SF2812

Project assignment 2A4

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1 Background

Varme is a power company with three producing units. The company is about to decide the production plan for its three units in the coming 24 hours. The task is to minimize the cost of production while satisfying the demand from consumers. Each day, the demand varies over five predefined time periods according to Table 1.

Table 1: The demand for each time period.

Period	Expected demand [MW]
00-05	50
05-10	60
10-15	80
15-20	70
20-24	60

For each of the producing units there are restrictions of producing capacity and associated costs given by Table 2. The initial cost reflects the cost of starting the unit after having been switched off for one period (warm) or more than one period (cold), where the initial cost increases by 50%. The production level must be between the minimum level and the maximum level for each unit in use. For technical reasons the units cannot be used for more than three consecutive periods, and the production plan must be cyclic i.e. the plan used must be feasible if used again the next day.

Table 2: Costs and production levels for each unit.

Unit	Initial cost (warm) [kkr]	Initial cost (cold) [kkr]	Running cost [kkr / MWh]	Minimum level [MW]	Maximum level [MW]
1	10	15	2.5	10	50
2	13	19.5	2.5	12	45
3	16	24	2.4	15	55

This report will consider the following cases:

1. The power demand is regarded as deterministic with the values according to Table 1.
2. The demand is assumed to be normally distributed with mean according to Table 1 and standard deviation 5 MW. The demand is constant and known at the beginning of each period. Additional power may be purchased for a price of 10 kkr/MWh form other suppliers.

2 Mathematical formulation

The mathematical modeling consist of defining the appropriate constraint equations and the objective function to be minimized.

2.1 Case 1 - A deterministic problem

For ease of notation, define sets for the periods and the units as

$$p = \{1, 2, 3, 4, 5\}$$
$$u = \{1, 2, 3\}.$$

As frequent reference to the values in Tables 1 and 2 will be made, the variables and parameters to be used are defined in Table 3.

Table 3: The notation used in this report for the variables and parameters in the problem. Refer to Tables 1 and 2 for the actual values of the parameters. Above the dashed line are variables and below are parameters.

Notation	Meaning
x_{up}	Production level unit u during period p
w_{up}	Indicator variable for unit u in use period p
warm_{up}	Variable indicating warm start
cold_{up}	Variable indicating cold start
d_p	Consumer demand period p
h_p	Number of hours period p
minL_u	Minimum production level unit u when in use
maxL_u	Maximum production level unit u
costR_u	Running cost for unit u
costI_u	Startup cost for unit u

2.1.1 Constraints

Since the units are either off, or having its production level between the minimum and maximum level, variables indicating the unit is in use are defined as

$$w_{up} = \begin{cases} 1, & \text{if unit } u \text{ is on during period } p, \\ 0, & \text{otherwise.} \end{cases}$$

With the production level of unit u during period p is denoted by x_{up}, the restriction on production level for each unit can now be stated as

$$w_{up}\text{minL}_u \leq x_{up} \leq w_{up}\text{maxL}_u, \quad \forall u, p. \tag{1}$$

Note that this constraint also requires x_{up} to be non-negative (the unit cannot produce negative energy). The requirement of the power production satisfying the demand d_p with equality, in every period p, is stated as

$$\sum_u x_{up} = d_p, \qquad \forall p. \tag{2}$$

Next the requirement that any unit may run for a maximum of three consecutive periods is considered. Suppose that this requirement is violated, then clearly at least four of the $w_{up} = 1$ for the relevant u, i.e. $\sum_p w_{up} \geq 4$. On the other hand, if $\sum_p w_{up} \leq 3$, then the worst case scenario is a unit running precisely three consecutive periods. Hence this requirement can be formulated as

$$\sum_p w_{up} \leq 3, \qquad \forall u. \tag{3}$$

These are all the constraints needed to guarantee a feasible production plan. However, to be able to reflect the cost of the cold starts or warm starts, additional inequality constraints will be needed for the two variables cold_{up} and warm_{up}. The goal is to have $\text{warm}_{up} = 1$ precisely when $w_{up} - w_{up-1} = 1$, and for all other cases $\text{warm}_{up} = 0$. Note in the expression $w_{up} - w_{up-1}$, the index $p-1$ is to be understood in the sense of modular arithmetic with modulus 5 (e.g. think of a clock which has modulus 12: $23 = 11$). This will satisfy the requirement that the production plan has to be cyclic as well. Now, the difference above will satisfy

