Contents

1 Question 1

This study introduces a scenario about a congestion charge programme, focusing on the revenue and the level of emissions, where traffic periods are divided into peak and non-peak. Data of the maximum willingness to pay (WTP) for peak and non-peak periods are collected by a survey among a 345-driver sample representing total 192,000 potential drivers in the charging zone. Because the main motivation of this programme is to realize emission reduction, as well as programme reinvestment, the average travel speed is introduced to create the emission function.

1.1 Q1.a Revenue maximisation under a single congestion charge for both peak and non-peak periods

In this scenario, a single congestion charge is applied for both peak and non-peak hours. The algorithm of revenue maximisation is formulated as below:

1. Maximum WTP in sample data is computed to set the upper bound for price search.
2. For each possible price point, sample demand can be calculated, according to clients' surplus. Record sample revenue for each price point.
3. Find maximum sample revenue, and confirm the corresponding price and sample demand.
4. Based on sample demand, calculate total cars and maximum total revenue.

The process of revenue maximisation is visualised in the Figure 1. The plot presents the association between price and total revenue, which increases at first but then decreases with the price going higher. The optimal price is £8, where the sample demand is 303 out of 345. Then, the total demand is 168,626 out of 195,000. Therefore, the maximum total revenue is £1,349,008 with optimal single price equal to £8.

With this price in effect, the total level of emissions can be calculated. The average speed is 19.46 km/h, following the given function. The emissions per car is then calculated as 292.50 g/km and the total level of emissions is 49,323,676 g/km.

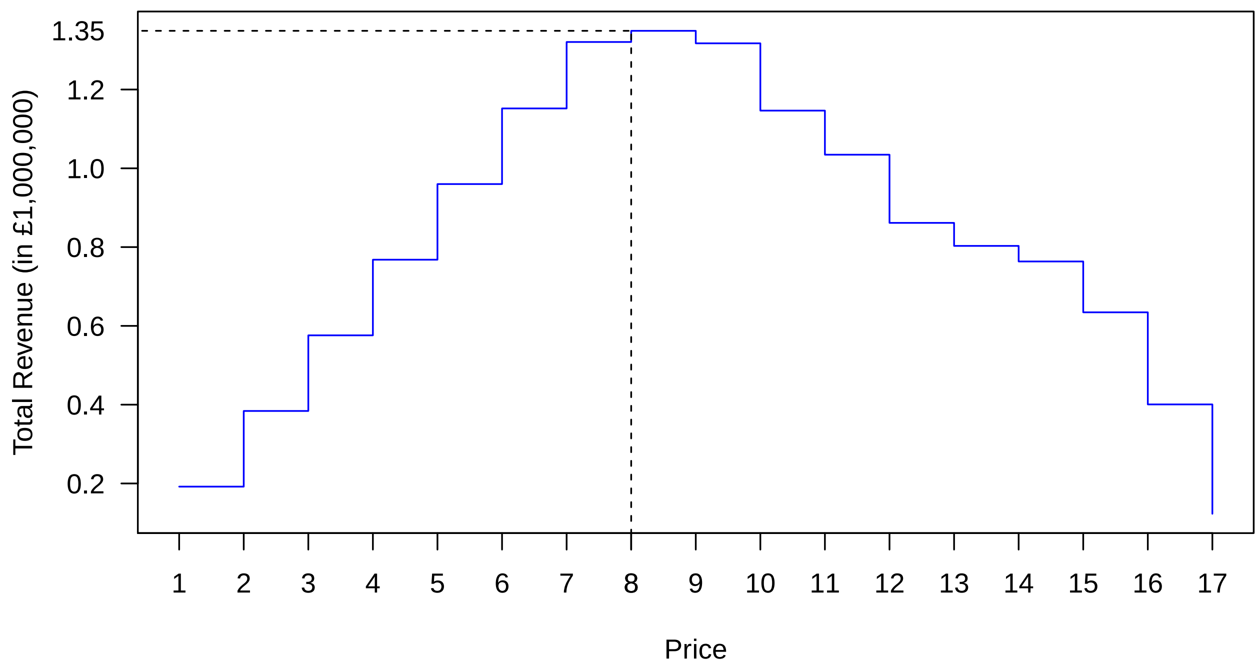

Figure 1: Changes of the total revenue against different single prices.

