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Laboratory Report

Evaluating the Spring Constant of Parallel and Series Spring Combinations Using Static and Dynamic Methods

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ABSTRACT

This experiment investigates the spring constant of two spring combinations: **parallel** and **series**, using both **static** and **dynamic methods**. The **parallel** combination was analysed using **Hooke's Law** via the **static method**, while the **series** combination was examined using the **dynamic method** through **simple harmonic motion**. The experimental spring constants were compared to the theoretical values derived from the standard combination formulas. The parallel combination yielded a spring constant close to the expected sum of the individual constants, while the series combination followed the reciprocal sum rule. The results confirmed that a **parallel** combination results in a **stiffer** system, while the **series** combination **weakens** the **overall stiffness**. Potential sources of error and improvements for precision are also discussed.

THEORETICAL BACKGROUND

OVERVIEW:

A **spring** is an elastic object that deforms under force and returns to its original shape when the force is removed. The **stiffness** of a spring is measured by its **spring constant** (**k**), which quantifies how much force is required to stretch or compress the spring by a unit length.

The relationship between force and displacement in a spring is described by **Hooke's Law**, which states:

$$F_S = -kx$$

where:

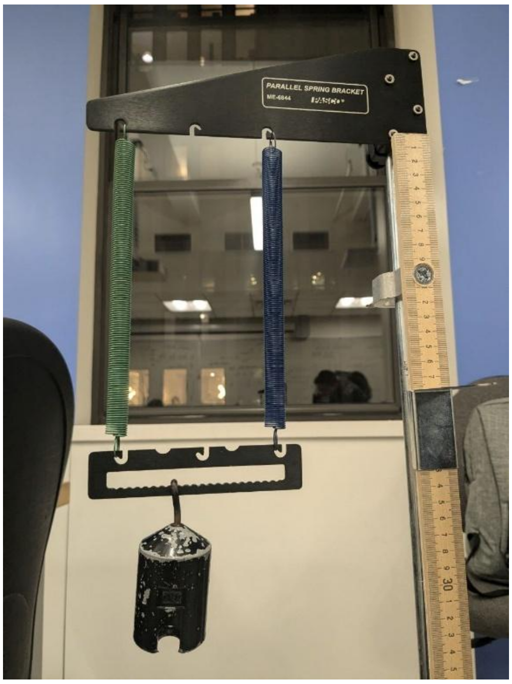
- **F_S** is the restoring force exerted by the spring (N),
- **k** is the spring constant (N/m),
- **x** is the displacement from the natural length of the spring (m).

The **negative sign** indicates that the restoring force is always directed **opposite** to the displacement. A **larger k** value means a **stiffer spring**, while a **smaller k** value means a more **flexible spring.**

In this experiment, we determined k for **two** different **spring combinations** using two different methods:

1. **Parallel Combination –** Static Method
2. **Series Combination –** Dynamic Method

PARALLEL COMBINATION – STATIC METHOD



In the parallel configuration, two springs are attached side by side, sharing the applied force equally (Figure 1). The equivalent spring constant for a parallel combination is given by:

$$k_{parallel} = k_1 + k_2$$

where k_1 and k_2 are the individual spring constants of the two springs in parallel.

FIGURE 1. *Experimental setup for the parallel spring combination. The green and blue springs are attached side by side to a parallel spring bracket, with a hooked mass suspended from a horizontal support bar.*

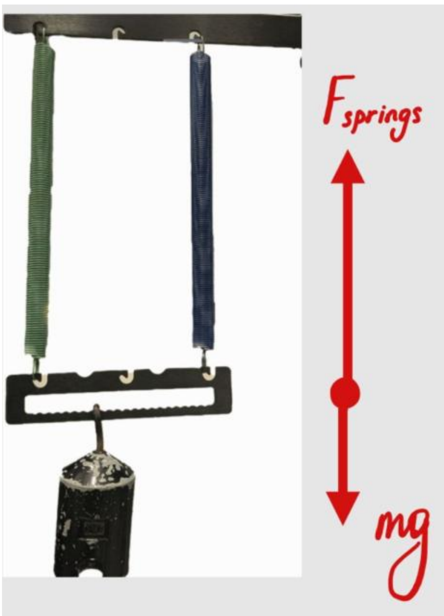
To experimentally determine $k_{parallel}$, we used the **static method** by measuring how much the springs **elongate under different weights**.

Since the weight of an object is the force due to gravity, we express it as:

$$F = mg$$

where:

- $\boldsymbol{m}$ is the mass of the hanging object (kg),
- $\boldsymbol{g} = 9.8\ m/s^2$ is the acceleration due to gravity.



