Thread-Weighted Projection from a Spindle–Torus Base

A Measured-Bundle Framework for Emergent Bulk Fields

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Abstract

We develop a compact holographic framework in which microscopic information resides on a two-dimensional spindle–torus base B and induces three-dimensional bulk fields by a *thread-weighted projection*. Threads are modeled as a nonnegative density $\lambda \in L^1(B)$ that weights local fiber data in a measured bundle $\pi : E \to B$. A transport kernel K carries fiber fields to bulk fields, with a minimal Gaussian factor enforcing locality around an embedding $X : B \hookrightarrow \mathbb{R}^3$. On a periodic T^3 toy model we prove stationarity of the induced field and derive a spectral transfer law that predicts a turnover at the fundamental mode and a Gaussian roll-off. We further incorporate a Hopf lift: a $U(1)$ connection from the Hopf fibration equips the transport with a gauge-consistent phase and quantized helicity, enabling parity-odd signatures.

The present work is kinematic and statistical in scope. It establishes well-posedness of the projection, spectral predictions, and a topological phase structure. Possible routes toward *emergent spacetime and quantum fields* are

System: are outlined at a speculative level only, as future directions.

Scope and contributions. We (i) formalize threads as a measurable density on a spindle–torus base, (ii) define a locality-preserving transport kernel that projects fiber data to bulk fields, (iii) prove stationarity and give an analytic power spectrum with a fundamental-mode turnover and Gaussian roll-off on T^3, and (iv) show how a Hopf $U(1)$ connection endows the construction with a gauge-consistent phase and quantized helicity. The paper is intentionally kinematic: we do not derive a bulk metric, causal structure, or quantum commutation relations. Any connection to *emergent spacetime or quantum fields* is reserved for speculative discussion in the conclusion.

1 Introduction

Holographic ideas suggest that bulk phenomena can be encoded on a lower-dimensional substrate. Here we formulate a mathematically explicit version of that intuition in which microscopic information lives on a two-dimensional *spindle–torus* base B and is transported to three-dimensional bulk fields by a *thread-weighted projection* (TWP).

The central modeling element is a measured bundle $\pi : E \to B$ whose fibers store local microdata and a nonnegative density $\lambda \in L^1(B)$, called *threads*, that weights how strongly base points contribute to bulk observables. A locality-preserving transport kernel K—minimal form: a Gaussian centered on the embedded base $X(B) \subset \mathbb{R}^3$—pushes forward fiber fields to bulk fields Φ.

On a toroidal bulk domain T^3 we prove that, under a mild normalization, the induced covariance is stationary (depends only on separations) and the power spectrum obeys a simple transfer law whose shape exhibits a turnover at the fundamental mode and Gaussian roll-off at high k. We then introduce a *Hopf lift*: a $U(1)$ connection inherited from the Hopf fibration induces a gauge-consistent phase in the transport, yielding quantized helicity and parity-odd signatures.

Related perspectives

Our construction sits at the interface of kernel-based integral geometry, statistical field modeling, and topological phases. The emphasis here is on a clean pipeline from base micro-data to bulk statistics with enough structure to support numerical simulation and comparison to observations.

Assumptions and limitations. We treat (i) the bulk geometry as fixed for analysis, (ii) threads as a static density on B, and (iii) transport as kinematic without equations of motion. All spectral results follow under these assumptions and do not rely on claims of emergent spacetime or quantization.

2 Geometry of the base and measured bundle

2.1 Spindle–torus base and embedding

Let B be a spindle–torus surface embedded in $\mathbb{R}^3$ by a smooth map $X : B \hookrightarrow \mathbb{R}^3$. Concretely, one may view B as the surface of revolution obtained by rotating a circle of radius $r > 0$ centered at distance $a \in (0, r)$ from the axis, yielding the spindle member of the torus family. The detailed parameterization will not be needed; we use only that B is compact, orientable away from the self-intersection locus, and admits a Borel measure ν_B induced by the surface element.

2.2 Measured fiber bundle and threads

Definition 1 (Measured bundle and threads)**.** *Let $\pi : E \to B$ be a measurable fiber bundle with typical fiber F and fiber measure ν_F. A* thread density *is a function $\lambda \in L^1_+(B, \nu_B)$, $\lambda \geq 0$, which we normalize by $\int_B \lambda \, d\nu_B = \Lambda > 0$. A* fiber field *is a measurable map $\varphi : E \to \mathbb{C}$ with finite second moment $\int_E |\varphi|^2 \, d\nu_E < \infty$, where $d\nu_E := d\nu_F \, d\nu_B$.*

The density λ encodes "where the information lives" on B; large λ indicates many or stronger threads. We assume either deterministic φ or a mean-zero random fiber field with known two-point function.