1.2 Q1.b Revenue maximisation under a peak period pricing strategy

In this scenario, a peak period pricing strategy is introduced. The base price for the non-peak period is set at £7. The peak price is not necessarily higher than non-peak price, for which no constraint between peak and base prices is set. The algorithm of revenue maximisation is formulated as below:

1. Maximum WTP in sample data is computed to set the upper bound for price search which is £17.

2. For each possible price point, sample demand for peak periods would be identified if clients' surplus for peak periods is greater than that for non-peak periods and greater than 0. Similarly, sample demand for non-peak periods would be identified if clients' surplus for non-peak periods is greater than that for peak periods and greater than 0. Record sample revenue for each price point of peak periods when non-peak price is £7.

3. Find maximum sample revenue, and confirm the corresponding peak price and sample demand for both.

4. Based on sample demand for peak and non-peak periods, calculate total demand for both periods and then maximum total revenue.

The association between peak period price and total revenue is visualised in Figure 2, where total revenue increases quickly at first but then slowly deceases with peak price going higher. According to the computation, the maximum total revenue is £1,371,826 when the optimal peak price is £9 and the base price for non-peak period is £7.

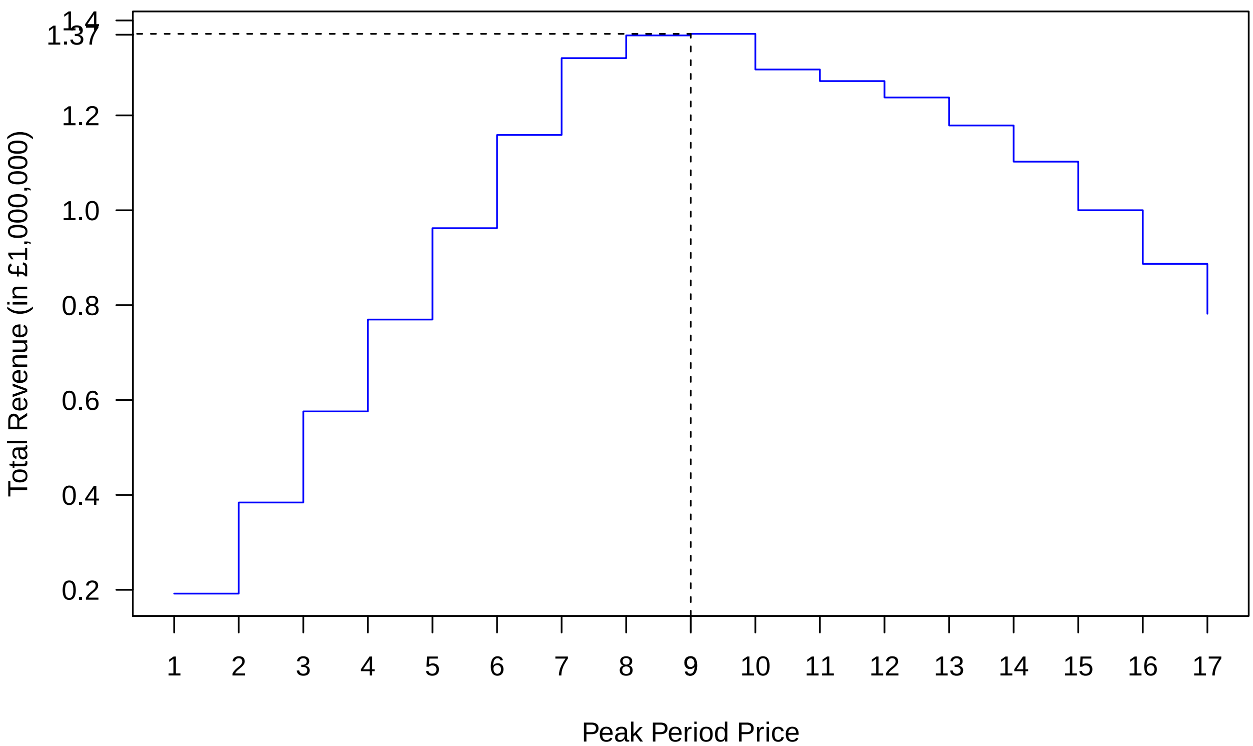

Figure 2: Changes of the total revenue against different peak period prices.

With peak price set at £9, the demands for peak period and non-peak period are 119,096 cars and 42,852 cars respectively, with a total of 161,948 cars out of 195,000. The average speed is 19.88 km/h and the CO2 emission per car is computed as 285.53 g/km. Therefore, the total level of emissions is 46,241,535 g/km.

