EEE 203 Continuous and Discrete Time Signals and System I

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1. Introduction

Electronic filter is a circuit consists of capacitance, inductance and resistance. This device intends to effectively remove the unwanted signals. In the field of electronic technology, the wave is commonly defined to describe process that values of various physical quantities fluctuate over time. Due to the fact that signals could be interfered or changed by environments in the stages of generating, transforming or transferring, which could cause serious problems. It is therefore the filter could avoid the above-mentioned problems. However, there are numerous type of filters could be used depending on different condition. For example, there has low-pass filter, high-pass filter, band-stop filter, band-pass filter and all-pass filter. Regarding to the low-pass filter, it is designed to allow the low frequency pass through and filter the high frequency so the noises could be avoided. As for the high-pass filter, it allows the high frequency pass through and filter the low frequency.

During this experiment, it requires to firstly build a low-pass filter and then build a high-pass filter. Consequently, a band-pass filter would be constructed. Finally, oscilloscope would be used to measure the waveforms (characteristics) of each filter.

2. Theory

1) Express the following values of the voltage gain in dB:

Gain	1	1/2	$1/\sqrt{2}$	$\sqrt{2}$	2	10	100
dB	0	-6.021	-3.01	3.01	6.021	20	40

Implement the relationship: $\mathrm{dB} = 20\log_{10} A_i$

2) Give formulae for (a) the transfer function, (b) the frequency response, (c) the gain response and (d) the phase response of a

second order Butterworth low-pass approximation with cut-off frequency 4 rad/sec.

Here, ω_c is 4 rad/sec and $\sqrt{2} = 1.414$

Transfer Function:

$$\mathrm{H(s)} = \frac{1}{1+\sqrt{2}\left(\frac{s}{\omega_c}\right)+\left(\frac{s}{\omega_c}\right)^2} = \frac{1}{1+1.414\left(\frac{s}{\omega_c}\right)+\left(\frac{s}{\omega_c}\right)^2}$$

$$\mathrm{H(s)} = \frac{1}{1+\sqrt{2}\left(\frac{s}{4}\right)+\left(\frac{s}{4}\right)^2} = \frac{1}{1+1.414\left(\frac{s}{4}\right)+\left(\frac{s}{4}\right)^2}$$

Frequency Response:

$$\mathrm{H}(j\omega) = \frac{1}{1+\sqrt{2}\left(\frac{j\omega}{\omega_c}\right)+\left(\frac{j\omega}{\omega_c}\right)^2} = \frac{1}{1+1.414\left(\frac{j\omega}{\omega_c}\right)+\left(\frac{j\omega}{\omega_c}\right)^2}$$

$$\mathrm{H}(j\omega) = \frac{1}{1+\sqrt{2}\left(\frac{j\omega}{4}\right)+\left(\frac{j\omega}{4}\right)^2} = \frac{1}{1+1.414\left(\frac{j\omega}{4}\right)+\left(\frac{j\omega}{4}\right)^2}$$

Gain Response:

$$\mathrm{G}(\omega) = \frac{1}{\sqrt{1+(\frac{\omega}{\omega_c})^{2n}}}$$

$$\mathrm{G}(\omega) = \frac{1}{\sqrt{1+(\frac{\omega}{4})^{2n}}}$$

Here, n is equal 2.

Phase Response:

$$\phi(\omega) = \arctan\left(\frac{Im(H(j\omega))}{Re(H(j\omega))}\right) + (\pi, \text{if } Re(H(j\omega)) < 0$$

$$= \begin{cases} \arctan\left(\dfrac{-4\sqrt{2}\omega}{16-\omega^2}\right) + \pi, & \omega \geq 4 \\ \arctan\left(\dfrac{-4\sqrt{2}\omega}{16-\omega^2}\right), & 0 < \omega < 4 \end{cases}$$

3) **Complete the missing entries in the following table for the 4 rad/sec cut-off low-pass filter referred to above:**

$\omega(rad/sec)$	0	0.5	1.0	1.5	4.0	10.0	20.0
Voltage gain	1	0.999	0.998	0.990	0.707	0.158	0.040
Gain in dB	0	-0.001	-0.017	-0.085	-3.012	-16.027	-27.966
Phase lead (deg)	0	-10.181	-20.663	-31.679	-90	-146.05	-163.58
Phase lag (rad)	0	0.178	0.361	0.553	1.571	2.549	2.855
Phase delay (sec)	0	0.356	0.361	0.369	0.393	0.2549	0.14275

4) **Referring to the table above, at frequencies greater than 4 radians/sec what is the reduction in gain in dB per octave (i.e. per doubling of frequency)?**

According to the table, when ω is at 10 rad/sec, the gain in dB is -16.027; when ω is at 20 rad/sec, the gain in dB is -27.959. Finally, the reduction in gain in dB is -27.966-(-16.027) = -11.939.

5) **At frequencies greater than 4 radians/sec what is the reduction in gain in dB per decade (i.e. increasing the frequency 10 times)?**

   When the ω is at 100 rad/sec, the gain could be calculated which is about -55.9 dB. It is therefore the reduction in gain in dB per decade = -55.9+16.027 = -39.873.

