Chapter 10
Applications in geosciences

10.1 ▪ Seismology and exploration geophysics

10.1.1 ▪ Presentation of the domain

In geophysics in general, and seismology[33] in particular, the objective is to obtain information about the earth's internal structure. The unknown earth structure is represented by a model, $\mathbf{m}$; the synthetic data by $\mathbf{u}(\mathbf{m})$; and the measurements by $\mathbf{u}^{\mathrm{obs}}$ (see Section 2.3). The signals are generated either by natural sources, i.e., earthquakes and other seismic activity, or by artificial sources that usually take the form of explosives, pneumatic hammers, or boomers (used in underwater seismic exploration). The reflected signals are measured by seismic sensors known as geophones (hydrophones in underwater acoustics—see below). Recently, there have been interesting attempts to use the earth's background seismic noise as the input source—see Shapiro et al. [2005], Larose et al. [2008], Verbeke et al. [2012], Daskalakis et al. [2016], and Zigone et al. [2015]. The model is some form of wave equation [Aki and Richards, 2002; Courant and Hilbert, 1989b; Brekhovskikh, 1980] that describes the propagation of acoustic, elastic, or even electromagnetic waves through the earth's crust.

The inverse problem can be solved by either deterministic or statistical/probabilistic approaches. The statistical approach was formalized by Tarantola [2005]. The deterministic approach, based on adjoint methods, is described in the excellent book of Fichtner [2011]. Other, more classical approaches that are still employed in the petroleum industry are based on a range of diverse stacking and migration methods—interested readers should consult Claerbout [1976, 1985] and Aki and Richards [2002].

To quote J. Claerbout: "There is not one theory of inversion of seismic data, but many—maybe more theories than theoreticians." (see his website at `http://sep.stanford.edu/sep/prof/`, where all his monographs are freely available).

The geophysical images that are computed by inversion techniques are then used to visualize petroleum and mineral resource deposits, subsurface water, contaminant transport for environmental pollution studies, archeology, etc.

[33] The study of the propagation of elastic waves in and through the earth's crust—often in relation to earthquakes.

10.1.2 ▪ Examples of DA problems in this context

10.1.2.1 ▪ Petroleum or seismic prospecting

Oil and gas exploration remains one of the major areas for inversion techniques. Often this is coupled with reservoir modeling—see Wikipedia [2014]. The expenditure on seismic data acquisition campaigns is estimated at billions of dollars per year [Bret-Rouzaut and Favennec, 2011]. The subsequent data processing costs alone, can amount to $1 million for a single seismic survey. There is clearly a need for reliable and efficient inversion methods and algorithms to process all this data and to generate meaningful images.

There is a lot of information to be found on the websites of the numerous commercial companies, either oil and gas or those that provide services to oil and gas exploration. Among these are Schlumberger, Total, CGG, Statoil, and Ikon.

Recent research on this subject has addressed more exotic aspects, such as complex geological structures [Liu et al., 2006], joint inversions [Gyulai, 2013], combination of Bayesian inversion with rock physics [Grana and Della Rossa, 2010], stochastic (Monte Carlo Markov chain) methods [Martin et al., 2012], high-dimensional seismic inverse problems [Zhang et al., 2016], and full waveform inversion (FWI; see below).

10.1.2.2 ▪ Geological prospecting

For geological and mineral prospecting, magnetic and electromagnetic approaches are most often employed. In time domain electromagnetics (TDEM), electric impulses are used to induce transient electric and magnetic fields. Recent references on this approach are Newman and Commer [2005] and Haber et al. [2007].

