A Distributed, Non-Oscillatory Ideal Magnetohydrodynamics Framework

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Introduction

Magnetohydrodynamics (MHD) is a mathematical framework that predicts the behavior of plasmas within certain physical limits. Our goal was to build a **one-dimensional MHD solver** based on the work of Balbas, et al (2004). The **computational significance** of Balbas's model is that it minimizes numerical diffusion and **without** relying on **Jacobian matrices**, thanks to a **staggered, predictor central differencing scheme**. We validate the model's accuracy by verifying our results against prior work and have gained significant physical insight into MHD wave propagation and confinement.

Scientific Questions:
- How does plasma behave in a quasi-1D Slow Shock? (Falle et al., 1998)
- How are slow shock plasma dynamics modified for a fast Shock Tube? (Brio & Wu, 1988)
- How do rarefaction waves propagate in a MHD plasma? (e.g., Falle et al., 1998)
- What is the mechanism for plasma confinement in the Tokamak reactor torus? (e.g., Chance et al., 1982)

Model

Our solver tracks the evolution of the following variables: **density**, **velocity**, **magnetic field**, and **energy**. We update the values of these quantities on the staggered mesh according to the following equation:

$$\bar{Z}^{n+1}_{j+\frac{1}{2}} = \frac{1}{\Delta x}\int_{I_{j+\frac{1}{2}}} Z(x,t_n)\,\mathrm{d}x - \frac{1}{\Delta x}\left[\int_{t^n}^{t^{n+1}} f(Z(x_{j+1},t))\,\mathrm{d}t - \int_{t^n}^{t^{n+1}} f(Z(x_j,t))\,\mathrm{d}t\right]$$

Key features of our code include:
- The **MinMod** function, which selects the minimum choice from centered, upwind and downwind methods of computing derivatives to ensure stability. MinMod relies on a parameter α, which we vary (see right) to investigate its influence on the propagation of physical quantities

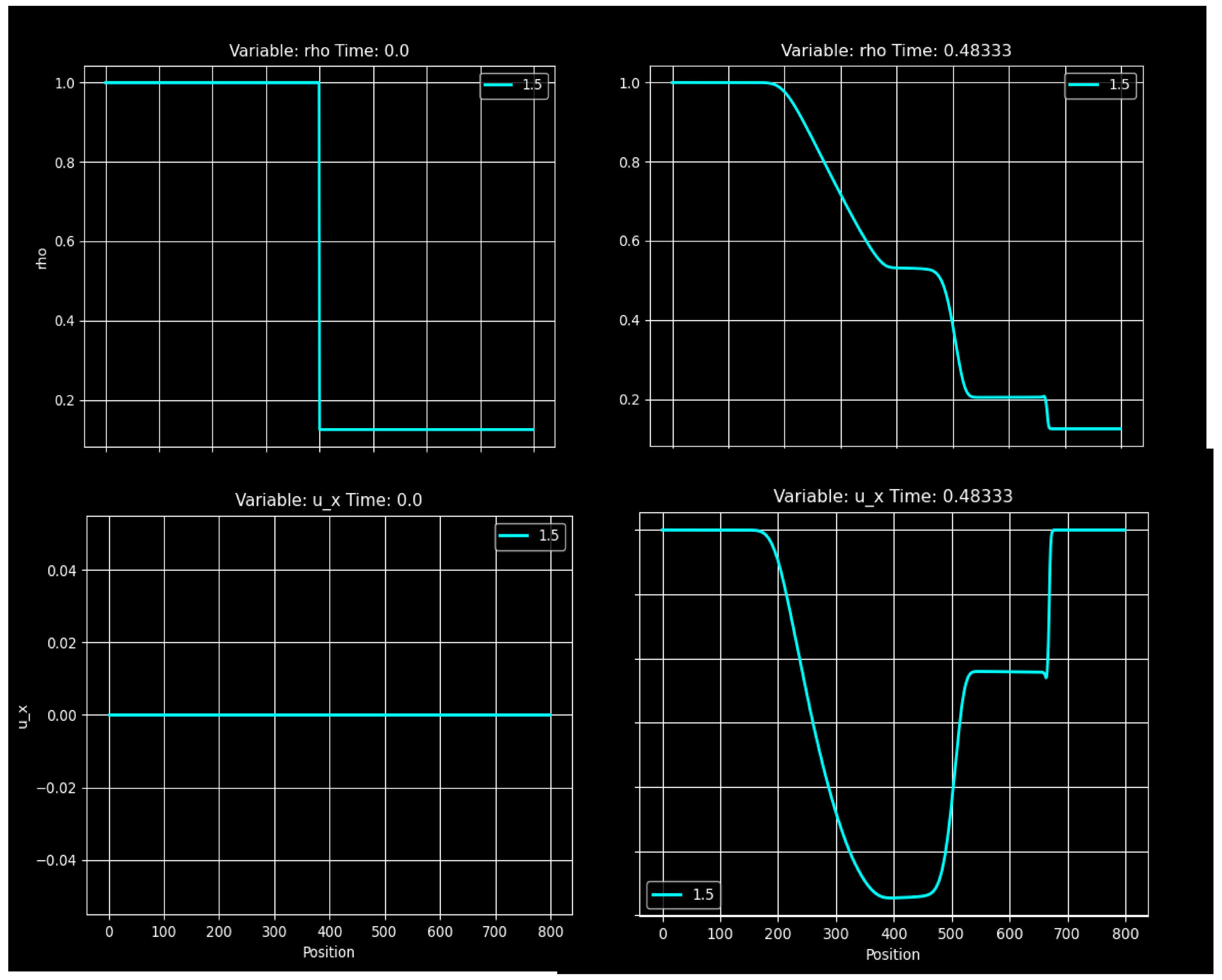
$$\mathrm{MinMod}(a,b,c)=\mathrm{sign}(a)\min(|a|,|b|,|c|) \qquad w'_j = \mathrm{MinMod}(\alpha\Delta_+\bar{w}_j, \Delta_0\bar{w}_j, \alpha\Delta_-\bar{w}_j), \quad 1 \leqslant \alpha < 4.$$