6) **A 1 radian/second sine wave will be delayed by about 0.8 seconds. TRUE/FALSE?**

   False. According to the table above, when the sine wave is 1 rad/sec, the delay is the phase delay which is 0.361s.

7) **Is the frequency response linear phase? YES/NO?**

   No. When the frequency is low, the frequency response could be assumed as linear phase. When the frequency is high, the frequency response could be assumed as nearly 0.

8) **Why is a linear phase response desirable?**

   Because in the real world, any physical devices system could have delay, it is common for signals to have phase delays after being through the filter. Linear phase response could guarantee that every frequency has the same phase delay and the relative phase relationships between each frequency remains fixed or unchanged. In conclusion, a linear phase

9) **Butterworth low-pass filters of all orders have –3dB gain (relative to their gain at 0 Hz) at $\boldsymbol{\omega = \omega_c}$. TRUE or FALSE?**

   True. According to the formula:

$$\mathrm{G}(\omega) = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2n}}}$$

   And $\omega = \omega_c$, so:

$$G(\omega) = \frac{1}{\sqrt{2}}$$

Then the voltage gain in dB is:

$$20\log_{10}(G(\omega)) = -3.01\text{dB}$$

10) Cascading three identical 2nd order Butterworth high-pass filters produces a 6th order Butterworth high-pass filter. TRUE or FALSE?

False.

The transfer function for 2nd order Butterworth high-pass filter:

According to the formula:

$$H(s) = \frac{s^2}{s^2 + 1.414\omega_c s + {\omega_c}^2}$$

The transfer function for ideal 6nd order Butterworth high-pass filter:

According to the formula:

$$H_1(s) = \left(\frac{s^2}{s^2 + 1.414\omega_c s + {\omega_c}^2}\right)^3$$

The transfer function for ideal 6nd order Butterworth high-pass filter:

According to the formula:

$$H_2(s) = \left(\frac{1}{1 + 1.932\, s\omega_C + (s\omega_C)^2}\right)\left(\frac{1}{1 + 1.414\, s\omega_C + (s\omega_C)^2}\right)\left(\frac{1}{1 + 0.518\, s\omega_C + (\omega_C)^2}\right)$$

Therefore, $H_1(s)$ and $H_2(s)$ are different. The answer for this question is False.

11) If a medium wave radio receiver were to be designed using a band-pass ceramic filter whose pass-band is centered on 455kHz, it could be turned to different stations by multiplying the incoming signal by a sine wave of correctly chosen frequency. The effect of this multiplication is to change the frequency of the input signal to make it pass through the filter. The filter will then remove other radio stations, leaving only the one passed by the filter. We can

understand this by remembering the formula:

$$2\cos(\omega_1 t)\cos(\omega_2 t) = \cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t$$

Assuming for simplicity that the incoming signal is a 750kHz sine-wave, what must be the frequency of the multiplying sine wave to make the frequency changed signal pass through the ceramic filter?

False.

Prove:

$$\omega_1 - \omega_2 = 455$$

And incoming signal is ω = 750kHz which could be assumed as ω_1 or ω_2, Therefore, there are two condition:

1. When $\omega_1 = 750\ kHz$, then $\omega_2 = 750 - 455 = 295\ kHz$
2. When $\omega_2 = 750\ kHz$, then $\omega_1 = 750 + 455 = 1205\ kHz$

3. Laboratory

1) This section intends to produce a non-inverting amplifier of gain 1.59 by constructing a circuit using an op-amp. Figure 1 is the used operational amplifier.

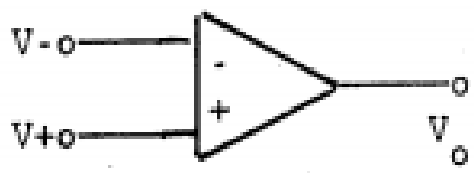

Figure 1: The used operational amplifier.

Due to the appointed device is non-inverting amplifier, it is therefore the designed circuit is shown in Figure 2.

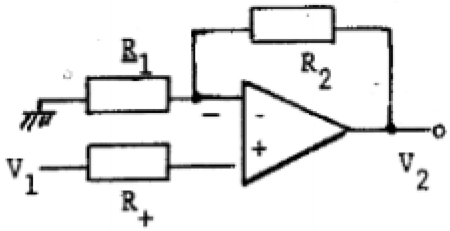

Figure 2: The circuit of non-inverting amplifier

Regarding to the gain of the non-inverting amplifier is 1.59, R_1 and R_2 need to be set in order to reach the design specification. According to the formulas:

$$A_V = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1}$$

Therefore, $\frac{R_2}{R_1}$ should be nearly to 0.59.

The next was to use the oscilloscope to the outputs when input signals are 100Hz, 1kHz, 10kHz, 100kHz and 1MHz correspondingly. Table 1 is the results after measuring and calculating.

Frequency	100Hz	1kHz	10kHz	100kHz	1MHz
Voltage gain	1.62	1.63	1.65	1.71	0.32
Gain in dB	4.19	4.24	4.35	4.66	-9.90

Implement the relationship: $\text{dB} = 20\log_{10} A_i$

The following figure are the measured results.