Compared with the results from part (a), maximum total revenue increases by £22,818.09. Due to the decrease of 6678 in the total number of cars, the average speed increases by 0.42 km/h. As a result, the total level of emissions decreases by 3,082,141 g/km. In conclusion, the fact that the total revenue increases while the number of cars running in the City and the level of emissions drop indicates the benefit of implementing peak period pricing strategy. Thus, the differentiated pricing strategy, which sets peak price at £9 and non-peak price at £7, is recommended to apply as it increases the programme revenue to facilitate programme reinvestment and at the same time reduces cars and emissions in the charging zone to realise less congestion and better carbon living.

1.3 Q1.c Emission minimisation with the revenue not below £1.1 million per day

In this scenario, the goal is to minimise the CO2 emissions rather than solely increase the total revenue. However, the total revenue should meet the minimum requirement of £1.1 million per day. The R package `nloptr` is applied to solve the optimisation problem. Following the format requirements of `nloptr`, the objective function is set to minimise emissions subject to constraints as below:

- The demand for both non-peak and peak periods should be greater than 0.
- The total revenue should not fall below £1.1 million.

In this study, peak price is not necessarily higher than non-peak price, for which no constraint is set. When setting the base price for non-peak period as £7, the optimal peak price is identified as £14. The minimum total level of emissions is thus calculated as 30,325,255 g/km, while the total revenue is £1,102,470.

Compared with the results from part (b), the optimal peak price is £5 higher while assuming the same non-peak price. As a result, the total level of emissions is 15,916,280 g/km lower than previously and the total revenue is £269,356 lower. In general, the increased peak price has positive influence on emission minimisation but negative impact on revenue maximisation. However, the total revenue, which is still above £1.1 million per day, meets the minimum operation requirement such that the programme can self-sustain and allocate a portion of revenue to reinvest in the public transportation. To conclude, with the goal of minimising the CO2 emissions and sustaining the minimum operation requirement, the differenced price strategy of setting peak price at £14 and non-peak price at £7 is recommended.

2 Question 2

This study involves the analysis of a sandwich shop's data of 4 new, different types of sandwiches (Avocado and Brie Small, Avocado and Brie Regular, Super Veggie Small, and Super Veggie Regular) in order to understand what are the optimal prices based on choice data that maximize returns and how shelf space is to be allocated. This is done using a type of logistic regression model called the MultiNomial Logit (MNL) model as it is most appropriate in a retail setting, such as the sandwich shop, where there are different alternatives at different price points to choose from. In this case, an experiment was conducted showing different combinations of prices on 80 participants to understand which of the sandwiches they would prefer to purchase or not purchase at all. Possible prices set for the small sandwich types were: £1.50, £1.75, £2.00, £2.25, and £2.50 while for the regular sandwich type were: £3.50, £3.75, £4.00, £4.25 and $4.50. Therefore, the sandwich shop data is used to identify the MNL model parameters to subsequently find the optimal price for different small and regular sandwiches respectively.

Initial analysis of the dataset indicates that preprocessing is not required since the data in the `choice` column presents customers' decision choices on regular and small sandwiches and the choice to not purchase at all. When exploring the dataset and understanding its summary statistics, we expect that customers would prefer a price point within the range of £1.75-£2.00 for small sandwiches, nothing more or less. Likewise, customers would prefer to not pay for regular sandwiches that are greater than or equal to £4.00.

The data is then converted from a wide format, where each customer choice is its own row, to a format suitable to be used with the `mlogit` function. Next, a Maximum Likelihood Estimation (MLE) is done with the transformed data fit onto the `mlogit` model using `price` to explain customer choices. The reference level is set to `DNB` where no purchase is made and this acts as a basis for selecting the parameters for all other alternatives. Next, we view the outcome of the model, with BrieAvoSm, BriAvoRg, SupVegSm and SupVegRg having an intercept of 7.380, 15.341, 7.380 and 15.970 respectively. The coefficient for price is -4.251. Subsequently, the following parameters are obtained:

- The MNL demand model is implemented by first calculating attraction values to obtain the parameters. This is the difference between the gross utilities of a sandwich and the price of a sandwich divided by the shape parameter μ and this is represented by the formula $v_j = \frac{u_j - p_j}{\mu}$.
- μ: This is the coefficient for price and is calculated by dividing -1 over -4.251 (coefficient value of price). The shape parameter is therefore 0.24.
- u_j: This is calculated by multiplying each intercept value of the sandwiches to the μ. The results for the gross utilities of `small.avo`, `regular.avo`,`small.veg` and `regular.veg` are 1.74, 3.61, 1.74 and 3.76 respectively.