3 Thread-weighted projection (TWP) and minimal locality

3.1 Transport kernel

Let $x \in \mathcal{M} \subset \mathbb{R}^3$ denote a bulk point (in this paper $\mathcal{M}$ will be either $\mathbb{R}^3$ or a 3-torus T^3). We define a transport kernel $K : \mathcal{M} \times B \times F \to \mathbb{C}$ of the form

$$K(x \,|\, b, f) \;=\; G_{\Sigma(b)}\big(x - X(b)\big)\; U(b, f; x), \tag{1}$$

where G_Σ is a Gaussian with covariance $\Sigma(b) \in \mathbb{R}^{3\times 3}$ positive-definite, and U is a bounded modulation factor. The *minimal kernel* takes $U \equiv 1$ and $\Sigma(b) = \sigma^2 I$.

Definition 2 (Thread-weighted projection)**.** *Given λ, φ, K, define the bulk field*

$$\Phi(x) \;=\; \int_B \int_F K(x \,|\, b, f)\, \varphi(b, f)\, \lambda(b)\, d\nu_F(f)\, d\nu_B(b). \tag{2}$$

Proposition 1 (Well-posedness)**.** *Suppose U is essentially bounded and $G_\Sigma \in L^1(\mathbb{R}^3)$ uniformly in b. If $\varphi \in L^2(E, \nu_E)$ and $\lambda \in L^1(B)$, then $\Phi \in L^2(\mathcal{M})$ and the map $(\varphi, \lambda) \mapsto \Phi$ is continuous.*

Proof. By Cauchy–Schwarz and Fubini,

$$|\Phi(x)| \le \int_B \lambda(b)\, \|U(\cdot, b, \cdot)\|_\infty \left(\int_F |\varphi(b,f)|\, d\nu_F \right) G_{\Sigma(b)}(x - X(b))\, d\nu_B(b).$$

Square and integrate over x, use $\|G_{\Sigma(b)}\|_{L^1} = 1$ after normalization (or a uniform bound), then Jensen/Minkowski yield the claimed continuity. □

3.2 Normalization on T^3 and periodized kernels

On $\mathcal{M} = T^3 = [0, L)^3$ we periodize G_Σ:

$$G_\Sigma^{(T)}(\chi) \;=\; \sum_{n \in \mathbb{Z}^3} G_\Sigma(\chi + nL), \qquad \chi \in T^3, \tag{3}$$

and impose the *stationarity normalization*

$$\int_{T^3} G_\Sigma^{(T)}(\chi)\, d^3\chi \;=\; 1. \tag{4}$$

4 Stationarity and the spectral transfer law on T^3

Assume φ is a mean-zero, second-order process on E with covariance

$$\mathcal{C}_\varphi((b,f),(b',f')) = \mathbb{E}\big[\varphi(b,f)\, \overline{\varphi(b',f')}\big]. \tag{5}$$

Write the induced bulk covariance

$$\begin{aligned} C_\Phi(x,y) &:= \mathbb{E}\big[\Phi(x)\, \overline{\Phi(y)}\big] \\ &= \iint_{B\times B} \iint_{F\times F} K(x|b,f)\, \overline{K(y|b',f')}\, \mathcal{C}_\varphi((b,f),(b',f'))\, \lambda(b)\lambda(b')\, d\nu_F d\nu'_F d\nu_B d\nu'_B. \end{aligned} \tag{6}$$

Proposition 2 (Stationarity on T^3)**.** *If K is of the periodized minimal form, $U \equiv 1$, and (4) holds, and if $\mathcal{C}_\varphi$ depends only on base separation along B (invariant under the isometries of B), then $C_\Phi(x,y) = C_\Phi(x-y)$ depends only on $\Delta := x - y \in T^3$.*

Sketch. Under the minimal kernel, the x and y dependence enters only via shifts $x - X(b)$ and $y - X(b')$. Periodization and normalization ensure that joint translations of (x,y) leave (6) invariant, with the residual dependence on Δ entering through the difference of Gaussians after integrating over base variables with isotropic statistics. □

Let $\hat{f}(k)$ denote the discrete Fourier transform on T^3 for $k \in (2\pi/L)\mathbb{Z}^3$, and define apologetic power spectrum $P_\Phi(k) = \mathbb{E}[|\hat{\Phi}(k)|^2]$.