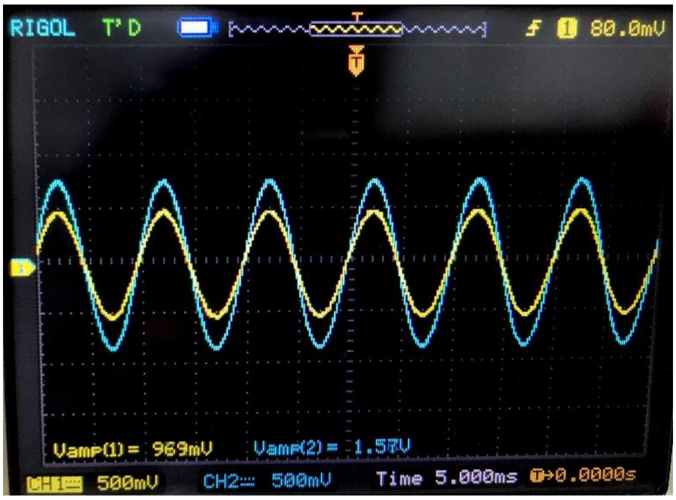

Figure 3: When input frequency is 100Hz

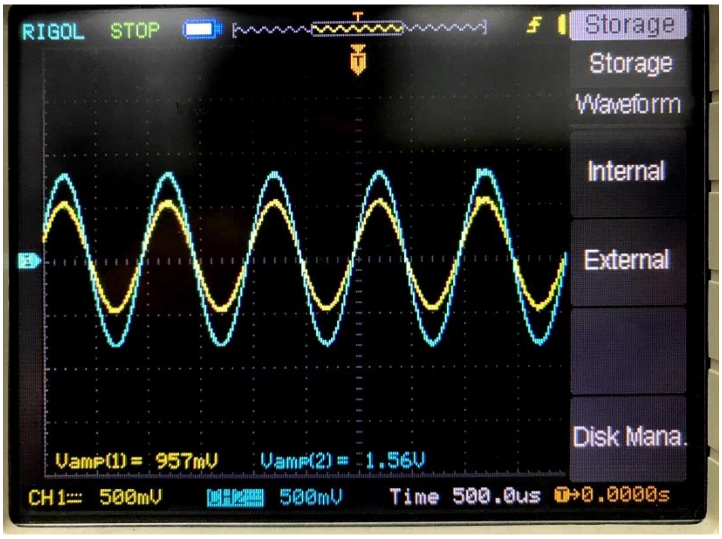

Figure 4: When input frequency is 1kHz

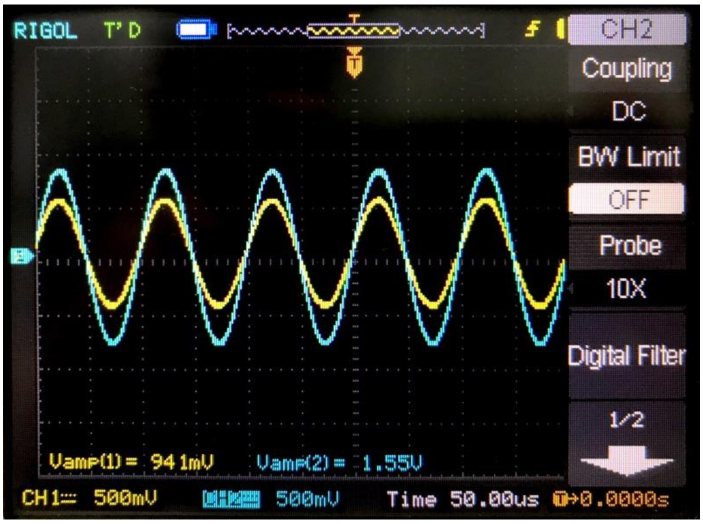

Figure 5: When input frequency is 10kHz

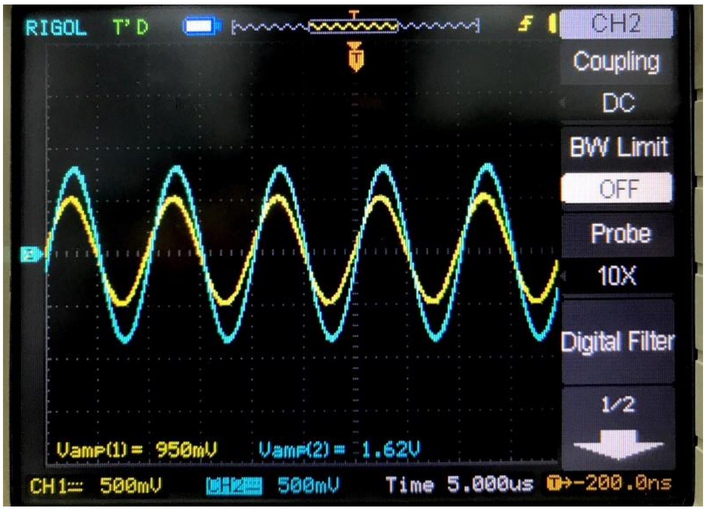

Figure 6: When input frequency is 100kHz

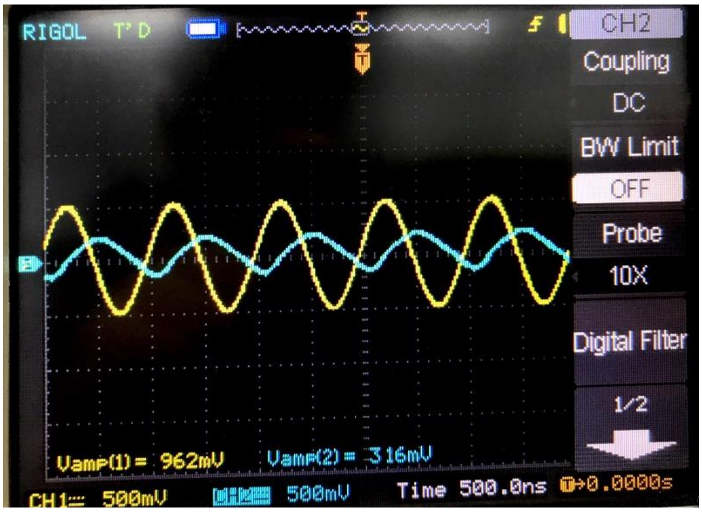

Figure 7: When input frequency is 1MHz

2) During this section, the non-inverting operational amplifier should be subsequently implemented in the design which is called 2nd order Butterworth filter and it is shown in Figure 8.

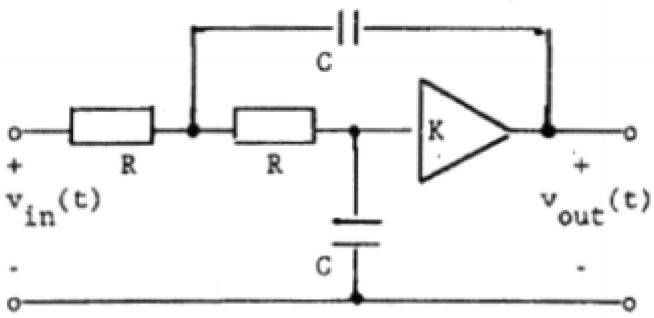

Figure 8: 2nd order Butterworth filter

Regarding to the cur-off frequency of the filter, it is assumed as 1592Hz. During this section, the voltage gains and phase responses of this design should be measure when the input signals are 100Hz, 1kHz, 1600Kz, 5kHz, 10kHz and 100kHz correspondingly.

The following deductions are to find R and C. It is assumed that R is 10 kΩ for convenience and from the transform equation:

$$\mathrm{H(s)} = \frac{1}{1 + (3 - K)RCs + (RCs)^2}$$

$$\mathrm{H(s)} = \frac{1}{1 + 1.414\left(\frac{s}{\omega_c}\right) + \left(\frac{s}{\omega_c}\right)^2}$$

So 3-K = 1.414. Then 3-K = 1.414 and $\mathrm{RC} = \frac{1}{\omega_c}$ where ω_c is 10000 rad/sec.

Therefore, C is 10nF.

Table 2 is the measuring results.

Frequency	100Hz	1kHz	1600Hz	5kHz	10kHz	100kHz
Voltage gain	1.53	1.38	1.09	0.19	0.045	0.016
Gain in dB	3.69	2.79	0.75	-14.42	-26.9	-35.9
Phase lead(deg)	0	57°	87°	144°	N/A	N/A

Table 2: The results for design in Figure 8.

The following figures are the measured results.