At equilibrium, the downward gravitational force (mg) is balanced by the upward restoring force of the springs (Diagram 1), so:

$$mg = k_{parallel}x$$

Diagram 1. *Free-body diagram of the parallel spring system*

Rearranging for $k_{parallel}$, we get:

$$k_{parallel} = \frac{mg}{x}$$

To determine $\boldsymbol{k}$ **graphically**, we can plot $\boldsymbol{mg}$ (force) on the **vertical axis (y-axis)** and **x** (elongation) on the **horizontal axis (x-axis)**. The **slope** of this graph represents the spring constant k:

$$Slope\ of\ \boldsymbol{mg}\ vs.\boldsymbol{x}\ = k_{parallel}$$

By applying **linear regression to our data**, we obtained a best-fit equation, **where the slope of the line** provided the **experimentally** determined value of $k_{parallel}$.

SERIES COMBINATION – DYNAMIC METHOD

In the series configuration, two springs are attached end to end (Figure 2), meaning that they experience the same force but different elongations. The equivalent spring constant for a **series combination** is given by:

$$k_{series} = \frac{k_1 k_2}{k_1 + k_2}$$

where k_1 and k_2 are the individual spring constants of the two springs in series.

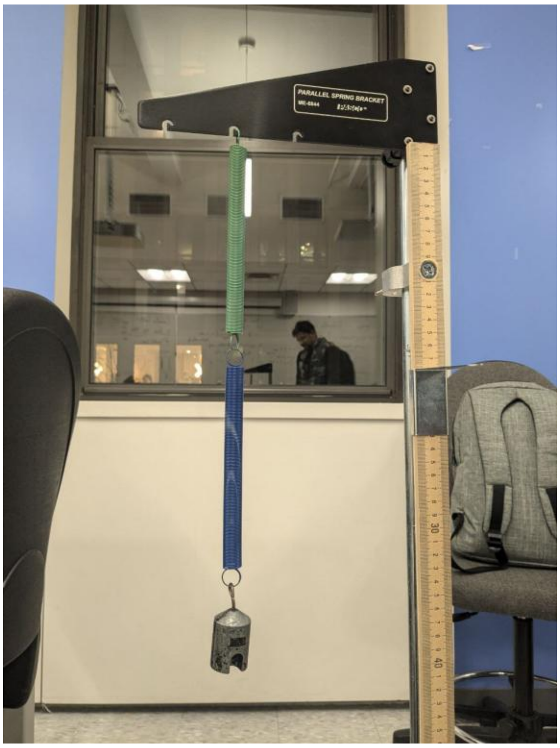


Figure 2. *Experimental setup for the series spring combination. The green and blue springs are connected end-to-end, forming a series system. A mass is attached at the bottom of the blue spring, causing both springs to stretch under the same applied force.*

To experimentally determine k_{series}, we used the **dynamic method**, analysing the **simple harmonic motion (SHM)** of a mass oscillating on the series spring system.

For a mass-spring system undergoing SHM, the period (T) is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Squaring both sides:

$$T^2 = \frac{4\pi^2 m}{k_{series}}$$

From this equation, if we plot **mass ($\boldsymbol{m}$)** on the **vertical axis** and $\boldsymbol{T^2}$ on the **horizontal axis**, the **slope of the best-fit line** of this graph should be:

$$Slope\ of\ \boldsymbol{m}\ vs.\boldsymbol{T^2} = \frac{k_{series}}{4\pi^2}$$

Thus, the spring constant for the series combination can be determined as:

$$k_{series} = 4\pi^2 \times Slope\ of\ \boldsymbol{m}\ vs.\boldsymbol{T^2}$$

By applying **linear regression** to our SHM data, we extracted the slope and calculated the **experimentally determined** value of k_{series}.

PROCEDURE

MATERIALS:

1. Green and blue springs (Figure 3)
2. Parallel spring bracket (Figure 4)
3. Parallel hook bar (Figure 4)
4. Support stand (Figure 5)
5. Meter stick
6. Set of known masses
7. Stopwatch
8. Data recording sheet

FIGURE 3. *Opened spring set.*

FIGURE 4. *Parallel spring bracket and hook bar.*

FIGURE 5. *Support stand.*

SETUP AND PROCEDURE:

1. PARALLEL COMBINATION (STATIC METHOD):

- The green and blue springs were attached to the parallel spring bracket (Figure 1).
- A hooked mass was attached to the bottom bar connecting both springs.
- The initial and elongated lengths were recorded for different masses.
- A graph of force vs. displacement was plotted, and the slope determined the experimental spring constant.