10.1.2.3 ▪ Earthquake detection and analysis

Detection and reconstruction (by simulation) of earthquakes and other seismic events is a vital domain for today's research and for decision-making by public authorities. In particular, there has been recent progress toward the goal of early detection and thus prediction of earthquakes (see below for the case of volcanic eruptions). In fact, a lot of hope has recently been placed in the ability of background seismic noise monitoring and inversion as a promising avenue. The first major paper on this subject was Shapiro et al. [2005], even though the statistical inversion technique was inspired by the earlier acoustics work of Lobkis and Weaver [2001]. These results were subsequently justified mathematically by Bardos et al. [2008] and have been applied in many concrete cases—see Rivet et al. [2015], Frank and Shapiro [2014], Campillo et al. [2011], Yao et al. [2011], Stehly et al. [2009], and Lin et al. [2008].

Note that seismic detection is also widely used for monitoring nuclear tests. The recent explosions in North Korea are a good example [Wikipedia, 2016a].

10.1.2.4 ▪ Volcanic eruption detection and analysis

The Whisper project[34] was a trailblazer in the utilization of ambient seismic noise to monitor property changes in the solid earth. They were the first to succeed in using seismic ambient noise recordings to extract deterministic signals that led to imaging of the earth's interior at high resolution. Their main goal was to apply the proposed noise-based inversion methods to study the transient processes related to volcanic and

[34] `https://whisper.obs.ujf-grenoble.fr`.

tectonic activity through continuous measuring of mechanical changes in deep parts of the earth. One of the main research targets was active volcanic regions. For studies of volcanoes, they processed data from the La Réunion vulcanological observatory and from other volcanic areas in Japan, the U.S. Pacific Northwest, etc.

Numerous striking results have been obtained. We can cite Mordret et al. [2015], Droznin et al. [2015], Obermann et al. [2013], Brenguier et al. [2008, 2007], and many more references that can be found on the project's website.

10.1.3 ▪ Focus: An adjoint approach for FWI

As explained in Fichtner [2011], FWI has recently been applied in seismic tomography. The method is characterized by the numerical simulation of the elastoacoustic vector wave equation [Aki and Richards, 2002; Landau and Lifschitz, 1975],

$$\rho \boldsymbol{u}_{tt} - (\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u}) - \mu \nabla^2 \boldsymbol{u} = \boldsymbol{f}, \tag{10.1}$$

where $\boldsymbol{u} = (u_1, u_2, u_3)^T$ is the displacement in the x-, y- and z-directions; λ and μ are the Lamé coefficients; ρ is the medium density; and $\boldsymbol{f}$ is an initial impulse that represents the acoustic source. The relations between these coefficients and the wave speeds are

$$c_p^2 = \frac{\lambda + 2\mu}{\rho} \qquad \text{and} \qquad c_s^2 = \frac{\mu}{\rho},$$

where c_p is the pressure (or primary) wave speed and c_s is the shear (or secondary) wave speed. The physical domain of interest is subdivided into layers that are either geological (rock, sand, etc.) or water. Thus, an acoustic layer is obtained in the model by simply setting $\mu = 0$ locally.

The advantage of the system (10.1) is that it intrinsically models all the different types of waves that can arise in layered media—compressional and shear waves in the bulk and Love, Stoneley, and Rayleigh waves along the interfaces. These equations must be completed with physically relevant boundary and initial conditions. On the surface we usually specify a zero pressure condition. Between layers, continuity conditions on the normal components of u and the stresses, which are related to the gradient of u by Hooke's law, must be satisfied. At the bottom-most level we give a suitable absorbing condition. On the lateral boundaries, suitable absorbing/radiating conditions need to be specified [Komatitsch and Martin, 2007; Xie et al., 2014]. The initial condition is usually a Ricker wavelet (`http://subsurfwiki.org/wiki/Ricker_wavelet`), with the desired frequency content, located at the source position—other initial conditions are possible. The geophones (or other measurement devices) are simulated by simply recording the solution at those points that correspond to their locations.