$$w_{up} - w_{up-1} \in \{-1, 0, 1\},$$

hence the goal is achieved if imposing on the (continuos) variable warm_{up} the following constraints:

$$\begin{aligned} \text{warm}_{up} &\geq w_{up} - w_{up-1}, \\ \text{warm}_{up} &\geq 0, \end{aligned} \tag{4}$$

and then letting the variable have a positive associated cost to be minimized. The minimizer will force the value of warm_{up} down to 0 in all cases but when a warm start is present, where it will force the variable to 1. Similarly

$$w_{up} - w_{up-1} - w_{up-2} \in \{-2, -1, 0, 1\}.$$

The constraints to achieve the goal for cold_{up} are then given by

$$\begin{aligned} \text{cold}_{up} &\geq w_{up} - w_{up-1} - w_{up-2}, \\ \text{cold}_{up} &\geq 0. \end{aligned} \tag{5}$$

At this point all the required constraints have been stated.

2.1.2 Objective function

The cost of running unit u during period p at the level of x_{up} is given by

$$x_{up} h_p \text{costR}_u \qquad \text{[kkr]}.$$

In addition to this are the costs of the warm starts and cold starts. If unit u makes a warm start at the beginning of period p the cost is

$$\text{costI}_u \qquad \text{[kkr]},$$

and if it is also a cold start the cost will be 50 % higher that is

$$(1 + \frac{1}{2})\text{costI}_u \qquad \text{[kkr]}.$$

These expressions are combined to form the total cost of the production plan:

$$z = \sum_{u,p} x_{up} h_p \text{costR}_u + (\text{cold}_{up} + \frac{1}{2}\text{warm}_{up})\text{costI}_u$$

2.1.3 Full formulation - Case 1

Summarizing the above constraints and the objective function gives the following problem:

$$
\text{(MIP1)} \quad
\begin{array}{lll}
\text{maximize} & z_1 = \sum\limits_{u,p} x_{up} h_p \text{costR}_u + (\text{cold}_{up} + \frac{1}{2}\text{warm}_{up})\text{costI}_u & \\
\text{subject to} & w_{up}\text{minL}_u \leq x_{up} \leq w_{up}\text{maxL}_u, & \forall u, p, \\
& \sum_u x_{up} = d_p, & \forall p, \\
& \sum_p w_{up} \leq 3, & \forall u \\
& \text{warm}_{up} \geq w_{up} - w_{up-1}, & \forall u, p, \\
& \text{cold}_{up} \geq w_{up} - w_{up-1} - w_{up-2}, & \forall u, p, \\
& x_{up} \geq 0, \text{warm}_{up} \geq 0, \text{cold}_{up} \geq 0 & \forall u, p, \\
& w_{up} \in \{0, 1\}, & \forall u, p.
\end{array}
$$

2.2 Case 2 - A model with stochastics

Most of the constraints used for case 1 above will be the same when considering the stochasticity. However, the discretization introduced below will require an additional set for scenarios, to be considered in the constraint equations. The compensation will also require an adjustment of the objective function.

2.2.1 Discretization

The demand during each period is assumed to be normally distributed with standard deviation $\sigma = 5$ [MW] and mean d_p (the expected demand). If $Z_1 \sim N(d_p, \sigma^2)$ then

$$Z_1 \sim d_p + Z_2, \qquad Z_2 \sim N(0, \sigma^2),$$

so the demand for each period may be modelled with a single random variable $Z_2 \sim N(0, \sigma^2)$. The random variable Z_2 is discretized to $S = 7$ scenarios with outcomes $\{-15, -10, -5, 0, 5, 10, 15\}$. Reasonable requirements are that the expected value and variance of this discrete distribution are the same as for the continous distribution and that it at least somewhat resembles a normal distribution. This imply outcomes far from the mean should be less likely than outcomes close to the mean. With the set of scenarios