2. SERIES COMBINATION (DYNAMIC METHOD):

- The green and blue springs were connected in series (Figure 2).
- A mass was suspended from the bottom spring, and oscillations were initiated.
- The time for 15 oscillations was recorded, and the period T was calculated.
- A graph of $\boldsymbol{m}\ vs.\boldsymbol{T}^2$ was plotted, and the slope determined the experimental spring constant.

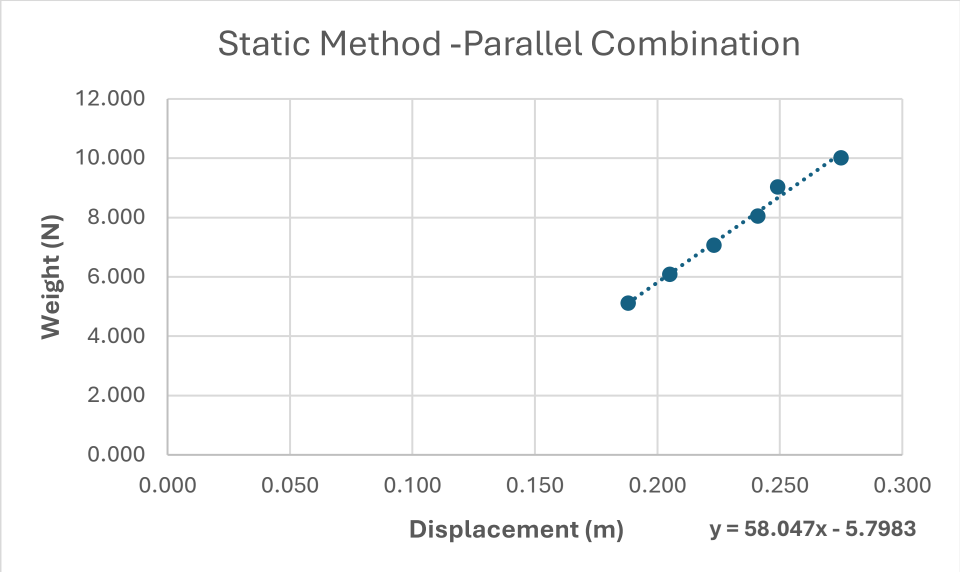
DATA & ANALYSIS

The collected data, calculations, and graphical analysis are presented below.

STATIC METHOD PARALLEL COMBINATION RESULTS:

Static Method – Parallel Combination					
Spring Combination	Data point	x (m)	m (kg)	Weight (N)	Natural length (m)
Green+Blue	1	0.188	0.520	5.111	0.130
	2	0.205	0.620	6.088	0.130
	3	0.223	0.720	7.070	0.130
	4	0.241	0.820	8.052	0.130
	5	0.249	0.920	9.034	0.130
	6	0.275	1.020	10.016	0.130

TABLE 1. This table *presents the elongation measurements for different masses*

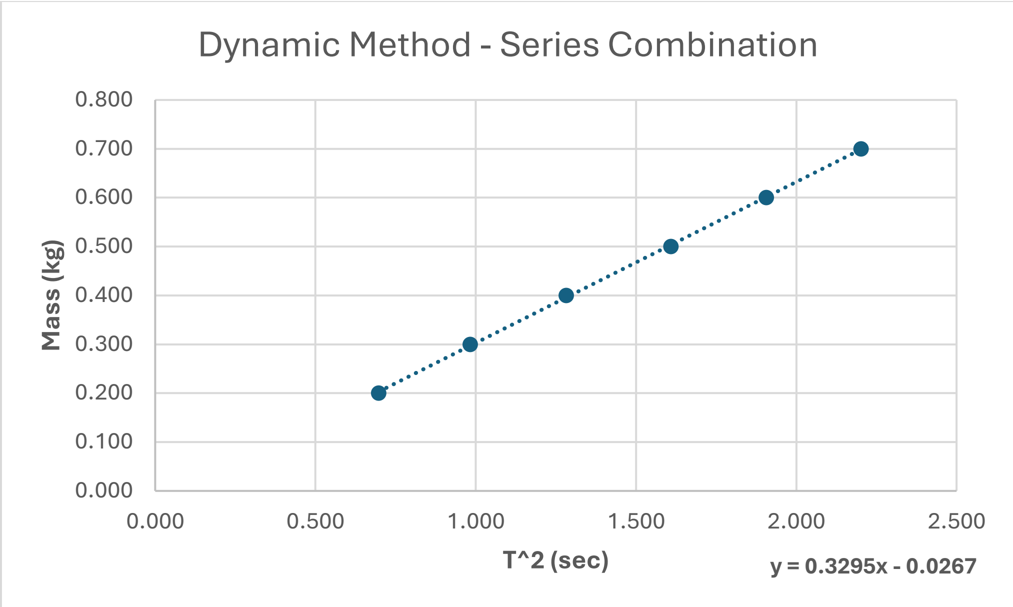


GRAPH 1. *Graph of applied weight versus displacement for the parallel spring combination using the static method. The linear trendline equation **y=58.047x−5.7983** indicates that the experimentally determined spring constant for the parallel system is given by the slope of the best-fit line, **58.047**.*