Using these estimated parameters, we aim to find and solve the optimal prices through nonlinear optimisation with the library `nloptr`. First, the objective function `eval_f` is created that goes through the following steps:

1. Define an array for `price` of all 4 sandwich types using one for small sandwiches and one for regular sandwiches.
2. Compute the attraction values using the `exp()` function where the power of `e` is calculated as the gross utility of a sandwich less the price of a sandwich, all divided by μ.
3. Based on the values calculated in step 2, the purchase probabilities are obtained by dividing the attraction for a sandwich with the total of all the attraction values (including the no purchase option). This is done for all 4 sandwiches.
4. Finally, the expected profit is determined by adding each products' multiplication of purchase probability and price. This is then multiplied with the total number of sample instances. Then, the objective function is defined as the negative of the expected revenue.

Next, the constraints are computed by creating a function `eval_g_ineq` which takes the argument of the decision variable and then produces the constraints. The regular and small sandwich prices should be greater than 0 and the difference between these prices should be no less than £1.5. Finally, the initial values are set and both the lower and upper bound values are set as 0 and 10 respectively. The optimisation options are fixed as `"algorithm"="NLOPT_LN_COBYLA"` as it does not require more inputs/derivative information, `"xtol_rel"=1.0e-9` which is the relative tolerance inputted on the decision variable `x`, and `"maxeval"=1000` which stops the algorithm at 1000 iterations for increased efficiency. The algorithm is then implemented using `nloptr()` using all the parameters set for the initial values, upper and lower bounds, the objective and constraints functions, and the optimisation options.

The optimal price for the small sandwich variant is £2 and the optimal price for the regular sandwich variant is £3.5. This matches our initial expectation where customers would prefer the price of small sandwiches within the range £1.75-£2.00 and regular sandwiches less than £4.00. Figure 3 shows the probabilities that customers would buy any of the 4 sandwiches at the optimal price of £2 for the small variants and £3.5 for the regular variants.

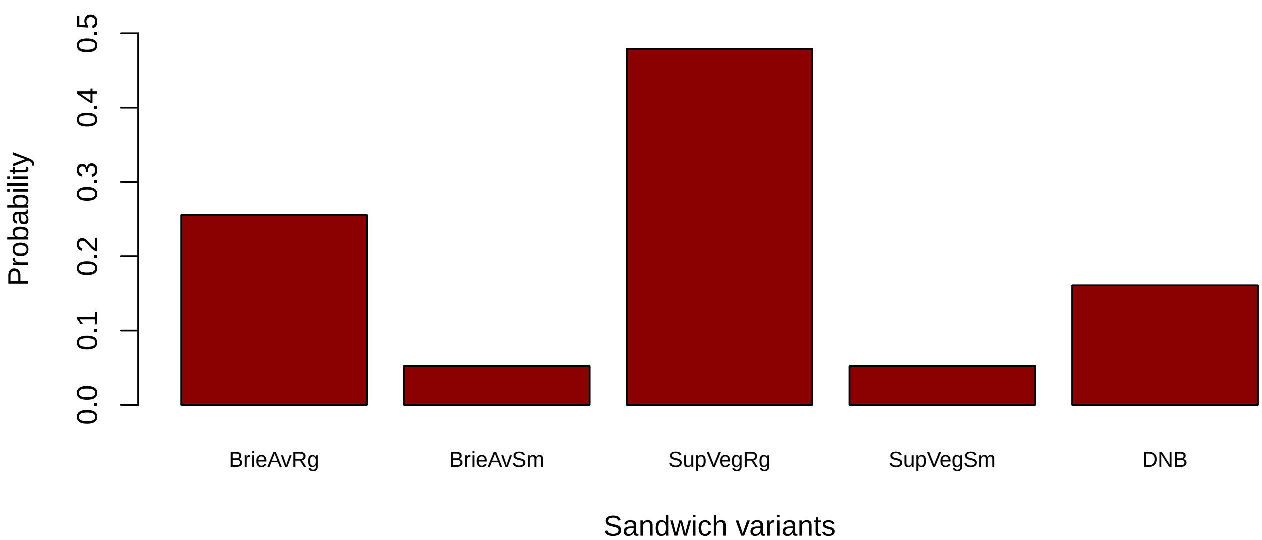

Figure 3: Purchase probabilities of small and regular optimal prices.

