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EEE 220 Instrumentation and Control System

Experiment 1
The Strain Gauge and Its Applications

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Experiment 1

Aims:

Construct the Wheatstone bridge arrangement which is shown in Figure 1 and measure its sensitivity. Then the theoretical expressions will be compared with the measured values.

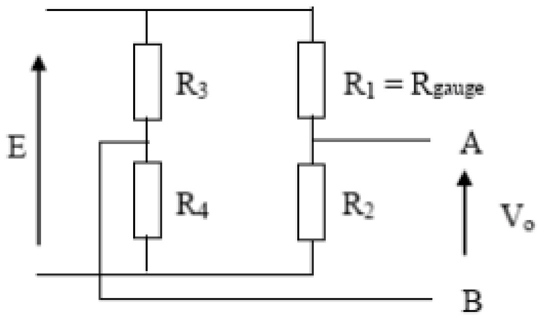

Figure 1: The circuit of the Wheatstone bridge

Analysis

During this experiment, the Wheatstone bridge were constructed on the Bridge box which is indicated in Figure 2.

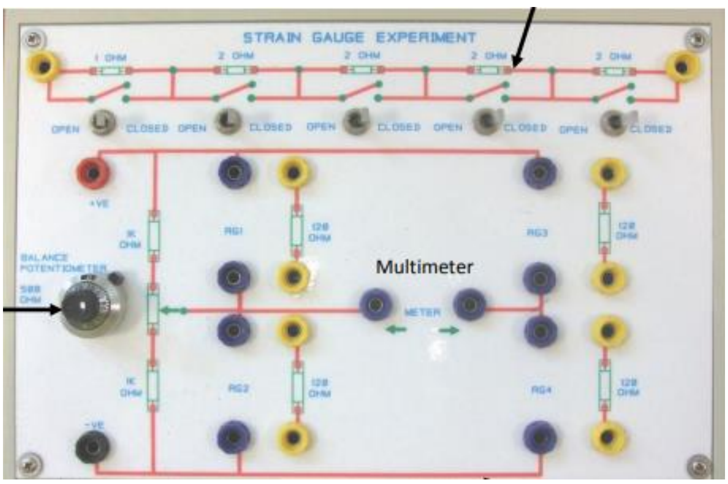

Figure 2: Bridge box

Due to the fact that it is difficult to obtain strain measurements, it is therefore the changes in resistances are desired to indirectly solve this problem.

Initially, R_3, R_2 and R_4 were set as 120Ω. R_3 should be in series with the resistor bank, which was convenient to set R_1 from 1Ω to 9Ω by setting five switches. The origin output voltage before the changes of R_1 was:

$$\mathrm{V} = \mathrm{E}(\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4})$$

Then the output voltage after the changes of R_1 ($R_1 \rightarrow R_1 + ER$)was:

$$\mathrm{V} + \delta V = \mathrm{E}(\frac{R_1 + \delta R}{R_1 + \delta R + R_2} - \frac{R_3}{R_3 + R_4})$$

Then,

$$\delta V = \mathrm{E}(\frac{R_2}{R_1 + \delta R + R_2} - \frac{R_2}{R_1 + R_2})$$

It is noted that $\delta R << R_1$, so

$$\delta V = -\mathrm{E}\left[\frac{R_2}{(R_1 + R_2)^2}\right] \times \delta R$$

In order to maximize the bracketed term, R_1 should be equal to R_2. Additionally, $R_2 = R_3 = R_4$, so $R_1 = R_2 = R_3 = R_4$, then δV could be verified as:

$$|\delta V| = \left[\frac{E}{4}\right]\frac{\delta R}{R_1}$$

Therefore, the sensitivity influenced by the change in resistance could be verified in the final $|\delta V|$ and R_1 is the key variable.

Answers

Question 1: <u>Determine the slope of the graph, representing the sensitivity of the bridge.</u>

Table 1 is the measured δV when R_1 varied.

δR	$\delta R/R_1$	δV
1	0.008333	22.121mV
2	0.01666667	42.840mV
3	0.025	63.152mV
4	0.0333333	84.090mV
5	0.0416667	104.150mV
6	0.05	124.810mV
7	0.05833	144.410mV
8	0.066667	164.435mV
9	0.075	183.590mV

Table1: The measured output of balance voltage δV

Figure 3 is the output voltage δV against the fractional change in resistance $\delta R/R_1$.

