Threads as Fiber Density, and Holographic Spindle-Torus Cosmology

A Measured-Bundle Foundation for Thread-Weighted Projection

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August 15, 2025

Abstract

We unify a rigorous notion of *thread* - a coordinate-free enhancement of fiber density in a measured bundle - with a holographic cosmology on a spindle-torus base. Mathematically, microscopic degrees of freedom form a smooth bundle $\pi : E \to B$ equipped with a positive density ν_E. When the structure group preserves a fixed fiber density ν_F, local trivializations push ν_E to $\nu_B \otimes \nu_F$ scaled by a scalar field $\lambda : B \to (0, \infty]$, the *fiber density. Threads* are loci where λ is elevated (or singular), invariantly defined via disintegration. We then introduce a *thread-weighted projection* that maps bundle fields on the two-dimensional spindle-torus (or hypertorus) base into bulk fields with a locality kernel K and weights λ. Spatial and temporal variation of λ acts as an effective source of curvature and stress-energy, providing a mechanism by which spacetime curvature is *emergent* from fiber density. In this picture, a singular thread at the node of the spindle torus models the big bang; slow evolution of the thread background yields late-time acceleration, and concentrated future threads support big-crunch/cyclic scenarios. We outline invariance properties, a minimal simulation pipeline, and qualitative observational signatures (finite-cell turnovers, small-scale damping, and anisotropies aligned with the thread).

1 Introduction

Classical fiber bundles encode local product structure but say nothing about how much of the total space lies over a region of the base. A measured-bundle enhancement records this as an invariant scalar field λ on the base. Regions of high λ are the *threads.* In parallel, holographic cosmology projects microscopic data defined on a compact base (e.g. a torus) into a three-dimensional bulk via a locality kernel.

This paper merges the two: threads furnish intrinsic weights for the projection, and the resulting bulk fields carry an imprint of λ. We interpret gradients and singularities of λ as sources of effective curvature, tying cosmic expansion history to thread dynamics.

Contributions. (i) A precise, trivialization-invariant definition of fiber density and *thread* in the smooth category; (ii) a thread-weighted holographic projection from a spindle-torus (or hypertorus) base to bulk fields; (iii) a phenomenological closure in which curvature emerges from thread structure; (iv) qualitative signatures and a simulation pipeline.

2 Measured bundles, fiber density, and threads

We work in the smooth category; all manifolds are smooth, second countable, and Hausdorff. Fix positive smooth density ν_B on B and ν_F on F. Let $\mathrm{Diff}(F, \nu_F)$ denote diffeomorphisms preserving ν_F.

Definition 2.1 (Measured fiber bundle)**.** Let $\pi : E \to B$ be a smooth bundle with fiber F and structure group contained in $\mathrm{Diff}(F, \nu_F)$. Equip E with a positive smooth density ν_E. We call (E, π, ν_E) *measured* if for every trivialization $\phi : \pi^{-1}(U) \to U \times F$ there exists a measurable $\lambda_U : U \to (0, \infty]$ such that

$$(\phi)_*\nu_E = \lambda_U(b)\, \nu_B|_U(b) \otimes \nu_F(f) \quad \text{on } U \times F.$$

Proposition 2.2 (Well-defined fiber density)**.** *The functions λ_U agree almost everywhere on overlaps, yielding a globally defined measurable function $\lambda : B \to (0, \infty]$, the* fiber density *of (E, π, ν_E).*

Remark 2.3 (Disintegration)*.* Locally, $d\nu_E(e) = \lambda(\pi(e))\, d\nu_B(\pi(e))\, d\nu_F(\text{fiber coordinate})$. Thus $\lambda = \frac{d(\pi_*\nu_E)}{d\nu_B}$ after normalization by ν_F.

Definition 2.4 (Thread locus, singular thread, thread mass)**.** For $c > 0$, the *thread locus at level c* is $\mathcal{R}_c = \{b \in B : \lambda(b) > c\}$. The *singular thread* is

$$\mathcal{R}_{\text{sing}} = \{b \in B : \lambda(b) = \infty \text{ or } \lambda \text{ is not locally essentially constant near } b\}.$$

The *thread mass* on a measurable $A \subset B$ is $\mathsf{M}_{\text{thread}}(A) = \int_A \lambda\, d\nu_B = \nu_E(\pi^{-1}(A))$.