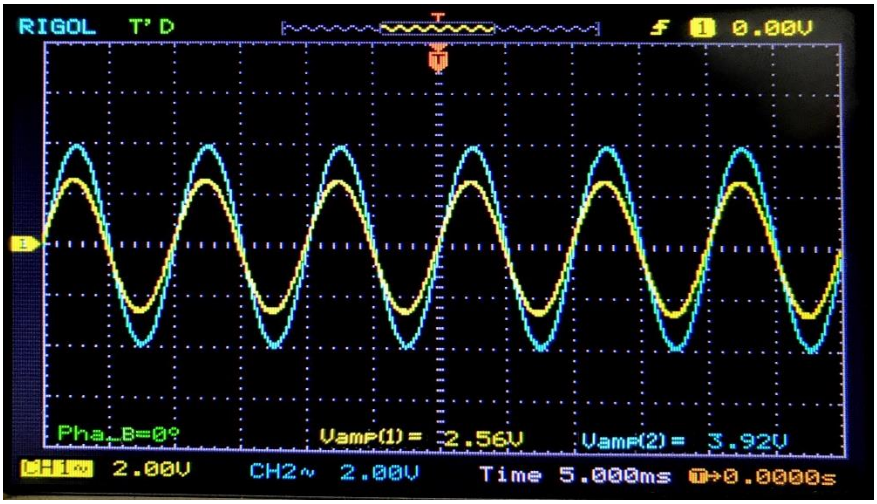

Figure 9: When input frequency is 100Hz

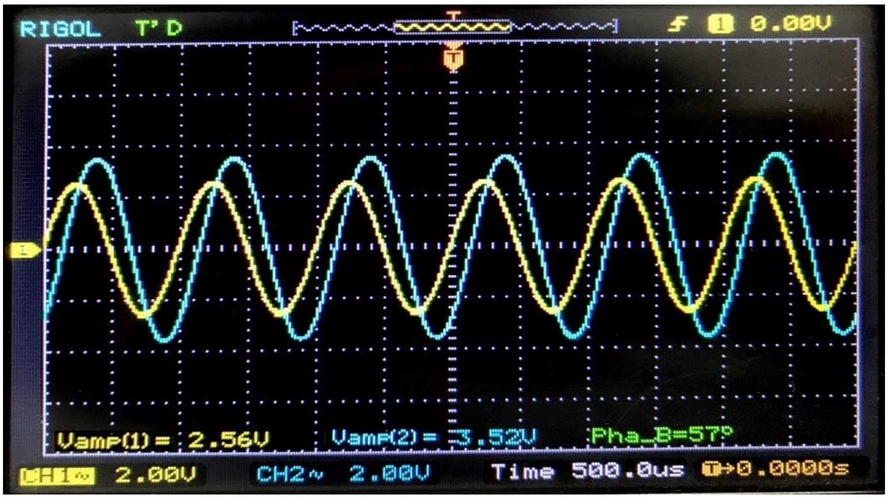

Figure 10: When input frequency is 1kHz

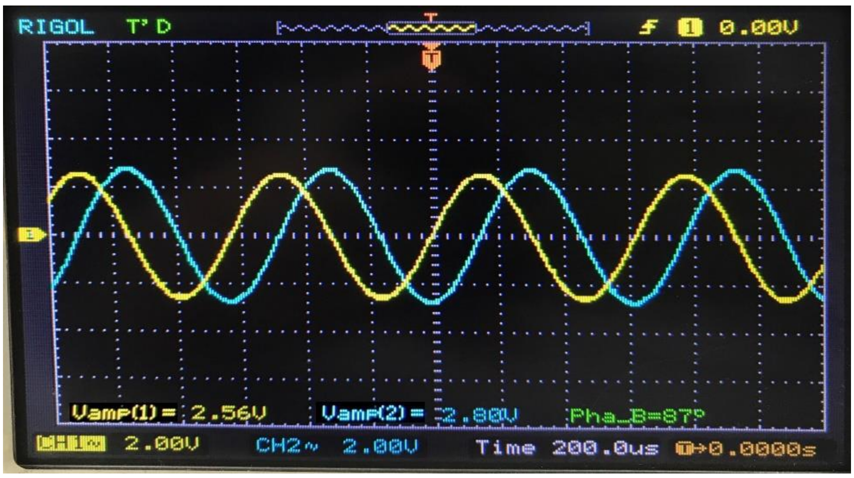

Figure 11: When input frequency is 1600Hz

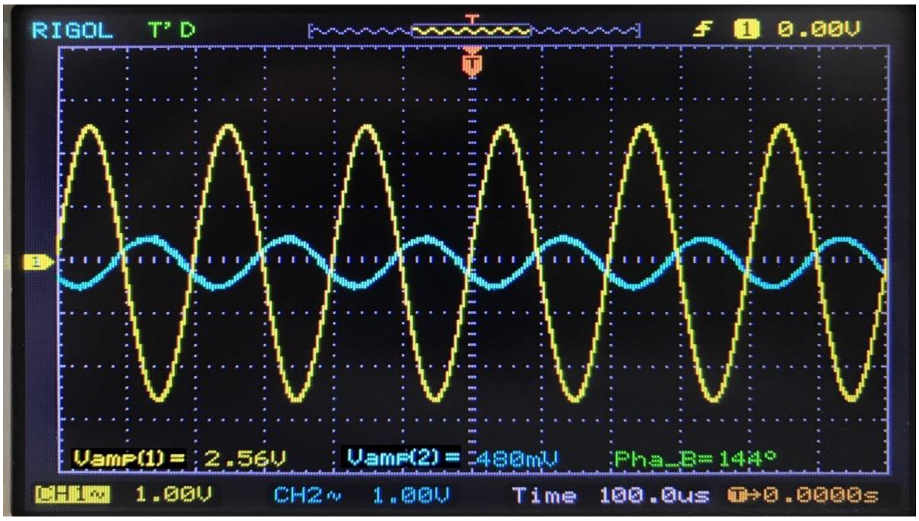

Figure 12: When input frequency is 5kHz

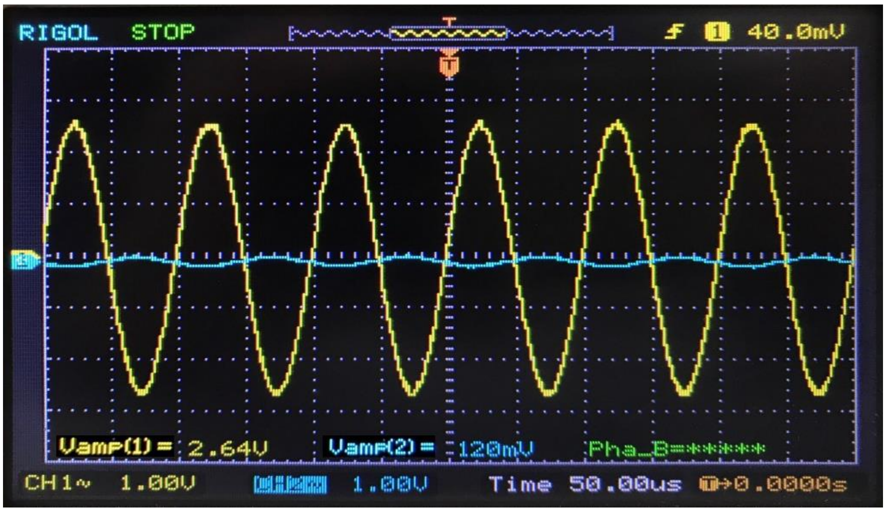

Figure 13: When input frequency is 10kHz

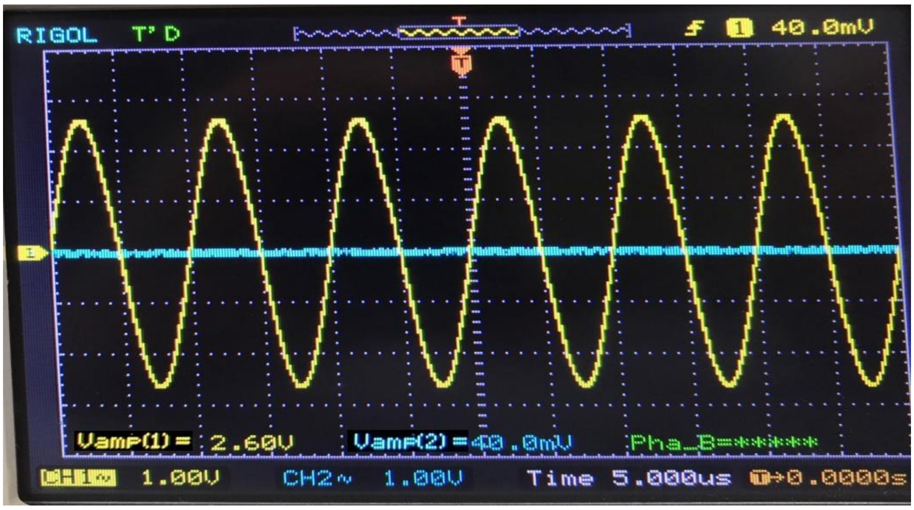

Figure 14: When input frequency is 100kHz

Moreover, in order to plot the graphs of the gain against a log scale for frequency and phase against linear scales, more data should be measured and collected. Table 3 is the extra measured results, while Figure 15 and Figure 16 are the graphs of the gain against a log scale for frequency and phase against linear scales, respectively.

