Tutorial 2 Problems

for discussion on Week 46

Conditional, Full probability and Bayes formula

1. What is the probability that the sum of numbers shown on two symmetric dice is 8 if we know that the sum is even? The same question, given the sum is odd.

2. Problem 1.7.1 from GS[*]

3. There are N lottery tickets lying on the table, among them $n < N$ are winning. The first person comes and picks one ticket at random, followed by the second and the third. Who has the largest chance of getting a winning ticket: the first, second or third person? Answering this question may help you to decide when it is better to come to an exam where you supposed to take a variant at random and you know answer to only 'lucky' few. Waiting could mean that your lucky tickets been already taken by other students who came earlier...

4. Work out the VLE studies 1.2-1.4.

Independence

1. Problem 1.5.1 from GS

2. Problem 1.8.7 from GS.

3. Problem 1.5.7 a)-c) from GS.

Distribution and its description

1. Work out Studies 2.1, 2.2 and 3.1 on the VLE.

2. Let $[X, \mathcal{X}]$ be a measurable space and let f be a mapping from some set Ω into X. Show that the system of inverse images $\{f^{-1}(B): \; B \in \mathcal{X}\}$ is a σ-field.
   Hint: What are $f^{-1}(B_1 \cap B_2)$ and $f^{-1}(B_1 \cup B_2)$?

[*]In what follows, GS stands for the course book: Geoffrey Grimmett and David Stirzaker, *Probability and Random Processes*, Oxford University Press, 3rd edition, 2001. Problem x.y.z means Problem z for Section x.y in this book

3. The σ-field in the previous example is called *generated* by the r.v. ξ. Describe the σ-fields generated by a constant and by a random variable taking two values: 0 and 1.

4. Show that any mapping $\Omega \mapsto \mathbb{R}$ is always measurable with respect to 2^Ω – the σ-field of its all subsets. This would imply that on discrete spaces when we usually consider 2^Ω as a standard σ-field, any mapping to $\mathbb{R}$ is a random variable.

5. The following example shows that random variables defined on different sample spaces may however have the same distribution. Consider rolling a symmetric die. An evident sample space is $\Omega = \{1, 2, 3, 4, 5, 6\}$ and ξ_0, the number shown on the face of the die, is the identity mapping into $\mathbb{R}$: $\xi_0(k) = k$ for $k \in \Omega$. Take another sample space $\Omega = [0, 1]$ with Borel σ-field of its subsets and the probability measure $\mathbf{P}$ which is the restriction of the Lebesgue measure onto $[0, 1]$ (so just the ordinary length). Define $\xi_1(\omega) = [6\omega]+1,\ \omega \in [0, 1]$, where $[x]$ is the integer part of x. Check that the distributions $\mathbf{P}_{\xi_0}$ and $\mathbf{P}_{\xi_1}$ are the same measures on the Borel sets of $\mathbb{R}$. What is this measure?

6. A random variable ξ has equal chances to take the following 3 values: $-1, 0$ and 1. Find its Probability Mass Function (p.m.f.), draw its Cumulative Probability Function (c.d.f.) and find p.m.f.'s for the following random variables:

   (a) $|\xi|$;

   (b) $\xi^2 + 1$;

   (c) 2^ξ;

   (d) $\xi_+ = \max\{\xi, 0\}$.

7. Problem 2.3.3 from GS. This property is the basis of computer simulations of random variables known as Monte-Carlo methods.