Proposition 2.5 (Functoriality)**.** *Measured isomorphisms preserve λ; pullbacks satisfy $\lambda_{f^*E} = \lambda \circ f$; products multiply density: $\lambda_{1\times 2}(b_1, b_2) = \lambda_1(b_1)\lambda_2(b_2)$.*

3 Spindle-torus base and thread profiles

Let B be (A) a *spindle torus* (an immersed T^2 with a distinguished node $\Gamma \simeq S^1$), or (B) a T^3 equipped with a closed curve $\Gamma \subset B$ playing the role of a node axis. The node is not a topological singularity of B; it marks a locus of enhanced throughput encoded by λ.

A convenient thread profile concentrating weight near Γ is

$$\lambda(b) = \lambda_0 \left[1 + \alpha \exp\left(-\frac{d_B(b, \Gamma)^2}{2\ell^2}\right)\right], \tag{1}$$

with amplitude $\alpha > 0$ and width $\ell > 0$. Singular threads may be modeled by allowing $\lambda(\cdot) \to \infty$ along parts of Γ.

4 Thread-weighted holographic projection

Microscopic bundle fields $\phi^a(b, f)$ induce bulk fields $\Phi^A(x)$ through a locality kernel K and thread weights:

$$\Phi^A(x) = \int_B d\nu_B(b)\, \lambda(b) \int_F d\nu_F(f)\ K^A{}_a(b, f; x)\, \phi^a(b, f). \tag{2}$$

We impose: (i) *Normalization* so constant base configurations yield homogeneous bulk fields; (ii) *Locality* with comoving width σ; (iii) appropriate symmetries/gauge covariance via a transport factor $U^A{}_a$.

A minimal kernel is

$$K^A{}_a(b, f; x) = \mathcal{N}\, \exp\left[-\frac{d_{\text{bulk}}(x, X(b))^2}{2\sigma^2}\right] U^A{}_a(b, f; x), \tag{3}$$

where $X : B \to$ (bulk comoving 3-space) sets the projection geometry.

5 Emergent curvature from fiber density

We view thread structure as a source of effective curvature through two complementary mechanisms.

Metric response. Introduce an effective bulk metric

$$\tilde{g}_{\mu\nu} = \Omega(\lambda)^2 \, g^{(0)}_{\mu\nu} + \Pi_{\mu\nu}(\nabla \log \lambda; \, \hat{n}), \tag{4}$$

where $g^{(0)}$ is a reference background (e.g. FRW), $\Omega(\lambda) > 0$ a scalar response (e.g. $\Omega = \lambda^{\beta}$), and $\Pi_{\mu\nu}$ an anisotropic correction aligned with the node direction $\hat{n}$ (tangent to Γ). Spatial/temporal variations of λ thus modulate local rulers and clocks.

Stress-energy response. Alternatively (or additionally), treat λ as inducing an effective stress-energy

$$T^{(\lambda)}_{\mu\nu} = \frac{\xi}{8\pi G} \left(\nabla_\mu \nabla_\nu \log \lambda - g_{\mu\nu} \Box \log \lambda \right) + \frac{\alpha_\lambda}{8\pi G} \left(\nabla_\mu \log \lambda \nabla_\nu \log \lambda - \frac{1}{2} g_{\mu\nu} |\nabla \log \lambda|^2 \right), \tag{5}$$

with phenomenological couplings ξ, α_λ. The Einstein equation becomes

$$G_{\mu\nu}(\tilde{g}) = 8\pi G \left(T^{\Phi}_{\mu\nu} + T^{(\lambda)}_{\mu\nu} \right), \tag{6}$$

where T^{Φ} is the stress-energy of projected fields (2). In regions where λ is locally constant, $\tilde{g} = g^{(0)}$ and $T^{(\lambda)}_{\mu\nu} = 0$, recovering standard GR.

Remark 5.1 (Interpretation). Eqs. (4)–(5) are a *closure*: they specify how thread structure back-reacts on the bulk. Different choices of $\Omega, \Pi, \xi, \alpha_\lambda$ encode distinct phenomenologies; data can constrain them.

6 Cosmological scenarios

Big bang as a singular thread. A singular thread at the spindle node Γ - $\lambda \to \infty$ in the past limit - focuses projection along $X(\Gamma)$ and yields divergent curvature in (5), modeling a big-bang-like initial condition.

Accelerated expansion. If λ grows slowly with cosmic time (e.g. $\partial_t \log \lambda > 0$ on large scales), then $T^{(\lambda)0}{}_0$ acquires an approximately constant positive component, acting as dark-energy-like pressure with $w \approx -1$; anisotropic corrections inherit the node direction.

Cyclic/big-crunch phases. Time-dependent thread mass concentrating again toward Γ can drive reversal/recollapse through sign changes in the effective pressure sourced by (5), yielding cyclic scenarios within the same holographic framework.