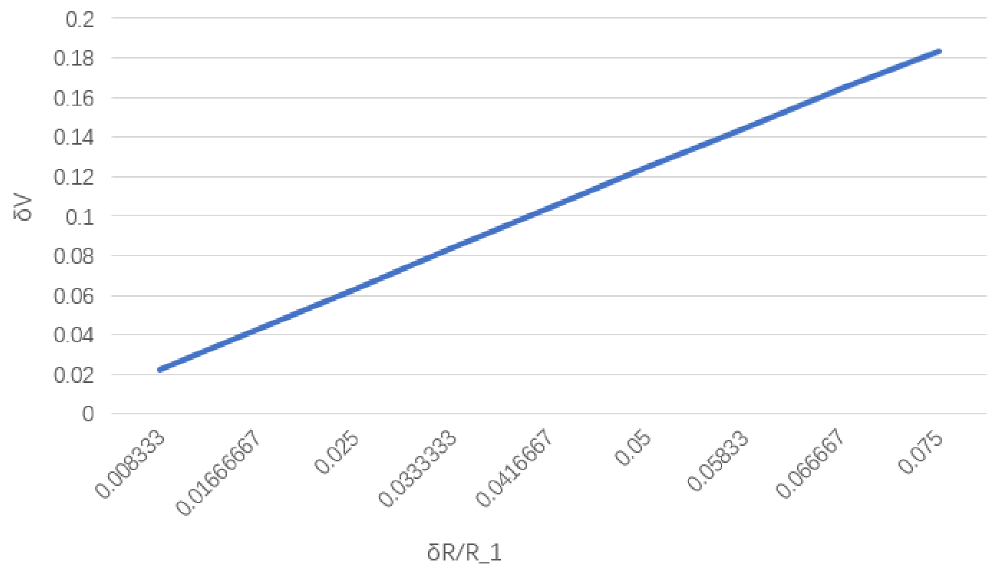

Figure 3: The relationship between δV and $\delta R/R_1$.

It could be found that the slope is $\frac{183.590\text{m}-22.121\text{m}}{0.075-0.008333} = 2.422$ which is the sensitivity.

In conclusion, the experimental sensitivity is **2.422**.

Question 2: Calculate the theoretical sensitivity of the bridge.

The theoretical sensitivity could be verified by the mentioned expression which is

$$|\delta V| = \left[\frac{E}{4}\right]\frac{\delta R}{R_1}$$

Where

$$\left[\frac{E}{4}\right] = \frac{5-(-5)}{4} = 2.5$$

Therefore, the theoretical sensitivity is **2.5.**

Question 3: Comment on the results.

Compared the experimental sensitivity with the theoretical sensitivity, they were almost the same. However, there would be some measurement error which is in the acceptable error range. In conclusion, the formula which was verified previously was correct and the experiment was successful.

Experiment 2 (Part A)

Aims: Explore the relationship between the resistance of the strain gauge and applied strain. Additionally, the applied strain would be generated by adding various weight s on both sides of the gauge bridge.

Analysis:

It is ideal that applied strain is the only factor influencing the resistance of the strain gauge. However, another factor which is temperature would affect the resistance of the strain gauge. Therefore, the pivotal method to eliminate the effect of temperature is to apply an equipment called temperature compensation gauge.

Considering the convenience for discussing the verifications later, it is noted that:

1. $x = \delta R/R$: the fractional change caused by strain
2. y: the fractional change caused by two gauges because of temperature
3. R_g' is the temperature compensation gauge

Additionally, R_1 could be influenced by temperature and strain simultaneously, that is

$$R_g = R_1(1+x)(1+y)$$

And for R_2:

$$R_g' = R_1(1+y)$$

Then the output signal of the bridge is:

$$V_o = \frac{ER_g}{R_g + R_g'} - \frac{E}{2} = \frac{x}{2(2+x)}E$$

As the strains would not beyond 10^{-3}, so the formula could be also written as:

$$V_o = \frac{x}{4}E$$

Therefore, it could be observed that V_o and the resistance x have the direct relation. Subsequently, V_o could be further verified as:

$$V_o = \left[\frac{\mathrm{E}}{4}\right] \times G \times \varepsilon$$

Because $\mathrm{E} = G \times \varepsilon$.