Frequency (kHz)	0.5	1	1.5	2	2.5	3	3.5	4
Voltage gain	1.69	1.38	1.19	0.87	0.63	0.45	0.35	0.27
Gain in dB	4.5	2.79	1.51	-1.2	-4.01	-6.93	-9.12	-11.3
Phase lead (deg)	21°	32°	81°	100°	122°	132°	141°	153°

Table 3: The extra measured results

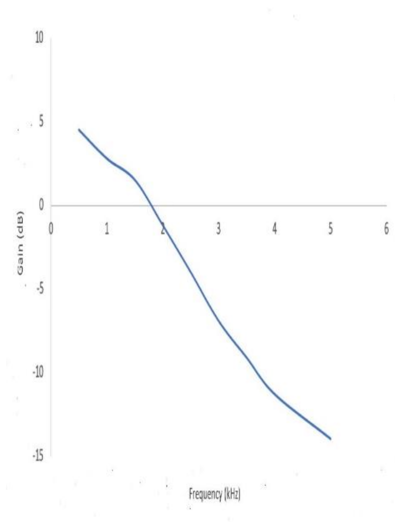

Figure 15: Gain response

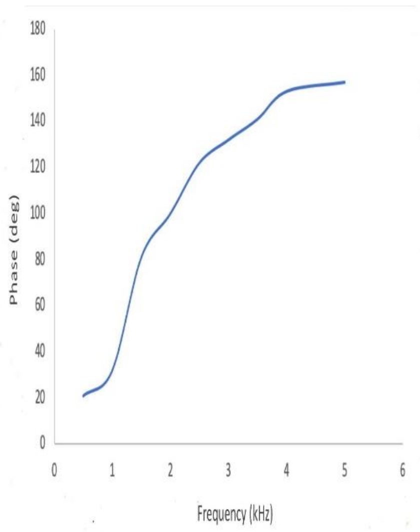

Figure 16: Phase response

3) This section intends to design an active 2nd order band-pass filter. The required pass band should be from 318.3Hz to 3.183kHz. It is therefore the filters used in this section were low-pass filter and high-pass filter so that the required pass band from 318.3Hz to 3.183kHz could be implemented.

   High-pass filter could be modified from the low-pass filter, it is therefore the used circuit in section is shown in Figure17.