- A single array object containing quantities of all variables on *both meshes* (staggered and un-staggered) over the whole domain
- Using the fluxes of quantities at the *gridpoints*, a (n^{th}-order) polynomial is used to reconstruct the values of the quantities at the *midpoints* (i.e., on the staggered grid)
- An **Animate** function that allows us to post-process the output, generating plots for a variety of test cases
- Integration with joblib's Parallel library, drastically decreasing runtime

Validation: Sod Shock Tube

A **shock tube** is the name for a class of systems in which a sharp change in the value of one of the parameters generates a force which drives evolution in a fluid. The **Sod Shock Tube** is such a system in which the magnetic field is zero, i.e., we are treating the plasma as a **hydrodynamic (unmagnetized) fluid.**

In our setup, the initial density jumps sharply, and this pressure gradient drives flow. This shock tube a helpful in validating the model, as it relies on simple initial conditions produces a variety of features.



Other Test Cases

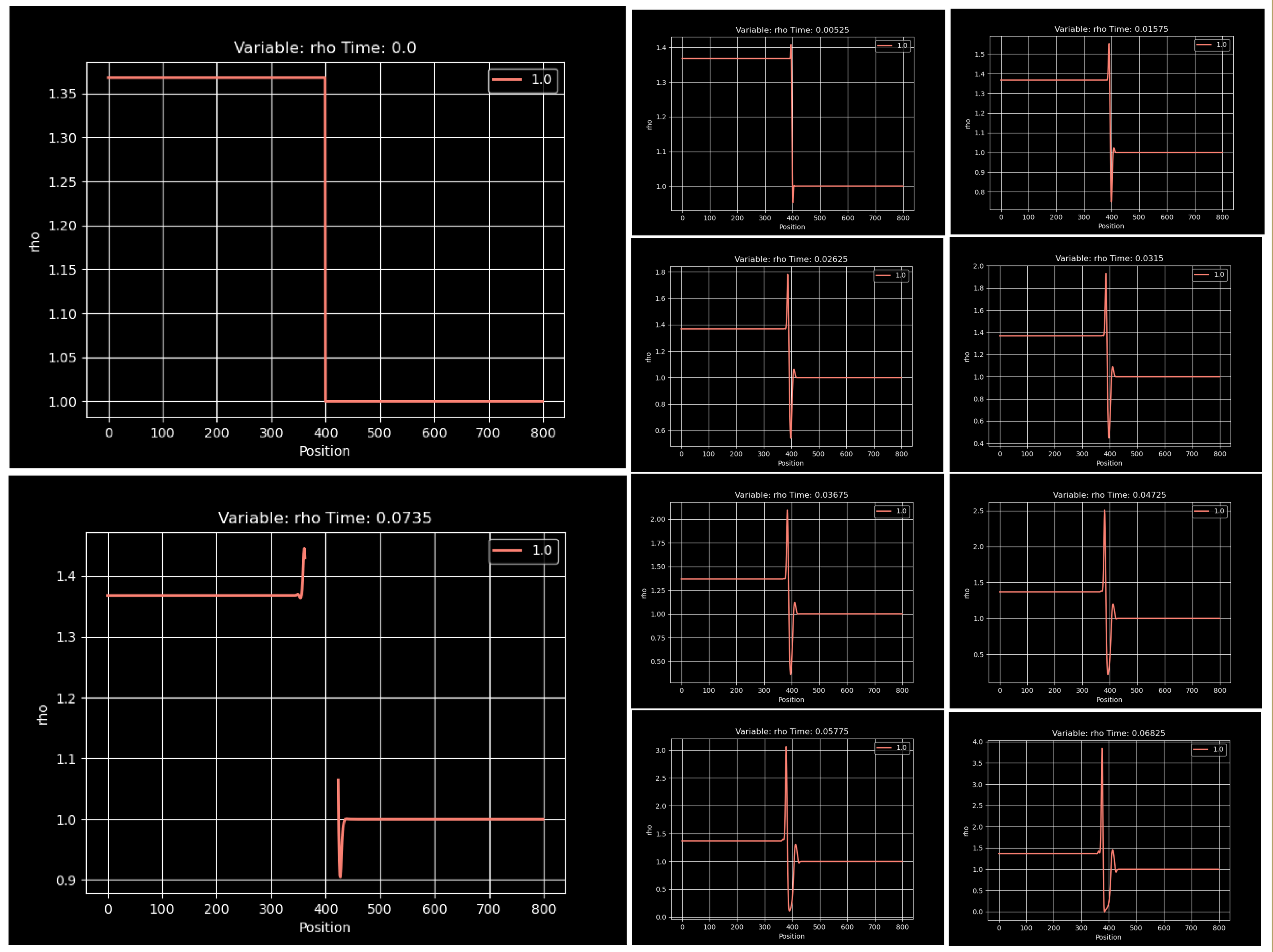
The Slow Shock Wave

The slow shock wave: a propagating surface across which all physical parameters jump- the shock front converts kinetic energy into thermal energy, generating entropy
Sharp gradients are slow to propagate outward => this specific shock causes the model to diverge, no matter the value of the stability parameter α.

Two solutions for future consideration are:
- Introducing an adaptive timestep
- A second-order polynomial reconstruction

Left State $(x<0)$:
$\rho_L = 1.386,\ p_L = 1.769,\ \vec{u}_L = (0.269, 1.0, 0.0),\ \vec{B}_L = (1.0, 1.0, 0.0)$

Right State $(x \geq 0)$:
$\rho_R = 1.0,\ p_R = 1.0,\ \vec{u}_R = (0.0, 0.0, 0.0),\ \vec{B}_R = (1.0, 1.0, 0.0)$