Answers

Question 1: <u>Why the temperature compensation strain gauge is used?</u>

Due to the fact that both strain and temperature have virtual influence on the performance of gauge. Furthermore, the fractional change of the resistance caused by strain is tiny. Then the pivotal method is to eliminate the effect of temperature. Another gauge would be used during this experiment basically depends on the temperature. In order to share the same temperature for these two strains, this extra gauge is placed close to the strain gauge. Consequently, the temperature compensation for strain gauge is applied then the influence caused by temperature could be solved.

Question 2: <u>Is there any variation in temperature? And why?</u>

The temperature is varied by different situation. During this experiment, the influence of temperature is eliminated by applying temperature compensation gauge. Additionally, the variation in temperature was not concerned. The reason of the temperature varies is the room temperature was distinguished from the one of the gauge. Moreover, the temperature might not be uniform. In conclusion, the variation in temperature exists and temperature compensation gauge is an appropriate method to avoid this error.

Experiment 2 (Part B)

Analysis

This experiment is the continuous part of last experiment. This experiment requires that the temperature compensation gauge which is notated R_4 should be replaced by the strain gauge indicated underside of the beam. Consequently, the sensitivity of the bridge to strain could be doubled because both two gauges are active in a half-configuration. Therefore, the four resistances of arms of the bridge could be further increased by activing strain gauges in a full-configuration.

The upside and underside of gauge are marked as R_{g1} and R_{g2} correspondingly. Then we have

$$R_{g1} = R_{g1}(1 + x)$$
$$R_{g2} = R_{g1}(1 - x)$$

And the voltage on R_2 is

$$\frac{R_1(1+x)E}{R_1(1+x)+R_1(1-x)} = \frac{E}{2}(1+x)$$

The output voltage of the bridge is

$$V_o = \frac{R_{g2}}{R_{g1}+R_{g2}} \times E - \frac{E}{2}$$

After simplification of the V_o, it will be

$$V_o = \frac{Ex}{2} = \left[\frac{\mathrm{E}}{2}\right] \times G \times \varepsilon$$

Therefore, it could be observed that the output voltage has no relationship with temperature and the sensitivity is doubled.

Answers

Question 1: <u>Compare the findings of bridge output against displacement for the two parts A and B. And the relative outputs with theory.</u>

The measured data is shown in Table 2.

Mass (kg)	Part A		Part B	
	δV	y(μm)	δV	y(μm)
0.0	0.057	0	0.062	0
0.5	0.244	77	0.459	78
1.0	0.479	165	1.078	163
1.5	0.762	263	1.552	241
2.0	1.079	308	2.262	321
2.5	1.376	394	2.704	387
3.0	1.571	448	3.235	479

Table 2: The measured data for Part A and Part B

From Table 2, it could be discovered that the displacement (y) would be the same when the weights in two parts are the same simultaneously.

In order to better analyze the measured from two groups of data, Figure 4 is the relationship of δV against y for both Part A and Part B.

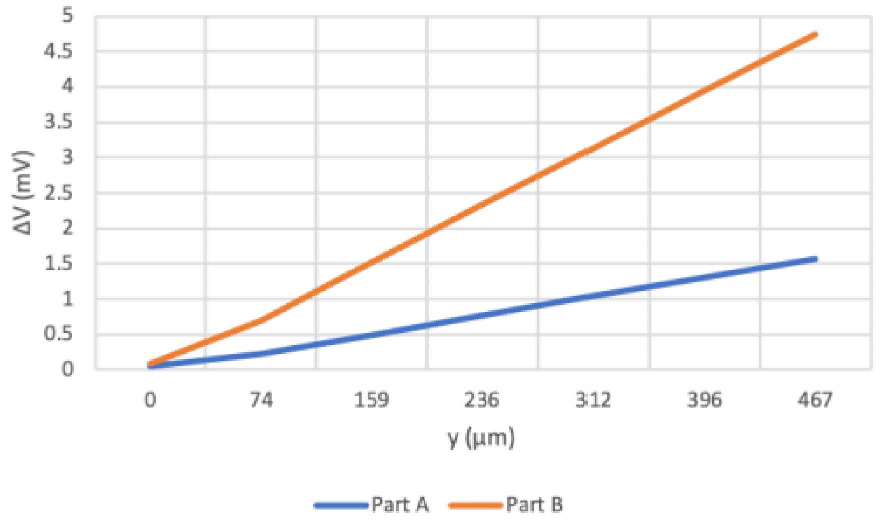

Figure 4: The relationship of δV against y for Part A and Part B

From Figure 4 and after calculating of the two slopes of lines, it could be found that the slope of output voltage in Part B is nearly twice larger than the one in Part A.