Proposition 3 (Spectral transfer law)**.** *Under the assumptions of Proposition 2 with $\Sigma(b) \equiv \sigma^2 I$, the bulk spectrum obeys*

$$P_\Phi(k) \;=\; \big|\widehat{G_\sigma^{(T)}}(k)\big|^2\, P_\varphi(k), \qquad \widehat{G_\sigma^{(T)}}(k) = e^{-\frac{1}{2}\sigma^2 \|k\|^2}, \tag{7}$$

where $P_\varphi(k)$ is the spectrum of the fiber-aggregated source after pushing to B and projecting onto T^3 modes.

Proof. Fourier transforming (2) and using convolution in $\mathbb{R}^3$ periodized to T^3 yields $\hat{\Phi}(k) = \widehat{G_\sigma^{(T)}}(k)\,\hat{\Psi}(k)$ where Ψ is the base-sourced field. Taking expectation of $|\hat{\Phi}(k)|^2$ and using independence of G from φ gives (7). $\square$

Corollary 1 (Turnover and Gaussian roll-off)**.** *On T^3 the lowest nonzero wavenumber is $k_{\mathrm{f}} = 2\pi/L$. If $P_\varphi(k)$ is finite at k_{f}, then $P_\Phi(k)$ turns over at k_{f} and decays as $\exp(-\sigma^2\|k\|^2)$ at large $\|k\|$.*

5 Anisotropic kernels and director fields

Threads can carry directional structure. Let $n : B \to S^2$ be a measurable director field and

$$\Sigma(b) \;=\; \sigma_\parallel^2\, n(b)n(b)^\top + \sigma_\perp^2\,\big(I - n(b)n(b)^\top\big), \quad \sigma_\parallel, \sigma_\perp > 0. \tag{8}$$

Then $G_{\Sigma(b)}$ elongates transport along $n(b)$.

Proposition 4 (Anisotropic spectral imprint)**.** *If n is slowly varying on the G_Σ support and $U \equiv 1$, then to leading order the spectrum acquires an angular dependence*

$$P_\Phi(k) \;\approx\; \exp\Big(-\tfrac{1}{2}(\sigma_\perp^2\|k\|^2 + (\sigma_\parallel^2 - \sigma_\perp^2)\,\mathbb{E}_B[(\hat{k}\cdot n)^2])\Big)\, P_\varphi(k), \tag{9}$$

where $\mathbb{E}_B$ denotes an average over B with weight $\lambda\, d\nu_B$ and $\hat{k} = k/\|k\|$.

6 Hopf lift: $U(1)$ connection, helicity, and parity odd signatures

To endow the transport with a topological phase, we incorporate a $U(1)$ connection from the Hopf fibration. Let $\pi_H : S^3 \to S^2$ be the Hopf map with connection A and curvature $F = dA$. Pull back along a map $h : B \to S^2$ to obtain $A_h = h^*A$ with quantized Chern number $\frac{1}{2\pi}\int_B h^*F \in \mathbb{Z}$.

6.1 Phase-modulated transport

Define

$$U(b, f; x) \;=\; \exp\big(iq\,\Theta(b; x)\big), \qquad \Theta(b; x) := \int_{\gamma(b,x)} A_h, \tag{10}$$

where $\gamma(b, x)$ is a chosen curve in an auxiliary space linking b to the bulk location x (for minimality, take a short geodesic in a tubular neighborhood of $X(B)$) and $q \in \mathbb{Z}$ is a charge. The precise choice of γ affects only a gauge; observables depend on the homotopy class via F.

Proposition 5 (Helical decomposition and parity odd power)**.** *With the phase-modulated U, the two-point function in Fourier space splits into helical components. In an isotropic limit on T^3,*

$$\langle \hat{\Phi}_i(k)\,\overline{\hat{\Phi}_j(k)}\rangle = P_S(k)\,\Pi_{ij}^{(S)}(k) \;+\; i\,P_H(k)\,\Pi_{ij}^{(H)}(k), \tag{11}$$

where $\Pi^{(S)}$ and $\Pi^{(H)}$ are the symmetric and helical projectors built from δ_{ij}, $\hat{k}_i\hat{k}_j$, and $\epsilon_{ij\ell}\hat{k}_\ell$. The pseudoscalar spectrum $P_H(k)$ is nonzero when $q \neq 0$ and h has nontrivial Chern number, generating parity-odd signatures.

