Test #6 – Midterm

1. Let $f(x) = 4x^2 + 5x$.

a) Find the average rate of change of f on the interval $[1,3]$.

b) Use the limit definition of the derivative to compute $f'(x)$.

c) Find the equation of the tangent line to f at $x = 3$.

2. Evaluate each limit:

$$\lim_{x \to 0} \frac{4x + 4\sin 4x}{2x}$$

$$\lim_{x \to -4} \frac{4x^3 + 16x^2 - x - 4}{x + 4}$$

3a) State the definition of continuity of $f(x)$ at $x = a$.

3b) Consider $f(x) = \begin{cases} 1 - 3x & x < -6 \\ 7 & x = -6 \\ x^3 & -6 < x < 1 \\ 1 & x = 1 \\ 2 - x & x > 1 \end{cases}$

Determine if $f(x)$ is continuous at $x = -6$ and $x = 1$?

4. Find y'' for $x^2 + y^4 = 10$ implicitly and express your answer in terms of x and y.

5. Evaluate each limit.

$$\lim_{w \to \infty} \sin^{-1}\left(\frac{\sqrt{3}w^3 + 5w^2 + 3w + 1}{2w^3 + 5w + 3}\right)$$

$$\lim_{x \to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$$

6. Find the equation of the line normal to $\cot^{-1}\sqrt{x}$ at $x = 1$.

7. Find the derivative and simplify as appropriate.

a) $f(\theta) = \cos^2(3\theta)$	b) $g(x) = \frac{4x^2 - 9x}{2x + 7}$

8. Use the information given in the table to evaluate the derivative of each function when $x = 2$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	1	12	½	-3

b) $\sqrt{2g(x)}$ c) $f(x) \cdot g(x)$

9. Water flows into a conical tank at a rate of 54π cubic feet per minute. The cone has radius 12 feet and a depth of 10 feet. Find the rate at which the height of the water is increasing at the moment when the depth of the water in the tank is 4 feet.	10. A 25-foot ladder slides down a wall at a rate of 2 feet per second. a) Find the rate at which the bottom of the ladder slides way from the ground at the moment when the top of the ladder is 7 feet above the ground. b) Find the rate at which the angle the ladder makes with the ground is decreasing at that same moment.

11. The graph of $f(x)$ is shown below. Sketch the graph of $f'(x)$ in the axes provided to the right.
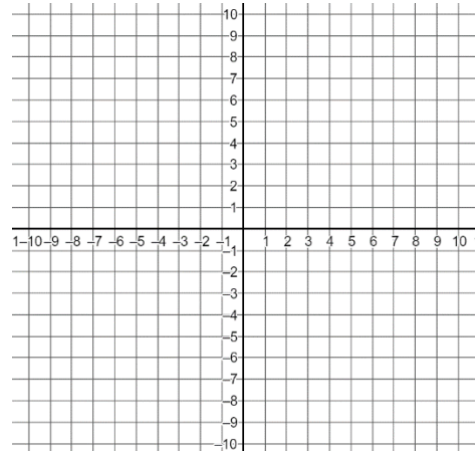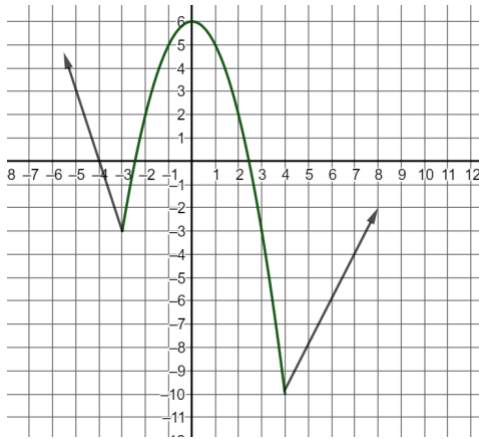
The function is not differentiable at which points? _____

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12. A train moves along a horizontal track. The position of the train (in miles) at any time in the interval $t \in [0,7]$ is given by the function $s(t) = 2t^3 - 21t^2 + 60t$ where t is given in hours.

      a) Find the equations of the velocity and acceleration in terms of t.

      b) Determine the intervals for which the train is moving left and right.

      c) Determine the intervals for which the train is speeding up and slowing down.