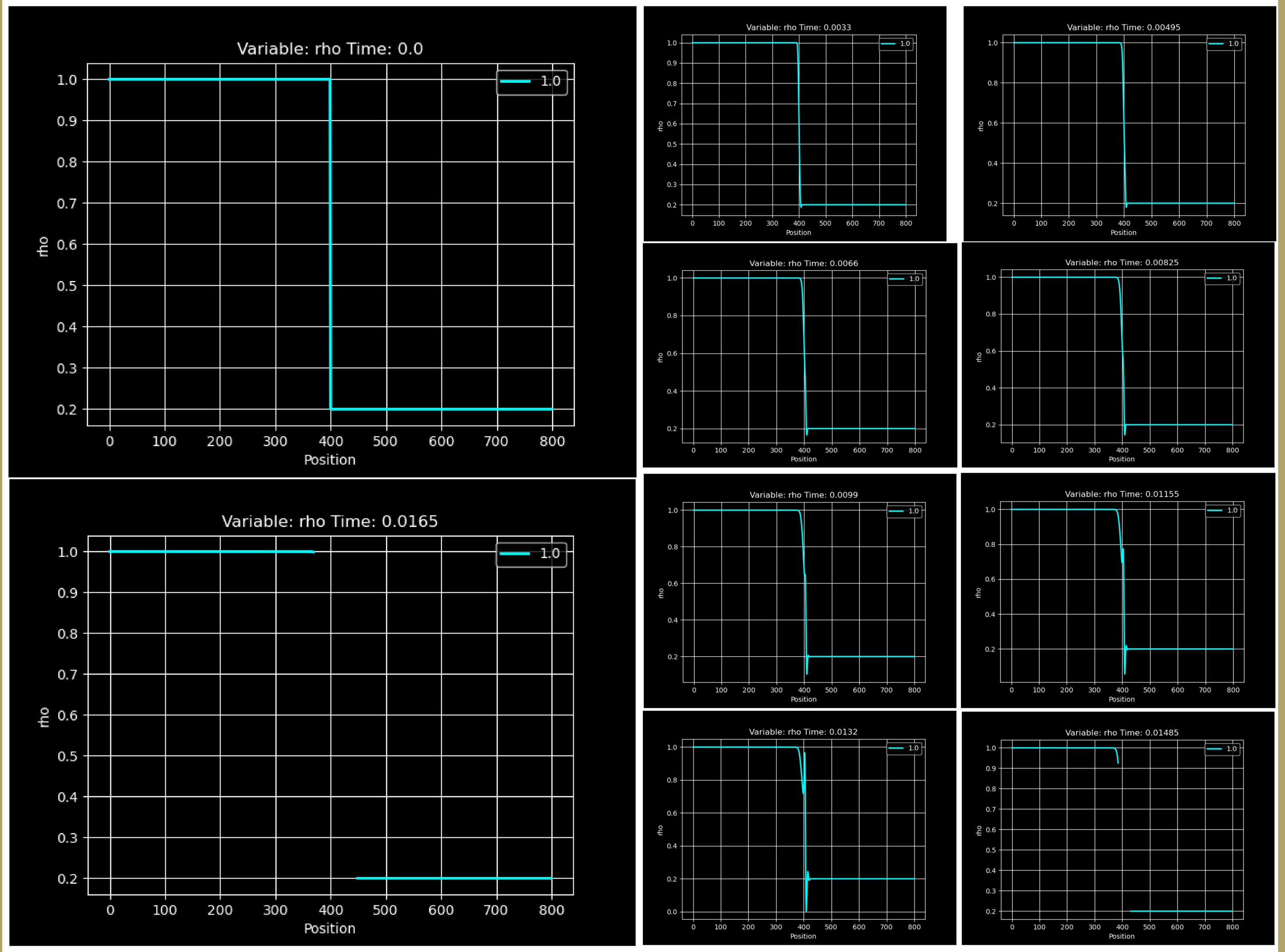


The Rarefaction Wave

The Rarefaction Wave: the progression of particles being accelerated away from a compressed/shocked zone
Propagates *away* from the direction of the acceleration of the particles
Unlike shocks: wave mode disperses over space & time

Left State $(x<0)$:
$\rho_L = 1.0,\ p_L = 2.0,\ \vec{u}_L = (0.0, 0.0, 0.0),\ \vec{B}_L = (1.0, 0.0, 0.0)$

Right State $(x \geq 0)$:
$\rho_R = 0.2,\ p_R = 0.1368,\ \vec{u}_R = (1.186, 2.967, 0.0),\ \vec{B}_R = (1.0, 1.6405, 0.0)$