$$s = \{1, 2, 3, 4, 5, 6, 7\},$$

the following distribution with discrete probabilities P_s and outcomes D_s satisfies the requirements;

$$
\begin{array}{lllllll}
D_1 = -15 & D_2 = -10 & D_3 = -5 & D_4 = 0 & D_5 = 5 & D_6 = 10 & D_7 = 15 \\
P_1 = 0.01 & P_2 = 0.06 & P_3 = 0.17 & P_4 = 0.52 & P_5 = 0.17 & P_6 = 0.06 & P_7 = 001.
\end{array}
$$

See Figure 1 which depicts the discretized and continuous distribution. For the discrete distribution the following holds which shows that the requirements stated above is fullfilled;

$$\sum_s P_s = 1, \quad \text{E}[D] = \sum_s P_s D_s = 0, \quad \text{Var}[D] = \sum_s P_s D_s^2 = 25 = 5^2.$$

The demand for each period and each scenario is now given by

$$d_{ps} = d_p + D_s, \qquad \forall p, s,$$

where d_p is the expected demand and D_s is the difference from this expected value, i.e. the outcome of the discrete random variable. Note that this assumes the demand in each period is correlated, meaning if the demand is increasing in the first period, it is also increasing in the other periods. When assuming the demand in the different periods are uncorrelated, the number of joint scenarios grow exponentially: For each outcome in period 1 (there are 7 of those), there are 7 outcomes in period 2, and for each of those 49 possibilities there are 7 different outcomes in period 3 and so on. In total there would be $7^5 = 16807$ joint scenarios. The model introduced below assume correlated demands.

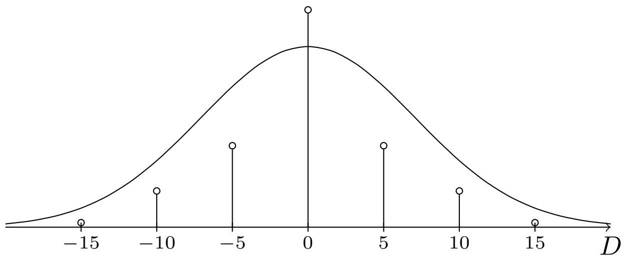

Figure 1: The probability density function of the assumed normal distribution $N(0, 5^2)$ and the probabilities for the discrete distribution. Note that the discrete probabilities are scaled by 1/5.

2.2.2 Constraints

The units to be used in each period have to be fixed before the outcome is known (due to technical reasons or similar), therefore w_{up} becomes stage-1 variables. However, once the outcome of the demand is known, at the beginning of each period, the production may be adjusted. For this reason denote the variables for the production level of unit u, during period p and for scenario s by

$$x_{ups}.$$

If adjusting the production level is not sufficient for any of the periods, more power may be purchased from other producers at the price 10 kkr/MWh. Denote this compensation by

$$y_{ps} \qquad \text{[MW]}.$$

The constraint for maximum and minimum production levels, Equation (1), has to be satisfied for each scenario giving the following updated constraint

$$w_{up}\text{minL}_u \le x_{ups} \le w_{up}\text{maxL}_u, \quad \forall u, p, s. \tag{6}$$

Similarly, the constraint for satisfying demand, Equation (2), must be adjusted to cover all scenarios. With the possibility of compensation, the constraint is adjusted to

$$\sum_u x_{ups} + y_{ps} = d_{ps}, \qquad \forall p, s. \tag{7}$$

The other constraints used in the deterministic model only affect the stage-1 variables w and the variables for warm and cold starts, since they only depend on w. Therefore the constraints (3) through (5) remain unchanged. Finally, since the purchase of additional power cannot be negative, a constraint is set for non-negativity

$$y_{ps} \geq 0, \qquad \forall p, s.$$

2.2.3 Objective function

The objective function is the expected cost, therefore the cost for the variables x_{ups} must be adjusted by the probability for each scenario. In addition, the cost of the compensation y_{ps} must be added. The expression for the objective function becomes

$$\begin{aligned} z = &\sum_{u,p,s} P_s x_{ups} h_p \text{costR}_u + \sum_{u,p} (\text{cold}_{up} + \frac{1}{2}\text{warm}_{up})\text{costI}_u \\ &+ 10 \sum_{p,s} P_s y_{ps} h_p. \end{aligned}$$

The factor 10 multiplied by the last sum comes from the price of compensation. Also note that since the initial costs are independent of the scenarios, this sum only runs over u and p.

