EN.601.454/654 Augmented Reality

Assignment 3

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1. Camera Distortion

1.1 What is the goal of undistortion? Why should we undistort an image before processing it? (1 point)

The goal of undistortion is to achieve an overal one-to-one mapping (linear image) Because the image processing is a linear transformation and a distorted image will have errors when applying image processing without undistortion

1.2 Given $f(r) = 1 + 0.057r^2 + 0.00014r^4$, compute and sketch the undistorted image of Fig. 1. (2 points)

According to Fig.1 , the distortion center $C = \begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

For point $\vec{P}_1 = [-0.9129, 0.9129]^T$

$r = \|(-0.9129, 0.9129)\| = 1.291$

$f(r) = 1 + 0.057r^2 + 0.00014r^4 = 1.095$

$\vec{P}_{1\,corrected} = f(r)\,\vec{P}_1 = 1.095 \cdot \begin{bmatrix} -0.9129 \\ 0.9129 \end{bmatrix} = \begin{bmatrix} -0.9996 \\ 0.9996 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

For point $\vec{P}_2 = [0, 0.9509]^T$

$r = \|(0, 0.9509)\| = 0.9509$

$f(r) = 1 + 0.057r^2 + 0.00014r^4 = 1.052$

$\vec{P}_{2\,corrected} = f(r)\,\vec{P}_2 = 1.052 \cdot \begin{bmatrix} 0 \\ 0.9509 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

For point $\vec{P}_3 = [0.9129, 0.9129]^T$

$r = \|(0.9129, 0.9129)\| = 1.291$

$f(r) = 1 + 0.057r^2 + 0.00014r^4 = 1.095$

$\vec{P}_{3\,corrected} = f(r)\,\vec{P}_3 = 1.095 \cdot \begin{bmatrix} 0.9129 \\ 0.9129 \end{bmatrix} = \begin{bmatrix} 0.9996 \\ 0.9996 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

For point $\vec{p}_4 = [-0.950, 0]^T$

$$r = \|(-0.950, 0)\| = 0.950$$

$$f(r) = 1 + 0.037r^2 + 0.0014r^4 = 1.052$$

$$\vec{p}_{4\,corrected} = f(r)\,\vec{p}_4 = 1.052 \begin{bmatrix} -0.950 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} -1 \\ 0 \end{bmatrix}}$$

For point $\vec{p}_5 = [0.950, 0]^T$

$$r = \|(0.950, 0)\| = 0.950$$

$$f(r) = 1 + 0.037r^2 + 0.0014r^4 = 1.052$$

$$\vec{p}_{5\,corrected} = f(r)\,\vec{p}_5 = 1.052 \cdot \begin{bmatrix} 0.950 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

For point $\vec{p}_6 = [-0.9129, -0.9129]^T$

$$r = \|(-0.9129, -0.9129)\| = 1.291$$

$$f(r) = 1 + 0.037r^2 + 0.0014r^4 = 1.095$$

$$\vec{p}_{6\,corrected} = f(r)\,\vec{p}_6 = 1.095 \cdot \begin{bmatrix} -0.9129 \\ -0.9129 \end{bmatrix} = \begin{bmatrix} -0.9996 \\ -0.9996 \end{bmatrix} = \boxed{\begin{bmatrix} -1 \\ -1 \end{bmatrix}}$$

For point $\vec{p}_7 = [0, -0.950]^T$

$$r = \|(0, -0.950)\| = 0.950$$

$$f(r) = 1 + 0.037r^2 + 0.0014r^4 = 1.052$$

$$\vec{p}_{7\,corrected} = f(r)\,\vec{p}_7 = 1.052 \cdot \begin{bmatrix} 0 \\ -0.950 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ -1 \end{bmatrix}}$$

For point $\vec{p}_8 = [0.9129, -0.9129]^T$

$$r = \|(0.9129, -0.9129)\| = 1.291$$

$$f(r) = 1 + 0.037r^2 + 0.0014r^4 = 1.095$$

$$\vec{p}_{8\,corrected} = f(r)\,\vec{p}_8 = 1.095 \cdot \begin{bmatrix} 0.9129 \\ -0.9129 \end{bmatrix} = \begin{bmatrix} 0.9996 \\ -0.9996 \end{bmatrix} = \boxed{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$$

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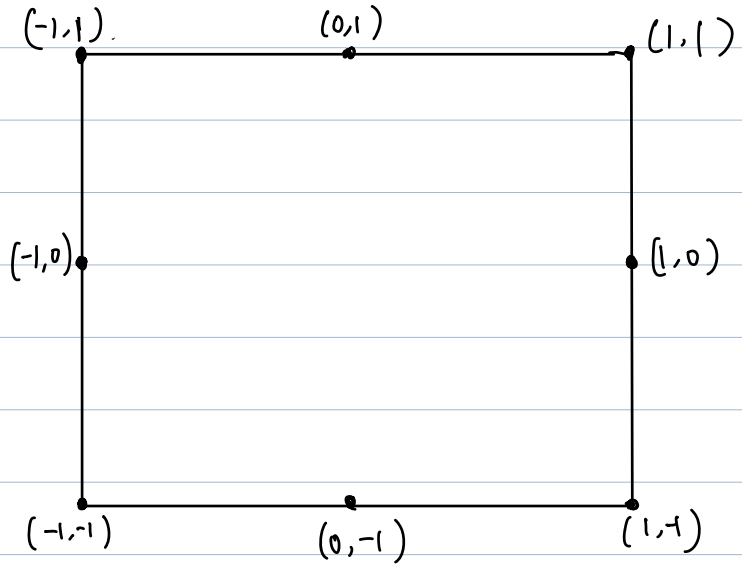
(-1,1)
(0,1)
(1,1)
(-1,0)
(1,0)
(-1,-1)
(0,-1)
(1,-1)

1.3 A fisheye camera is a camera that can capture images with a field of view of more than 180^o. (More details on *Wikipedia*). We are also interested in undistorted images from fisheye camera. Will Eq. 1 work for modeling the distortion? Why or why not? (2 points)

The equation 1 doesn't work for modeling the distortion. Because in the fisheye camera, there's existing tangential distortion. But in Eq.1, it doesn't contain the expression for correcting tangential distortion

1.4 *(UG Optional)* Typically, we use $f(r) = 1+k_1r^2+k_2r^4+k_3r^6$ to model the radial distortion. Note that only even power terms are used in the polynomial. Why do we not use the odd order terms? (1 point)

As far as I'm thinking, because the function domain r is $[0, r_{max}]$ and is always positive, there's no need to use odd term