As verified previously, it has:

$$V_{oA} = \left[\frac{\mathrm{E}}{4}\right] \times G \times \varepsilon \quad \text{(Part A)}$$

$$V_{oB} = \left[\frac{\mathrm{E}}{2}\right] \times G \times \varepsilon \quad \text{(Part B)}$$

Then

$$\frac{V_{oB}}{V_{oA}} = \frac{\left[\frac{\mathrm{E}}{2}\right] \times G \times \varepsilon}{\left[\frac{\mathrm{E}}{4}\right] \times G \times \varepsilon} = 2$$

It is therefore the theory could examine the experimental result, then the experiment was successful.

Question 2: For your best set of measurements

a) **Calculate the strain.**

By the formula of strain $(\varepsilon) = \frac{4hy}{l^2}$, where h = 6.27mm and l = 20cm. In terms of y, it is chosen as the data when the Mass = 3kg. Then, y is 542.

So

$$\varepsilon = \frac{4hy}{l^2} = \frac{4 \times 6.27mm \times 542\mu m}{20cm \times 20cm} = 3.398 * 10^{-4}$$

So the strain $\boldsymbol{\varepsilon = 3.398 * 10^{-4}}$

b) **Calculate the gauge factor (G).**

According to the formula verified previously,

$$G = V_o\left[\frac{2}{E \times \varepsilon}\right] = \frac{2 \times 4.105 \times 10^{-3}}{10 \times 3.396 \times 10^{-4}} = 2.416$$

So the gauge factor $\boldsymbol{G = 2.416}$.

c) **What is the effect of using multiple strain gauges on sensitivity?**

From Table 2, the slope for **Part A =** $\frac{1.952-0.12}{0.531} = \mathbf{3.45}$ and **the slope for Part B =** $\frac{\mathbf{4.105-0.333}}{\mathbf{0.542}} = \mathbf{6.96}$. **Therefore, the sensitivity could be increased by using multiple strain gauges.**

Experiment 4

Analysis

This experiment intends to explore the distribution of strain supports with a two-point loaded beam. Then Young's modules would be determined by computing stresses from the known loading conditions.

According to Hooke's Law, it shows the relationship between stress and strain.

$$\sigma = Y \times \varepsilon$$

It is noted that σ is stress, Y is Young's Modulus and ε is strain. σ could be further verified as:

$$\sigma = \frac{3W(L-l)}{h^2 b}$$

Where l is the distance between supports, L is the distance between the loading points at the extremity of the beam, W is the loaded applied at each point, h is the height of the beam perpendicular to the bending axis and b is the width of the beam.

Answers

Question 1: <u>Calculate the strain for the corresponding loads and show sample calculation.</u>

As it verified previously,

$$V_o = \frac{Ex}{2} = \left[\frac{\mathrm{E}}{2}\right] \times G \times \varepsilon$$

So

$$\varepsilon = \frac{2 \times V_o}{G \times \mathrm{E}}$$

Table 3 is the measured output voltage V_o and its relationship with mass is indicated in Table 3.

Mass (kg)	δV
0.0	0 mV
0.5	0.654 mV
1.0	1.533 mV
1.5	2.165 mV
2.0	2.823 mV
2.5	3.436 mV
3.0	4.008 mV

Table 3: The measured output voltage V_o and its relationship with mass

Sample calculation:

2.0kg for mass has been chosen for presenting sample calculation. Moreover, G has been calculated previously in Experiment 2 (Part B) which is equal to 2.416. Then

$$\varepsilon = \frac{2 \times V_o}{G \times \mathrm{E}} = \frac{2 \times 2.823mV}{2.416 \times 10} = 2.32 \times 10^{-4}$$

Therefore, the whole table could be figured out by calculating as above when mass varies.

