-	-	-	-	-	-	-
Årsta 3	49.6	45.6	-	24	51.7	-	-	-	-	25	-	-	-	-	-
Älv. 4	-	-	31.9	-	42.5	-	-	-	-	5	10.9	-	-	-	-
Liljeh.5	48.3	-	35	20	-	60.4	-	-	-	-	1	-	-	-	-
Brom. 6	41.6	-	-	-	49.9	-	46.2	-	-	-	-	-	-	19.2	-
Solna 7	42.8	-	-	-	-	46.3	-	37.2	-	-	-	55.8	-	-	-
Djursh. 8	55.8	-	-	-	-	-	27.9	-	-	-	-	-	-	-	-
Lid. 9	35	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Farsta 10	-	-	63.7	6.4	-	-	-	-	-	-	-	-	5	-	-
Skärh. 11	-	-	-	10	26.9	-	-	-	-	-	-	-	-	-	-
Kista 12	-	-	-	-	-	-	40	-	-	-	-	-	-	0	35
Han. 13	-	-	-	-	-	-	-	-	-	30	-	-	-	-	-
Järf. 14	-	-	-	-	-	25	-	-	-	-	-	19.2	-	-	-
Arl. 15	-	-	-	-	-	-	-	-	-	-	-	15	-	-	-

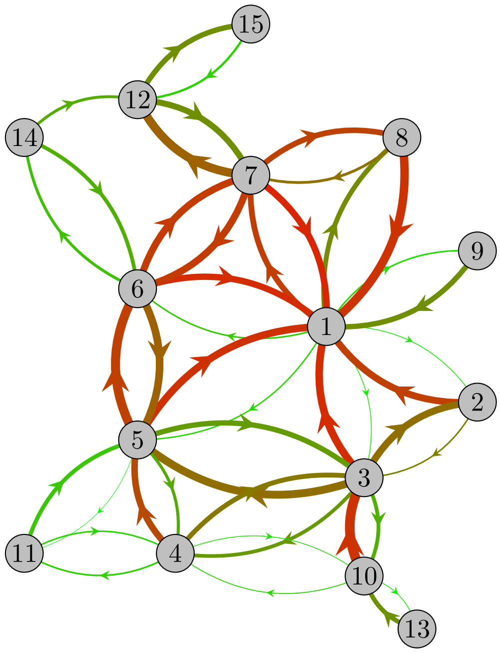

Figure 3: The result of the non linear model (NLP). Note how more traffic is directed around the city center in this solution as compared to (LP). Clearly the flow on e_{35} and e_{56} has increased when taking non linear congestion into account.

3.3 Capacity increase

The most negative values of the gradient $\nabla_u f(\hat{\xi}, u)$ is given by Table 7. Clearly it is desired to increase the capacity on roads to/from the city centre. A more naïve approache than taking the gradient w.r.t. u would be to simply compare the traffic with the maximum capacity. As can be seen in Table 7 this would yield the same result. This approach, however, would not enable the interpretation of reduced time per increased capacity.

Table 7: The most negative components of the gradient. The first edge not terminating in the city centre on the list is e_{72} on eight place.

edge	$[\nabla_u f(\hat{\xi}, u)]_{ij}$	ξ_{ij}/u_{ij}
e_{71}	-71	0.86
e_{61}	-47	0.83
e_{31}	-46	0.83
e_{51}	-34	0.81

By computing optimal solutions to the problem with changed capacities, tha accuracy of this prediction may be analyzed. Increasing the capacity with 10×10^3 cars on edge e_{71} results in a total travel time reduction of 538×10^3. minutes. An increased capacity by 10^3 cars gives a reduction of 69×10^3 minutes and an increase of 0.1×10^3 cars results in a time reduction of 7×10^3 minutes. From table 7 one can expect that if the capacity on edge e_{71} is increased by 1000 cars the objective function value should be decreased with 71×10^3 minutes. Clearly this prediction is only accurate for small changes in the capacity of the edges. This is because the gradient calculated is only valid as a local analysis.

4 Discussion

The model of the Stockholm area used in this work is very simplified. Even so, the problem becomes very large, for example, the number of x_{ijkl} variables are over 50000 (15^4). This have to kept in mind when designing the model used.

To facilitate a quick solution, in the implementation many of these variables were set to identically zero. This solution makes marginal analysis of roads not yet built impossible. If this information is of interest, the implementation needs a slight modification.

As a current example, the construction of *Förbifart Stockholm* could be analyzed using this technique. In that case, some traffic from south of Stockholm to north and vice versa would probably be needed to make the analysis realistic. This could be implemented using a node an additional node south of Skärholmen (11) with heavy traffic to Arlanda (15). Similarly, one could analyze roads connecting the east suburbs (Nacka, Lidingö, Djursholm).

An additional modification to the model, to avoid division by zero, was made. The capacity constraint $\xi_{ij} \leq u_{ij}$ was modified to $\xi_{ij} \leq 0.999u_{ij}$. It was deemed highly unlikely to end up this close to the maximum capacities and the performance increase was substantial. Indeed, at the optimal solution, none of the capacity constraints are close to being active.

At first, it may seem that the linear model initially considered would give the solution obtained by separate shortest path problems. This is not the case in the current implementation (LP) since the maximum capacities are considered as linear constraints. The optimal solution to (LP) would give an infinite time if plugged into the objective function in (NLP) since several roads are on their maximum capacity. If this was not the case, one could compare the solution arising if every commuter simply choose to drive the quickest way to its destination with the solution where commuters collectively agrees on a solution to minimize the sum of their travel time.

Another problem could be to minimize the maximum travel time among the commuters, or somehow penalize longer travel times over short, since it is quite unrealistic that the commuters would choose such a path that would minimize the total travel time for all commuters.

A final note on the data used in the problem. This data was mainly taken from experience. There is no claim that the data is accurate, this was neither the objective of the work, rather it serves to explore the methods used.

5 Conclusion

A model for the traffic flow in Stockholm was solved using non linear optimization tools. The problem grows quickly with the number of nodes used in the network. The solution behaves as expected when taking congestion into account.

SF2822 Applied Nonlinear Optimization, 2015/2016
Project assignment 2
Self assessment form

Please fill out this self assessment form and submit with your report.

Name:

Moa Englund

Group number:

2A1

My contribution to the work:

Problem formulation and solving the problem,
GAMS-implementation and
Writing on the report.

I have contributed to the advanced exercises (yes/no):

No

Amount of my time spent on this project assignment (hours):

15

....
(Signature)

SF2822 Applied Nonlinear Optimization, 2015/2016
Project assignment 2
Self assessment form

Please fill out this self assessment form and submit with your report.

Name:

Anna Lundemo

Group number:

2A1

My contribution to the work:

Working with the formulation
of the problems
Discussing model improvements
and capacity increase

I have contributed to the advanced exercises (yes/no):

Yes

Amount of my time spent on this project assignment (hours):

11

. . . .
(Signature)

SF2822 Applied Nonlinear Optimization, 2015/2016
Project assignment 2
Self assessment form

Please fill out this self assessment form and submit with your report.

Name:

Mikael Persson

Group number:

2A1

My contribution to the work:

Problem formulation.
Mathematical model.
GAMS-implemention.
Report writing.

I have contributed to the advanced exercises (yes/no):

Yes

Amount of my time spent on this project assignment (hours):

30

....
(Signature)