DYNAMIC METHOD SERIES COMBINATION RESULTS

Dynamic Method – Series Combination					
Spring	Data point	m (kg)	Time for 15 Periods (s)	Period T (s)	T^2 (s)
Green + Blue	1	0.200	12.530	0.835	0.698
	2	0.300	14.870	0.991	0.983
	3	0.400	16.990	1.133	1.283
	4	0.500	19.025	1.268	1.609
	5	0.600	20.710	1.381	1.906
	6	0.700	22.260	1.484	2.202

TABLE 2. *This table presents the period measurements for different masses.*



GRAPH 2. *This graph illustrates the relationship between mass and the period T^2 for the series spring system. The trendline slope, 0.3295, when multiplied by $4\pi^2$, gives the experimental spring constant.*

The error percentage was calculated using the formula:

$$\%\ error = \left|\frac{Theoretical\ Value - Experimental\ Value}{Theoretical\ Value}\right| \times 100$$

Which led us to a table below:

Spring Combination	Standard k (N/m)	Experimental k (N/m)	Error %
Green	40.00	-	-
Blue	20.00	-	-
Parallel(Static)	60.00	58.05	3.26%
Series (Dynamic)	13.33	13.01	2.44%

TABLE 3. *Comparison of standard and experimentally determined spring constants for parallel and series combinations. The percentage error indicates the deviation between the experimental and theoretical values, showing that both methods yielded results within a reasonable accuracy range, with the dynamic method for the series combination having the lowest error.*

DISCUSSION

VERIFICATION OF PARALLEL AND SERIES COMBINATIONS:

- The results support the theoretical expectations. The **parallel** combination produced a **higher $\boldsymbol{k}$** than either individual spring, while the **series** combination had a **lower $\boldsymbol{k}$** than either spring alone.
- The mathematical proof follows from:

$$k_{parallel} > max\,\{k_1, k_2\}$$

and

$$k_{series} < min\,\{k_1, k_2\}$$

since **summation increases the stiffness** in **parallel**, while **reciprocation reduces stiffness** in **series**.

CONSIDERATION OF EQUILIBRIUM CONDITIONS:

- The first condition of **equilibrium** (sum of forces = 0) was satisfied as the system remained at **rest** when weights were applied.
- The second condition (sum of **torques = 0**) was not a significant factor since forces were applied **uniformly in a vertical motion**.

CONCLUSION

This experiment successfully determined the spring constants of parallel and series combinations. The **parallel** combination was **stiffer** than the individual springs, while the **series** combination was **weaker**, confirming theoretical predictions. The experimental values were in good agreement with the theoretical values, with errors of **3.3%** for the **parallel** combination and **2.4%** for the **series** combination. Minor deviations were likely due to measurement precision and manual timing in the dynamic method. Future improvements could involve digital sensors to increase accuracy. Overall, this experiment successfully demonstrated the validity of Hooke's Law and the relationship between oscillation period and spring stiffness, reinforcing fundamental principles of mechanics in elastic systems.

ACKNOWLEDGMENTS

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FIGURES

Tuiachieva, Z. (2025). *Photo of the experimental setup for the parallel spring combination* [Figure 1].

Tuiachieva, Z. (2025). *Photo of the experimental setup for the series spring combination* [Figure 2].

Tuiachieva, Z. (2025). *Photo of the springs set* [Figure 3].

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Tuiachieva, Z., & Villafan, J. (2025). *Graph of applied weight versus displacement for the parallel spring combination using the static method* [Graph 1].

Tuiachieva, Z., & Villafan, J. (2025). *Graph of mass versus T^2 for the series spring combination using the dynamic method* [Graph 2].

TABLES

Tuiachieva, Z., Villafan, J, Rahman J. (2025). *Elongation measurements for different masses* [Table 1].

Tuiachieva, Z., Villafan, J, Rahman J. (2025). *Period measurements for different masses [Table 2].*

Tuiachieva, Z., Villafan, J, Rahman J. (2025). *Comparison of standard and experimentally determined spring constants for parallel and series combinations* [Table 3].