Mass (kg)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
$\boldsymbol{\delta V}$ **(mV)**	0	0.654	1.533	2.165	2.823	3.436	4.008
$\boldsymbol{\varepsilon}$ $(\times 10^{-5})$	0	5.41	12.69	17.92	23.37	28.44	33.18

Table 4: Completed table for ε when mass varies.

Question 2: <u>Compute stresses by using Equation (13) from the known loading conditions. Show a sample calculation.</u>

Since σ could be further verified as:

$$\sigma = \frac{3W(L - l)}{h^2 b}$$

Sample calculation:

Then when Mass = 2.0kg,

$$\sigma = \frac{3W(L - l)}{h^2 b} = \frac{3 \times 2kg \times g \times (59cm - 20cm)}{(6.27mm)^{-4} \times 25.4mm} = 2.34 \times 10^7$$

Eventually, the completed table for calculating σ when mass varies is shown in

Table 5.

Mass (kg)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
$\boldsymbol{\delta V}$ **(mV)**	0	0.654	1.533	2.165	2.823	3.436	4.008
$\boldsymbol{\varepsilon}(\times \mathbf{10^{-5}})$	0	5.41	12.69	17.92	23.37	28.44	33.18
$\boldsymbol{\sigma}(\times \mathbf{10^{7}})$	0	0.58585	1.1717	1.758	2.343	2.929	3.515

Table 5: The completed table for calculating σ when mass varies

Question 3: Plot a graph of stress versus strain.

Figure 4 is the graph of stress versus strain.

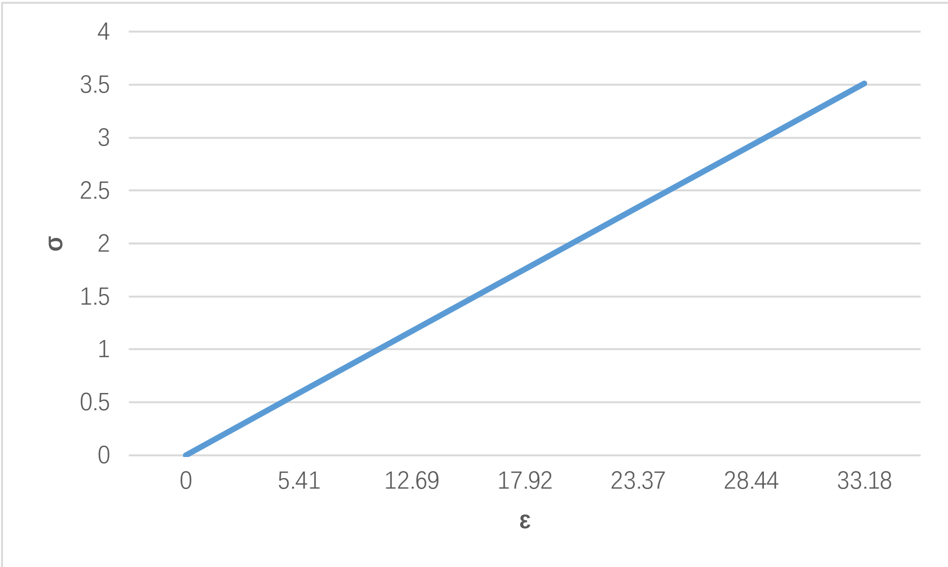

Figure 4: The graph of stress versus strain.

Question 4: <u>Determine the value of Young's modulus for the beam material and compare your result with the $\mathbf{Y = 1.05 \times 10^{-4} Nm^2}$ for brass.</u>

From Hooke's Law, Y could be obtained

$$Y = \frac{\sigma}{\varepsilon}$$

Combined Table 5 with Figure 4, the slope of the line could be calculated:

$$Y = \frac{3.515}{33.18} \times 10^{12} = 1.0594 \times 10^{11}$$

Therefore, the experimental value of Young's modulus was $\underline{\mathbf{1.0594 \times 10^{11}}}$.

In order to demonstrably analyze the accuracy of the experimental value, it has

$$error = \frac{1.0594 - 1.05}{1.05} = 0.895\%$$

Therefore, the experimental Young's modulus was highly closed to the theoretical one and the experiment was successful.

Question 5: <u>How does the strain measured near the pivots compare with that found for the same loading near the center in Experiment 2-B?</u>

Table 6 is the comparison of ε near center and the one near pivots.

