MLCS - Homework 2

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1 EF-games and (non)-expressibility

Exercise 1.6. Consider the following two graphs:

1. $\mathfrak{G}_1$ is a line of length $4n$;
2. $\mathfrak{G}_2$ consists of a line of length $2n$ and a cycle of length $2n$ (the two components are disjoint).

Analyze the EF games on these structures for $n = 1, 2$ and write a sentence of minimal quantifier rank distinguishing the two structures for $n = 1, 2$.

BONUS: Formulate a generalization of your observations and prove that the Acyclicity query is not expressible in the language of graphs over finite graphs, using EF-games.

Solution. Let's discuss every point separately.

Part 1: $n = 1$.

For $n = 1$, $\mathfrak{G}_1$ is a line of length 4, while $\mathfrak{G}_2$ is the disjoint structure composed by a line of length 2 and a cycle of length 2.

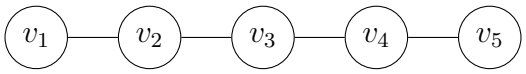

Figure 1: Graph $\mathfrak{G}_1$, consisting of a line of length 4.

Figure 2: Graph $\mathfrak{G}_2$, consisting of a line of length 2 and cycle of length 2.

Part 2: $n = 2$.

For $n = 2$, $\mathfrak{G}_1$ is a line of length 8, while $\mathfrak{G}_2$ is the disjoint structure composed by a line of length 4 and a cycle of length 4. □

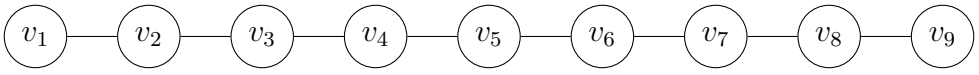

Figure 3: Graph $\mathfrak{G}_1$, consisting of a line of length 8.

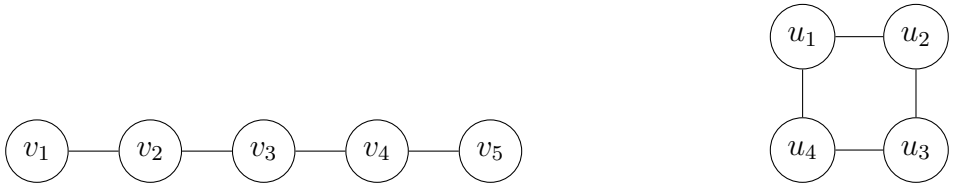

Figure 4: Graph $\mathfrak{G}_2$, consisting of a line of length 4 and cycle of length 4.