Remark 1. *Operationally, P_H scales with $\int_B \lambda\, A_h \wedge dA_h$ at leading order in the phase, giving a topological control knob for handedness in the induced bulk.*

7 Numerical pipeline and observational handles

7.1 Minimal simulation loop

1. Sample φ on E with chosen fiber covariance.
2. Sample or prescribe λ on B; optionally, a director field n and map $h : B \to S^2$ for Hopf phase.
3. Assemble K using $G_{\Sigma(b)}$ and U.
4. Compute Φ via (2); on T^3 use FFTs and (7).
5. Measure $P_\Phi(k)$, anisotropy, and (if vector/tensor fields) the helical spectra.

7.2 Observable features

- **Turnover scale**: $k_\mathrm{f} = 2\pi/L$ sets the first peak or turnover.
- **High-k damping**: Gaussian roll-off $\sim e^{-\sigma^2\|k\|^2}$.
- **Anisotropy**: Angular modulation governed by $\sigma_\parallel/\sigma_\perp$ and distribution of n.
- **Parity odd signals**: Controlled by q and the Chern number of h through $P_H(k)$.

8 Extensions: identifiability and inverse problems

Proposition 6 (Identifiability in a tight-frame limit (informal))**.** *If the kernel family $\{K(\cdot \mid b, \cdot)\}_{b\in B}$ forms a tight frame for the bulk function space with frame bound $A > 0$, then low-band components of λ are stably estimable from P_Φ (or higher correlators) with error bounded by A^{-1} times the noise level.*

Remark 2. *A precise statement requires specifying the class of φ and measurement noise; we leave the full inverse analysis to future work.*

9 Conclusion

We presented a measured-bundle construction in which a nonnegative *thread* density on a spindle–torus base weights local fiber data that is transported to bulk fields by a locality-preserving kernel. On T^3 this yields a stationary bulk process with an analytic spectral transfer law exhibiting a fundamental-mode turnover and Gaussian high-k damping. A Hopf lift adds a gauge-consistent phase with quantized helicity, enabling parity-odd observables.

Speculative outlook: toward emergent spacetime and quantum fields

The thread-weighted projection provides a mathematically controlled map from base data to bulk fields, together with testable spectral features and a parity-odd handle via the Hopf lift. While we have not attempted it here, one can speculate about three extensions.

(a) **Emergent geometry.** A bulk effective metric might be constructed from second moments of the kernel family, e.g., by a Fisher-information or covariance-based tensor averaged over λ. A self-consistency condition between K and the induced metric could close the geometry in the infrared.

(b) **Causal transport.** A 3+1D retarded kernel could introduce finite signal speed and, under appropriate scaling, yield approximate Lorentz invariance at long wavelengths.

(c) **Quantum structure.** With Gaussian base statistics and a quadratic effective action, the projected two-point function may satisfy positivity and spectral conditions characteristic of free quantum fields, suggesting a route to emergent correlators.

These items are hypotheses. They are outside the scope of the current paper and are stated only to motivate future work and potential observational tests.

A Spindle–torus parameterization (optional)

A convenient implicit equation for a torus family in $\mathbb{R}^3$ is

$$\left(\sqrt{x^2+y^2}-a\right)^2+z^2=r^2,$$

with $0<a<r$ corresponding to the spindle member. Local charts avoiding the self-intersection suffice for measure-theoretic constructions.

B Periodized Gaussian on T^3

Let $G_\sigma(\chi)=(2\pi\sigma^2)^{-3/2}\exp(-\|\chi\|^2/2\sigma^2)$ on $\mathbb{R}^3$. The periodization

$$G_\sigma^{(T)}(\chi)=\sum_{n\in\mathbb{Z}^3}G_\sigma(\chi+nL)$$

satisfies $\int_{T^3}G_\sigma^{(T)}(\chi)\,d^3\chi=1$ and has discrete Fourier transform $\widehat{G_\sigma^{(T)}}(k)=e^{-\frac{1}{2}\sigma^2\|k\|^2}$ for $k\in(2\pi/L)\mathbb{Z}^3$.

C Helical projectors

For vector fields, define the transverse projector $\Pi^\perp_{ij}=\delta_{ij}-\hat{k}_i\hat{k}_j$ and the helical projectors

$$\Pi^{(H)}_{ij}(k)=\epsilon_{ij\ell}\,\hat{k}_\ell,\qquad \Pi^{(S)}_{ij}(k)=\Pi^\perp_{ij}.$$

Then a general statistically isotropic, parity-violating covariance has the form $P_S(k)\Pi^{(S)}+iP_H(k)\Pi^{(H)}$.

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