7 Observational signatures

Isotropic baseline (finite cell size).

- Turnover in $\Delta^2(k)$ near $k_{\text{fund}} \approx 2\pi/L$ for a fundamental cycle L of B.
- High-k damping controlled by the kernel width σ in (3).
- CMB topology tests (matched circles) in hypertorus variants.

Thread-induced effects.

- Anisotropic power $P(\boldsymbol{k})$: quadrupole-like modulation aligned with $\hat{n}$ (the tangent to Γ).
- Directional damping: effective $\sigma_\parallel \neq \sigma_\perp$ if K narrows along Γ.
- Cross-survey correlations with characteristic scale $\sim \max\{\ell, \sigma\}$ set by (1).

8 Simulation pipeline

1. Sample base fields $\phi(b, f)$ on B from a chosen spectrum.
2. Choose a thread profile $\lambda(b)$ (e.g. (1)) and parameters $\{\alpha, \ell, \hat{n}\}$; optionally a time profile $\lambda(b, t)$.
3. Project to $\Phi(x)$ using (2)–(3) on a 3D grid; compute $T^{\Phi}_{\mu\nu}$.
4. Evolve $\tilde{g}_{\mu\nu}$ via (6) with $T^{(\lambda)}_{\mu\nu}$ from (5); iterate if back-reaction alters X.
5. Derive observables: $\Delta^2(k)$, CMB spectra, anisotropy estimators aligned with $\hat{n}$; fit $\{L, \sigma, \alpha, \ell, \hat{n}, \xi, \alpha_\lambda\}$ to data.

9 Parameters and priors

Geometry: fundamental lengths L_i of B; spindle parameter; node orientation $\hat{n}$.
Kernel: locality width σ; normalization $\mathcal{N}$.
Threads: strength α, width ℓ, optional multiple loci $\{\Gamma_j\}$.
Back-reaction: β in $\Omega = \lambda^\beta$, anisotropy model in Π, and ξ, α_λ in (5).
Noninformative or weakly informative priors on $\log \sigma$, $\log \ell$, and angles for $\hat{n}$ are convenient.

10 Mathematical examples

Example 10.1 (Trivial bundle). $E = B \times F$ with $\nu_E = \nu_B \otimes \nu_F$ has $\lambda \equiv 1$; all thread loci at $c \geq 1$ are empty.

Example 10.2 (Branched coverings). For $p : \Sigma \to B$ of degree d with discrete fibers (counting measure), $\lambda \equiv d$ off the branch locus; at a branch point b, $\lambda(b)$ equals the number of distinct preimages (typically $< d$). Thus branch points are *lower*-density regions; for $d - 1 < c < d$, $\{\lambda > c\}$ picks out the regular region.

Example 10.3 (Disk with central thread). Take $E = D^2 \times S^1$, $\nu_E = (a \circ \pi)(\nu_B \otimes \nu_F)$ with $a > 1$ near 0 and $a \equiv 1$ away from 0. Then $\lambda = a$; for $1 < c < \max a$, $\mathcal{R}_c = \{a > c\}$. Allowing $a(0) = \infty$ produces a singular thread at the center.

11 Existence and normalization

Proposition 11.1 (Existence). *For any smooth bundle $\pi : E \to B$ with structure group in $\mathrm{Diff}(F, \nu_F)$ and any positive smooth $\lambda : B \to (0, \infty)$, there exists a smooth ν_E making (E, π, ν_E) measured with fiber density λ.*

Sketch. Choose a reference density $\tilde{\nu}_E$ that pushes forward to $\nu_B \otimes \nu_F$ in trivializations and set $\nu_E = (\lambda \circ \pi)\tilde{\nu}_E$. □

Proposition 11.2 (Uniqueness up to base scaling). *If ν_E and ν'_E induce the same λ, then $\nu'_E = (c \circ \pi)\nu_E$ for some positive function c on B.*

12 Consistency and outlook

The thread concept is rigorously defined and invariant under changes of trivialization. Weighting holographic projection by λ provides a clean route from base microphysics to bulk fields. Simple closures show how curvature can emerge from thread structure, linking early-time singularities, late-time acceleration, and possible recollapse to the dynamics of λ. Next steps include: computing two-point functions on B and propagating through K; implementing anisotropic likelihoods for LSS/CMB; and exploring gauge sectors on B projected via $U^A{}_a$.

Acknowledgments

This work synthesizes a measured-bundle formalism for fiber density with a spindle-torus holographic kernel framework.