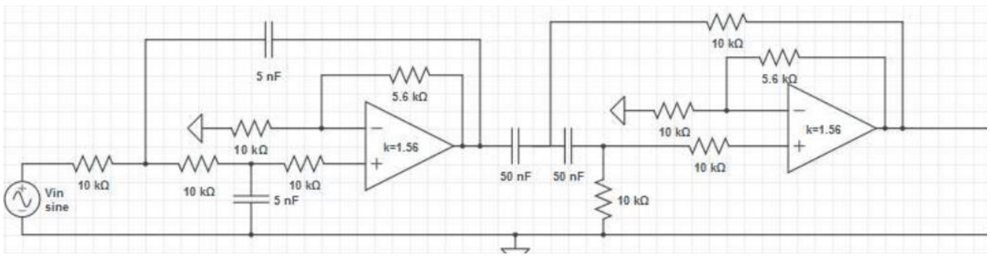

Figure 17: The design of active 2nd order band-pass filter.

The following figures are the measured results.

Figure 18: When input frequency is 100Hz

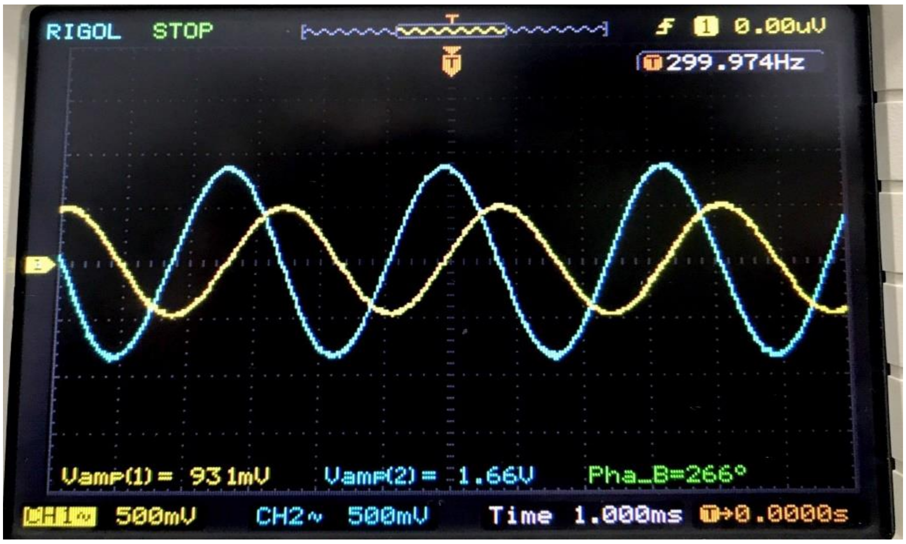

Figure 19: When input frequency is 300Hz

Figure 20: When input frequency is 1600Hz

Figure 21: When input frequency is 1600Hz

Figure 22: When input frequency is 3kHz

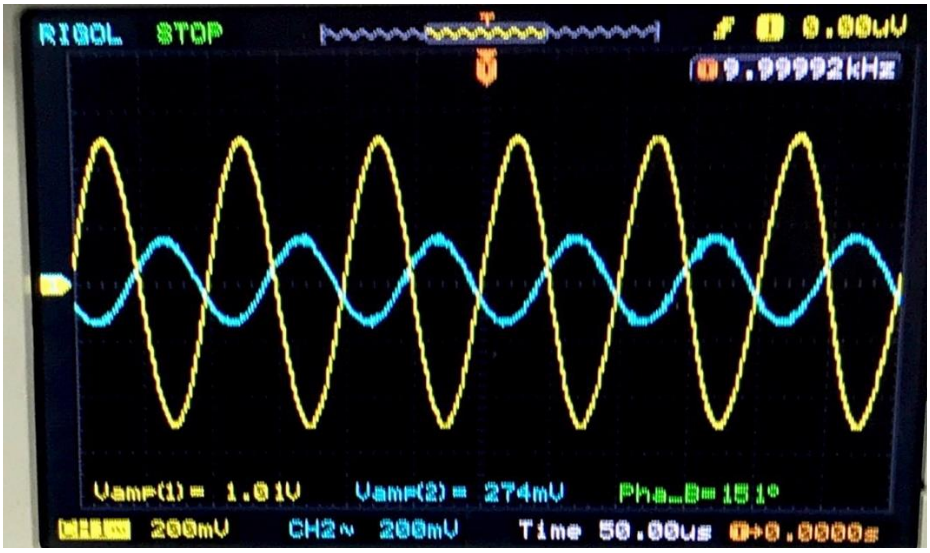

Figure 23: When input frequency is 10kHz

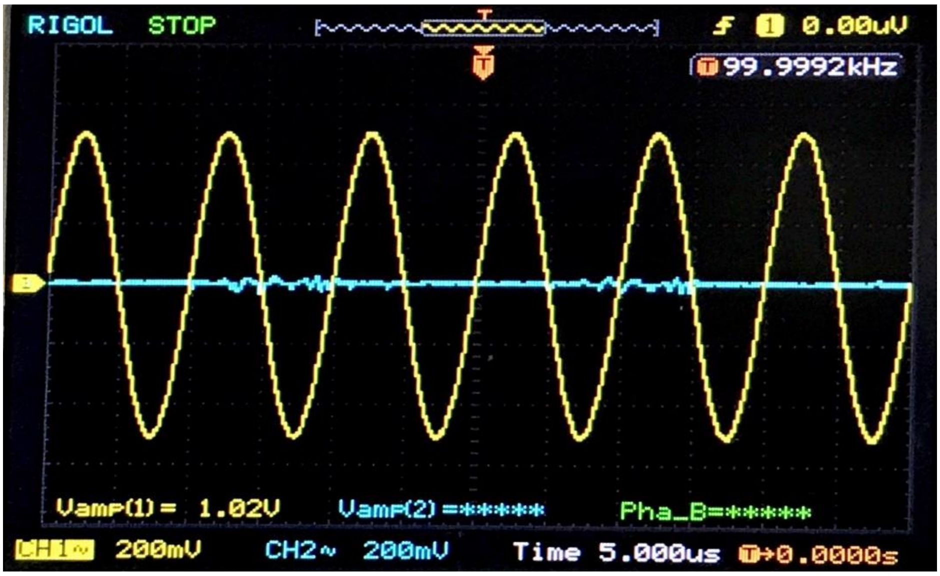

Figure 24: When input frequency is 100kHz

Table 4 is the measured and calculated results.

Frequency	100Hz	300Hz	1600Hz	3kHz	10kHz	100kHz
Voltage gain	0.22	1.4	2.91	2.51	0.27	N/A
Gain in dB	-13.15	5	9.27	7.99	-11.37	N/A
Phase lead(deg)	-155°	-134°	20°	72°	151°	N/A

Table 4: The measured and calculated results.

Moreover, in order to plot the graphs of the gain against a log scale for frequency and phase against linear scales, more data should be measured and collected. Table 5 is the extra measured results, while Figure 25 and Figure 26 are the graphs of the gain against a log scale for frequency and phase against linear scales, respectively.