2.2.4 Full formulation - Case 2

The above constraints and the objective function gives the following problem:

$$\text{(MIP2)} \quad \begin{array}{lll} \text{maximize} & z_2 = \sum_{u,p,s} P_s x_{ups} h_p \text{costR}_u + \sum_{u,p} (\text{cold}_{up} + \frac{1}{2}\text{warm}_{up})\text{costI}_u & \\ & \quad +10 \sum_{p,s} P_s y_{ps} h_p & \\ \text{subject to} & w_{up}\text{minL}_u \leq x_{ups} \leq w_{up}\text{maxL}_u, & \forall u, p, s, \\ & y_{ps} + \sum_u x_{ups} = d_{ps}, & \forall p, s, \\ & \sum_p w_{up} \leq 3, & \forall u \\ & \text{warm}_{up} \geq w_{up} - w_{up-1}, & \forall u, p, \\ & \text{cold}_{up} \geq w_{up} - w_{up-1} - w_{up-2}, & \forall u, p, \\ & x_{ups} \geq 0, & \forall u, p, s, \\ & y_{ps} \geq 0, & \forall p, s, \\ & \text{warm}_{up} \geq 0, \text{cold}_{up} \geq 0 & \forall u, p, \\ & w_{up} \in \{0, 1\}, & \forall u, p. \end{array}$$

2.2.5 EVPI and VSS

Suppose the company somehow would be able to foresee the outcome of the stochastic demand. Then the production plan may be optimized for this outcome. The so called "Wait-and-See" problem gives the expected cost if all variables (including stage-1) are set after the outcome is known. Denote this cost z^*_{WS}. EVPI is then defined by

$$\mathrm{EVPI} = z^*_2 - z^*_{\mathrm{WS}}.$$

where z^*_2 is the optimal objective value from (MIP2). EVPI reads "Expected Value of Perfect Information".

The VSS ("Value of Stochastic Solution") gives the expected improvement in expected cost from using the stochastic model as compared to using the primitive so called EV-model. In this model, the expected values of the stochastics are used to generate a production plan. The stage-1 variables from this solution are then fixed to give the EEV-model, "Expected value of EV-solution". Denote the optimal cost from this model by z^*_{EEV}. Then the VSS is given by

$$\mathrm{VSS} = z^*_{\mathrm{EEV}} - z^*_2.$$

3 Result

3.1 Case 1

The optimal objective value from the solution of (MIP1) above is

$$z^*_1 = 3828.5 \qquad [\text{kkr}].$$

The optimal production plan achieving this is given by Table 4.

Table 4: The optimal production plan and levels x_{up} for each unit u during period p in the deterministic problem.

	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$u=1$	50	10	-	-	48
$u=2$	-	-	25	15	12
$u=3$	-	50	55	55	-

As can be seen in Table 4, two units are used during each time period except the first time period, when only unit 1 is active. Also note that once a unit has started to produce, it remains active for another two time periods. Since the plan is periodic, all units will be switched off during two consecutive time periods. This means that the initial cost will increase by 50% for all units in the planning schedule according to Table 4.

3.2 Case 2

The minimized expected cost in the stochastic problem, i.e. z^*_2 from (MIP-2) is

$$z^*_2 = 3851.4 \qquad [\text{kkr}].$$

The optimal production plan (the stage-1 variables w_{up}) is given in Table 5. Note that only the stage-1 variables are presented as the actual production levels will be set once the outcome is known. Therefore, the actual production levels for each scenario are in some sense of low relevance.

Table 5: The optimal production plan for the stochastic model. The values indicated are those of w_{up}, the indicator for unit use.