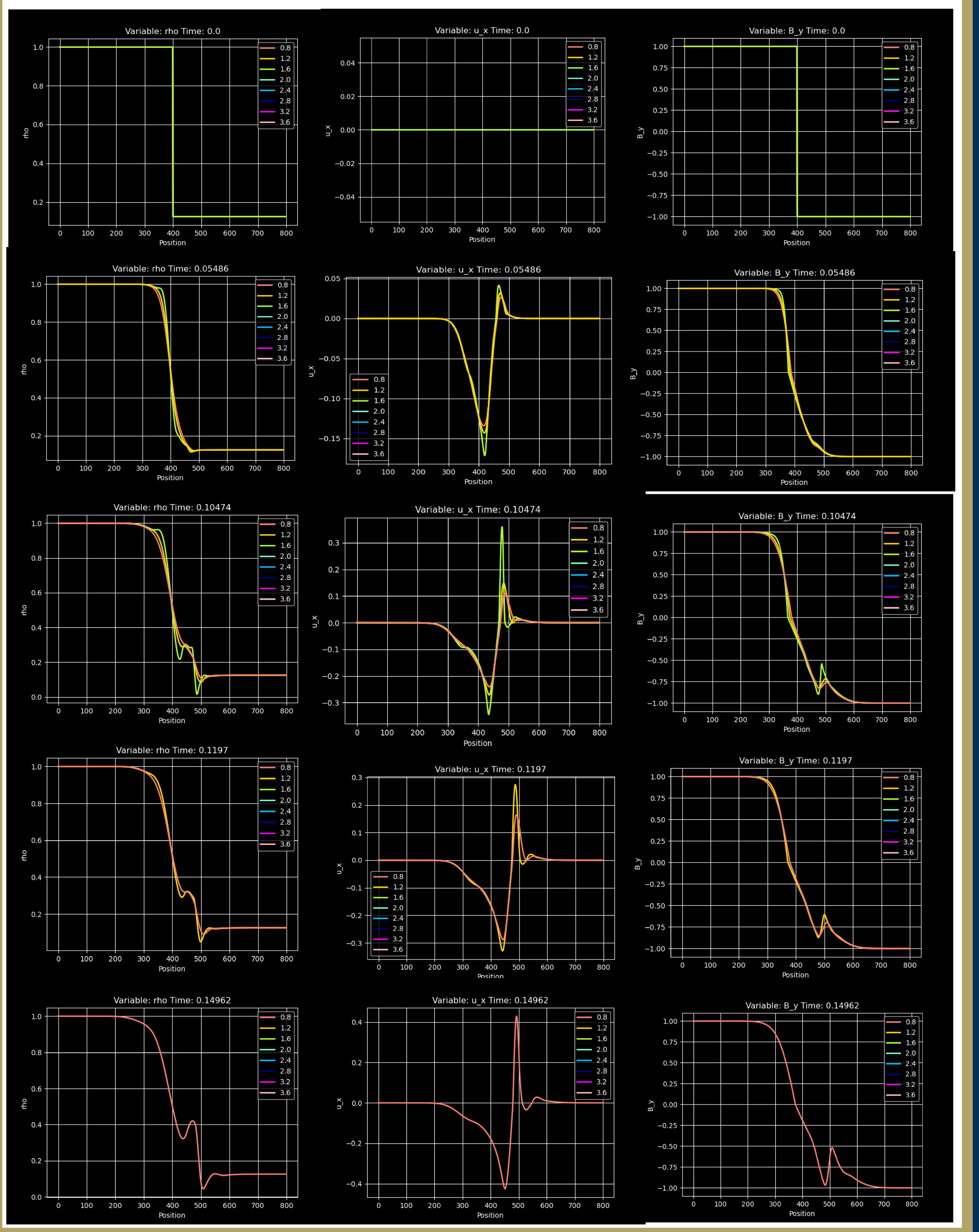
Alpha – Time Evolution Study

Brio-Wu Shock Tube

The Brio & Wu shock tube initializes the system in the same state as in the sod shock tube, only this time the magnetic field is nonzero. We test the evolution for many α values, demonstrating how lower values increase stability.

Left State $(x<0)$:
$\rho_L = 1.0,\ p_L = 1.0,\ \vec{u}_L = (0.0, 0.0, 0.0),\ \vec{B}_L = (0.75, 1.0, 0.0)$

Right State $(x \geq 0)$:
$\rho_R = 0.125,\ p_R = 0.1,\ \vec{u}_R = (0.0, 0.0, 0.0),\ \vec{B}_R = (0.75, -1.0, 0.0)$



Future Work

In addition to the those presented, we also have promising results for a comparison to another Riemann problem (Dai & Woodward, 1998). Our next task from the proposed work involves applying this MHD model to investigate plasma dynamics in a Tokamak nuclear reactor. Other improvements include:
- Experimenting with different types of boundary conditions
- Addressing numerical diffusion near discontinuities
- Extending the model to higher dimensions

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