Numerical solutions of the above system enable accurate modeling of seismic wave propagation through heterogeneous, realistic, geophysical earth models. When these direct solutions are combined with adjoint methods, exactly as presented in Chapter 2, we obtain excellent tomographic resolution of the subsurface structure. Numerous real-world applications can be found in the last three chapters of Fichtner [2011] and also in Tape et al. [2009], Virieux and Operto [2009], Peter et al. [2011], Monteiller et al. [2015], and Fichtner and Villasenor [2015].

10.2 ▪ Geomagnetism

10.2.1 ▪ Presentation of the domain

Dynamo theory [Wikipedia, 2015b] describes the generation of a magnetic field by a rotating body. The earth's dynamo (also called the geodynamo) is a geomagnetic source that gives rise to long-term variability—this is known as geomagnetic reversal, or "flip." On shorter time scales, of about one year, there are changes in declination. The understanding of the dynamical causes of this variability, based on available data, is a long-standing problem in geophysics with numerous fundamental and practical implications. DA has been applied to this problem thanks to the increasing quality and quantity of geomagnetic observations obtainable from satellites, coupled with our ability to produce accurate numerical models of earth-core dynamics. First attempts were based on the optimal interpolation approach [Kuang et al., 2008]. Reviews can be found in Fournier et al. [2010], Kuang and Tangborn [2011], and most recently Hulot et al. [2015].

Depending on whether we seek to understand fast or slow variability, we can resort to two-dimensional (quasi-geostrophic) or three-dimensional numerical models to assimilate geomagnetic observations. The former are well suited for describing the fast variability and are less demanding in resources. The latter are more complete and able to correctly represent the variability on longer time scales, but the computational cost is higher. More details can be found in Canet et al. [2009] and Aubert and Fournier [2011].

10.2.2 ▪ Focus: An EnKF for time-dependent analysis of the geomagnetic field

In Fournier et al. [2013], an ensemble Kalman filter (EnKF; see Chapter 6) is used to assimilate time-dependent observations based on a three-dimensional dynamo model. Twin experiments led to the choice of an ensemble size equal to 480 members. Forecasting capabilities of the assimilation were promising but required a full observation error covariance matrix to ensure good results in some cases. Note that the state vector in the calculations had a dimension of almost 10^6.

10.3 ▪ Geodynamics

10.3.1 ▪ Presentation of the domain

In earth sciences the knowledge of the state of the earth mantle and its evolution in time is of interest in numerous domains, such as internal dynamics, geological records, postglacial rebound, sea level change, ore deposit, tectonics, and geomagnetic reversals. Convection theory is the key to understanding and reconstructing the present and past state of the mantle. For the past 40 years, considerable efforts have been made to improve the quality of numerical models of mantle convection. However, these models are still sparsely used to estimate the convective history of the solid earth, especially when compared to ocean or atmospheric models for weather and climate prediction. The main shortcoming is their inability to successfully produce earth-like seafloor spreading and continental drift in a self-consistent way.

The ultimate goal is to reconstruct the deep earth from geological data and thus be able to image the deep earth from ancient times up until today. This is termed *hindcasting* and is simply forecasting in the direction of decreasing time; i.e., we are "predicting" the past.

10.3.2 ▪ Examples of DA problems in this context

In Coltice et al. [2012], convection models were used to predict the processes of seafloor spreading and continental drift. The results obtained prove that the combination of high-level DA methodologies and convection models together with advanced tectonic datasets can retrieve the earth's mantle history.

There is now hope to understand the causes of seismic anomalies in the deep earth. Another application is the understanding of geomagnetic phenomena—in particular the evolution of the earth's magnetic field (see above). In fact, knowledge of the earth's paleogeography has potentially wide-ranging applications in water resource research, mineral resource research, and even paleontology.

10.3.3 ▪ Focus: Sequential DA for joint reconstruction of mantle convection and surface tectonics

An extended Kalman filter (EKF) was employed in Bocher et al. [2016] and was able to recover the temperature field of a convective system with plate-like tectonics at its surface over several hundred megayears. The only observations used were surface heat fluxes and surface velocities.