Mass(kg)	0.0	0.5	1.0	1.5	2.0	2.5	3.0
ε near center	0	5.41	12.69	17.92	23.37	28.44	33.18
ε near pivots	0	5.58	12.63	17.37	22.45	25.89	32.87

Table 6: The comparison of ε near center and the one near pivots.

It could be observed that ε are very closed to each other.

Experiment 5

Analysis

Similar to previous experiments, however, this experiment required to apply the uniform beam with a hole at its center. In addition, there were two places of strain gauge whose further position could be referred in Figure 5.

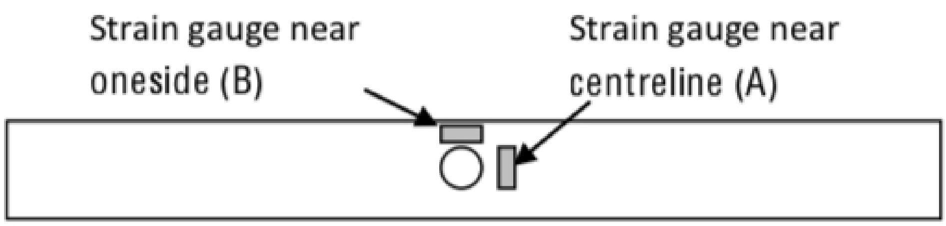

Figure 5: Two places of strain gauge to be measured.

Answers

Question a): <u>Compare between:</u>

1. <u>the two strains at those two positions and explain the reason,</u>

The steps for this experiment were similar to the previous experiments. Table 6 is the measured output voltages for Position A and Position B.

Positon	δV_{max}
Positon A	1.701 mV
Positon B	2.628 mV

Table 6: The measured output voltage for Positon A & Positon B.

As it has been verified previously, it has

$$V_o = \left[\frac{E}{4}\right] \times G \times \varepsilon$$

Therefore, the strains for A and B could be calculated:

$$\varepsilon_A = \frac{4 \times V_{oA}}{G \times E} = \frac{4 \times 1.701\text{mV}}{10V \times 2.416} = 2.816 \times 10^{-4}$$

$$\varepsilon_B = \frac{4 \times V_{oB}}{G \times E} = \frac{4 \times 2.628\text{mV}}{10V \times 2.416} = 4.351 \times 10^{-4}$$

Obviously, ε_B is larger than ε_A. <u>Due to the fact that both sides of the beam had carries which resulted the strain on the horizontal were greater the one on the vertical, which is consistent with the experimental result.</u>

2. <u>the strain under these conditions with that at the uniform beam at the same loading.</u>

It is suggested that the strain should be taken when the mass is equal to 3kg which is the maximum weight. For the strain of this experiment, the equation could be used:

$$\varepsilon = \frac{4hy}{l^2} = \frac{4 \times 6.27mm \times 1219\mu m}{20cm \times 20cm} = 7.67 * 10^{-4}$$

In experiment 2, the strain is:

$$\varepsilon = \frac{4hy}{l^2} = \frac{4 \times 6.27mm \times 542\mu m}{20cm \times 20cm} = 3.398 * 10^{-4}$$

Therefore, Table 7 is the comparison table for the strain between Experiment 5 and Experimental 2 (when mass = 3kg).

	ε
Beam with hole	$7.67 * 10^{-4}$
Beam without hole	$3.398 * 10^{-4}$

Table 7: The comparison table for the strain between Experiment 5 & Experimental 2 (when mass = 3kg).

Question b): <u>Find the stress concentration factor K at the center of the beam.</u>

According to the formula K:

$$K = \frac{\sigma_{max}(beam\ \ with\ \ whole)}{\sigma_{max}(beam\ \ without\ \ whole)} = \frac{7.67 * 10^{-4}}{3.398 * 10^{-4}} = 2.257$$

Question c): <u>Does the presence of the hole affect stresses and strains at points remote from the hole? And Why?</u>

<u>The presence of the hole does not affect stresses and strains at points from the hole.</u> Stress concentration is the phenomenon that the local stress increases evidently when the sudden change of section size. Additionally, due to the characteristics for the stress concentration, the stress is increased locally, which means it is effective in limited area. <u>Because the points are far away from the presence of the hole, stress concentration will not occur at these points.</u> For example, when glass is to be cut, people usually slice shallowly on the surface of the glass. Then after the stresses on both sides of the glass, the glass could be cut neatly.