Overall, it seems that when setting the small variant prices as £2 and regular variant prices as £3.5, the purchase probability for the small sandwiches are roughly 0.05. For the regular avocado and brie sandwich, it is 0.25 and the regular super veggie sandwich is 0.48. The probability that customers would not buy any sandwich is 0.15. The large difference between the small and regular sandwiches suggests that there is a higher chance that customers are willing to pay an extra £1.5 for the regular sized sandwiches. Additionally, customers are more likely to not purchase any sandwich than actually purchase one of the small variants. While the prices of small sandwiches could be adjusted to increase its purchase probability, these are the optimal prices that will maximise returns to the sandwich shop. Based on these findings, the sandwich shop can allocate shelf space, prioritising the regular sized sandwiches.

3 Question 3

In this study, the scenario of the congestion charge programme is introduced, where traffic periods include peak and non-peak hours, corresponding to peak and non-peak prices. Data are from the file, CongestionPricing.csv, which is generated from a survey among a 345-driver sample representing total 192,000 potential drivers in the charging zone. Further, revenue and the level of emissions are used to evaluate the peak and non-peak pricing strategies. And average travel speed and emissions per car are introduced to help create the emission function.

Based on given settings, assumptions are relaxed to allow a sufficiently rich variation and extension, where peak and non-peak prices would remain dynamic without further constraints. In order to improve the programme's revenue, each possible price combination of peak and non-peak prices would be attempted till the optimal price combination is found.

In this scenario, a price combination strategy is to be confirmed. The base price for the non-peak period is unset, as well as the peak price. What is more, it is not required that peak price exceeds base price, because the goal at current stage is to model the demand in terms of prices. To this end, further comparison can be made with the former findings to see whether there is any possible room to improve the revenue, which is the main target in this study. The algorithm of revenue maximization is formulated as below:

1. Maximum WTP in sample data is used to compute surplus. Initial prices and price bounds are set to make results traceable and comparable.

2. For each possible price point, sample demands for peak price and for non-peak price would be identified. And record sample revenue for each possible price combination.

3. Find maximum sample revenue and confirm the corresponding peak price and sample demand for both.

4. Based on sample demand for peak and non-peak periods, calculate total demand for both periods and then maximum total revenue.

Table 1: Revenue matrix (in £1,000,000) of under pricing strategy for different price combination.

	p=£1	p=£2	p=£3	p=£4	p=£5	p=£6	p=£7	p=£8	p=£9	p=£10	p=£11	p=£12	p=£13	p=£14	p=£15	p=£16	p=£17
b=£1	0.19	0.21	0.21	0.20	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19
b=£2	0.33	0.38	0.40	0.40	0.39	0.39	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
b=£3	0.44	0.52	0.58	0.59	0.59	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58	0.58
b=£4	0.50	0.63	0.72	0.77	0.78	0.78	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77	0.77
b=£5	0.51	0.69	0.82	0.91	0.96	0.97	0.97	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
b=£6	0.51	0.71	0.88	1.02	1.10	1.15	1.17	1.17	1.16	1.15	1.15	1.15	1.15	1.15	1.15	1.15	1.15
b=£7	0.48	0.70	0.89	1.06	1.20	1.28	1.32	1.33	1.33	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32
b=£8	0.45	0.67	0.88	1.05	1.22	1.33	1.36	1.35	1.36	1.34	1.33	1.33	1.33	1.33	1.33	1.33	1.33
b=£9	0.35	0.63	0.84	1.05	1.22	1.35	1.36	1.33	1.32	1.27	1.25	1.24	1.24	1.24	1.24	1.24	1.24
b=£10	0.29	0.54	0.79	0.98	1.20	1.30	1.27	1.23	1.19	1.15	1.11	1.09	1.09	1.09	1.09	1.09	1.09
b=£11	0.24	0.48	0.70	0.93	1.13	1.26	1.23	1.20	1.16	1.10	1.03	0.99	0.99	0.99	0.99	0.99	0.99
b=£12	0.23	0.42	0.64	0.86	1.10	1.21	1.19	1.12	1.06	1.00	0.91	0.86	0.86	0.86	0.86	0.86	0.86
b=£13	0.21	0.42	0.58	0.81	1.04	1.17	1.11	1.04	1.01	0.95	0.86	0.80	0.80	0.80	0.80	0.80	0.80
b=£14	0.20	0.40	0.57	0.74	0.97	1.09	1.06	0.97	0.96	0.91	0.82	0.76	0.76	0.76	0.76	0.76	0.76
b=£15	0.19	0.38	0.54	0.72	0.90	1.02	0.95	0.86	0.82	0.80	0.71	0.63	0.63	0.63	0.63	0.63	0.63
b=£16	0.19	0.38	0.53	0.69	0.87	0.92	0.84	0.72	0.66	0.59	0.50	0.40	0.40	0.40	0.40	0.40	0.40
b=£17	0.19	0.38	0.53	0.69	0.87	0.92	0.77	0.59	0.51	0.43	0.24	0.12	0.12	0.12	0.12	0.12	0.12