Frequency	0.1	0.2	0.3	0.35	0.4	0.45	0.5	3.1	3.5	4
Voltage gain	0.2	1.45	1.78	2.06	2.38	2.5	2.76	2.47	2.2	1.76
Gain in dB	-13.85	3.27	4.99	6.32	7.52	7.95	8.8	7.8	6.8	4.9
Phase lead (deg)	-152°	-105°	-92°	-78°	-62°	-57°	-44°	80°	88°	96°

Table 5: The extra measured results

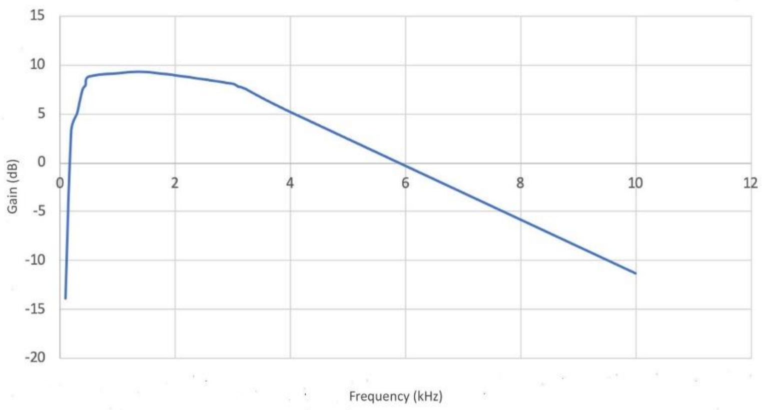

Figure 25: Gain response

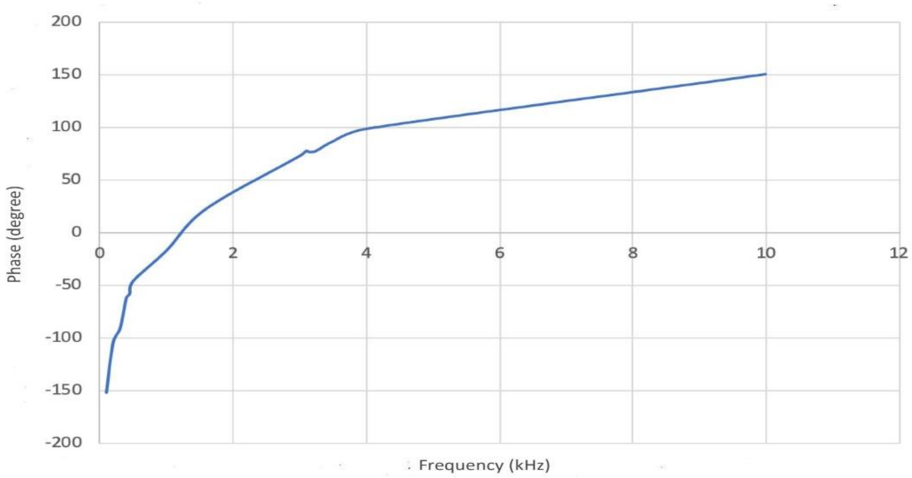

Figure 26: Phase response

4. Conclusion

From this lab experiment, the knowledge and theories for the basic electrical circuit about filters have been familiar with and mastered. Before the experiment, it is required to preview and review the relevant knowledge, which helps students further understand the steps to construct the circuits and, moreover, the theory could be applied in practice. In conclusion, the designs for low-pass filter, high-pass filter and band-pass filter were successfully constructed and their output characteristics were accurately measured and the results were all in the accessible error range.