	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$u=1$	-	1	-	1	1
$u=2$	-	-	1	1	1
$u=3$	1	1	1	-	-

The Expected Value of Perfect Information and Value of Stochastic Solution are given by

$$\text{EVPI} = 7.7 \qquad \text{[kkr]}$$
$$\text{VSS} = 37.7 \qquad \text{[kkr]}$$

4 Discussion

The two different cases are compared and as can be seen from the result tables 4 and 5, the production plans are not similar. In the deterministic case, all the units begin with a cold start and run for three consecutive time periods. However, in the stochastic case, unit 1 has only warm starts whereas unit 2 and 3 still have cold starts. During period 1, unit 1 generates power in the deterministic production plan but in the stochastic case unit 3 generates power instead. Varme has a change in the production plan at period 4 as well: in the deterministic case, unit 2 and 3 generate power but in the stochastic case, unit 1 and 2 generate power. This can be explained by using a unit with a higher maximum capacity during period 1 were only one unit is active. In other words, unit 3 can handle increased demand better compared to unit 1, due to its higher capacity. Due to the constraint that one unit only can run for three consecutive time periods, there will be at least one period were only one unit is producing. In this case it is period 1 when the demand is smallest. Furthermore, by switching production for unit 1 and unit 3, unit 1 will have two warm starts instead of only one cold start. The initial cost of two warm starts is more expensive than one cold start but this gives a lower expected cost since the expected additional purchase is lowered. Unit 1 is chosen to have two warm starts because it has the lowest initial cost of all the units.

With the opportunity to purchase additional power from other suppliers, there will not be any infeasible solutions. If the demand increases, Varme can simply purchase more power. If the demand is lower than the minimum level of any of the three units, power can be purchased as well to meet the demand.

4.1 Sensitivity analysis

It could be of interest to Varme to know the change in optimal objective value if adjusting the constraints, e.g. increasing the maximum level or lowering the minimum level for a producing unit. By inspecting the marginal values for constraints (1) one can see that the optimal objective function value is reduced by 0.5 kkr for each MW reduction on the minimum level for unit 1. Similarly the optimal objective function value is reduced by 1 kkr for each MW in increased capacity of unit 3. Hence these are of primary interest to adjust for Varme. Another interesting question is how the optimal objective value would change if for example letting the units run for 4 consecutive periods (e.g. by employing another shift of workers). This can not be answered by simply inspecting the marginal for (3) since w are binary. Rather a new optimization has to be run which shows that if all units may run for 4 consecutive periods the optimal objective value is reduced by 38.5 kkr. If only one (instead of all three as above) unit may run for 4 periods, or if one unit can only run for 2 periods (e.g. due to maintenance) deeper analysis is required.

4.2 Potential model improvements

Including the opportunity of purchasing additional power from other suppliers in the deterministic case could potentially provide another solution. For example, whenever unit 2 is switched on, it produces power close to its minimum level. Perhaps Varme would be better off purchasing power externally rather than using unit 2.

Assuming demand in different time periods are uncorrelated would substantially increase the model. For example, using seven outcomes would give over 16 000 joint scenarios and approximately 500 000 variables. Interestingly, solving the stochastic model with uncorrelated demand provides similar results as using correlated demand. This is possibly due to the fact that the worst case joint scenarios are covered by the correlated scenarios.

The discretization of the normal distribution is a rough approximation. One would prefer to discretize the continuous distribution into many more scenarios with corresponding probabilities. This would provide a more accurate result for the stochastic approach.

A final suggestion on how to refine the model would be to allow a unit to produce a surplus. If the demand is lower than the minimum level of any unit, Varme would simply buy additional power externally and let all the three units be switched off. To improve the model, Varme should instead be allowed to use one unit to produce at minimum level and get a power surplus to be sold on the market, possibly at a lower price.

5 Conclusion

This report investigated a strategy of production planning for the power plant company Varme. First, a deterministic case was considered followed by a stochastic case. The problems were solved with a mixed integer programming approach. The results are presented in Table 4 and 5.

Integer programming is an effective way of determining the optimal production plan in such situations where units are either off or above a minimum level. Using uncorrelated scenarios dramatically increases the number of variables and equations in the model.