According to the computation, the revenue matrix under different possible price combinations is created. As in the revenue matrix, lower and upper bounds for both base price and peak price are set from £1 to £17, by the unit of £1. So, it is much clearer that, for each possible base price point of non-peak periods, the revenues increase with the peak price going higher mainly from £1 to £10. However, for each certain base price, when the corresponding peak price goes higher than £10, the revenues stop growing, which means there is no more clients would pay for the higher charge. In addition, for each possible price point of peak periods, the revenues increase with the non-peak price going higher mainly from £1 to £9. However, for each certain peak price, when the corresponding base price goes higher than £9, the revenues cease to grow, which means there is no more clients would pay for the higher charge. In general, the dynamic study on both peak and non-peak prices is very necessary. It provides more possible solutions for potential congestion pricing strategies, which might be taken into consideration when it comes to different real-world scenarios and policy constraints. From another side, the revenue matrix gives the study a new view, where interesting price combinations of high base price and low peak price are presented, although it is not reasonable that low peak price would ease the traffic congestion. Exceptions exist, when it comes to certain celebration or ceremony holidays. To be specific, for these days, the traffic tends to be heavy due to tourists and people going for visit. Rather than increase peak price in these periods, the government would be more willing to lower down traffic charges for clients, in which way the motivation to travel would be stimulate and consumption would boost as well.

Therefore, making no difference between peak and non-peak prices or timely implementing lower peak price can be an efficient way to facilitate economy growth in holidays.

For this study, exploring extra room for revenue improvement is the main target. Following the given methodology, the view is extended from sample respondents into the total potential drivers, from which the extent of how revenue would be influenced by different price combinations in a macro way can be presented more accurately. According to demands of peak and non-peak prices, revenues are recorded. The maximum total revenue is £1,363,365, when the optimal price combination is £9 for peak price and £8 for non-peak price, which is consistent with what is showed in the revenue matrix. So, in this dynamic model, the optimal solution for revenue maximization exists regardless of emissions constraint.

Compared with one single congestion charge for both peak and non-peak periods, the maximum total revenue increases from £1,349,008 to £1,363,365 by £14,357, while the total level of emission climbs from 49,323,676 g/km to 55,767,736 g/km by 6,444,060 g/km. That is to say, peak and non-peak prices can be flexibly set for the purpose of higher revenue, which would ensure the programme self-sustainability and a sufficient portion of revenue to reinvest in the public transportation infrastructure. However, while the programme is ongoing, the total level of emissions is uncontrolled. It means that constraints on emission should be set further and meanwhile the revenue should not fall below a certain level if the programme's objective is more to minimize emissions rather than maximizing revenue. Therefore, for the optimal price combination to balance both emission minimization and revenue maximization, a more accurate algorithm should be designed to explore more possible choices with the primary objectives of this study maintained.

Based on the study above, parametric approach is introduced. This parametric analysis can produce reliable results even when continuous data are non-normally distributed. Following this method base price is set at £7 for the purpose of further comparison with previous findings. The main steps of this method are as below:

1. Construct model for peak price.

2. Record demands for peak and non-peak periods by fitted linear model.

3. Identify maximum revenue based on recorded demands and corresponding prices.

4. Compute optimal peak price.

After the process of data, the optimal peak price is £10 while the total revenue is £1,226,317 which decreases by £145,509, compared with previous finding, £1,371,826, where the base price is also set at £7 for revenue maximization. As a result of the total level of emissions, it decreases by 12,498,284 g/km, from 46,241,535 g/km to 33,743,251 g/km. In conclusion, this optimal solution has disadvantage of lower total revenue but advantage of less emissions. If the programme self-sustainability can be guaranteed, less emissions will be the important objective for which the government implement congestion charge programme. Besides, compared with another previous study where base price is set at £7 for emission minimization, a lower total level of emissions can be achieved in sacrifice of total revenue. Therefore, it is vital to know how effective the strategy works in the real world, from which suitable minimum revenue can be confirmed for the ongoing reinvestment and minimum emissions.