BEST PRACTICES FOR ONLINE TEACHING PROJECT

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I found David Salomon's course on Online Teaching very thought-provoking and helpful in many ways. For my project, I wanted to explore two areas that he highlighted during the course - setting up a virtual classroom at home, and alternative forms of assessment. For my project, I have attempted to work out some practical ways in which I could implement some of his suggestions. The results of this exploration can be found below.

The Home-Virtual Classroom. An important question that needs to be answered is how to migrate a live, in-person class to an online setting quickly, without needing to redesign the entire class. One important aspect of the migration is developing a sensible and effective physical space from which to teach. Having good and reliable equipment as well as a setup that makes students feel like they are in classroom is extremely important. It helps students to see their teacher working out of a space that looks professional and orderly. This helps to set the tone for the online classroom and helps in increasing engagement.

During this summer, I purchased and built my own equipment to set up my home office in a way that would be optimal for virtual teaching in the fall (and potentially in the future). I explored two main options that I felt would give me flexibility, versatility and clarity.

Option #1: "Paper-Based" Teaching. In this setup, a camera is pointed to a piece of paper, and the teacher lectures and writes on the piece of paper to convey content. Students are able to see the piece of paper but not the teacher directly. I adapted this method slightly. Rather than use a piece of paper, I built a glass whiteboard into my desk which can be written on with dry-erase markers. This avoids wasting a lot of paper every day and keeps my work space clean organized. Instead of using a traditional camera pointed at the board, I have been using the VZ-R HDMI Document Camera. One helpful feature of this camera is that it integrates seamlessly with Zoom and has different options for zoom, rotation, focus and lighting. One important additional feature that I added to my setup was to connect external monitors to my laptop. This allows me to use Zoom in dual monitor mode, and lets me see my students on one screen, and then my board on the other screen. On the second monitor, I can look at what view the students see and so this is extremely helpful for making adjustments when the camera goes out of focus or I start writing off screen. I plan to use this setup for office hours or meetings as it is less formal. Pictures of this setup are available on the next page.

Option #2: Whiteboard Teaching. In this setup, the camera is pointed at the teacher and a whiteboard. Lecturing is done in the "traditional" way by writing on the board. To use this particular technique, I mounted two boards on walls in my home office. The boards are large enough that a substantial amount of work can be

done on them without constantly needing to erase. One important piece of equipment that is essential to making this setup effective is an external webcam with good video quality. Most integrated webcams in laptops record at 720P resolution. But to use a large whiteboard setup where the camera is away from the board by some distance, it helps to have a camera that can give a crisper and clearer image than an integrated laptop camera. After researching several of the options David suggested, I settled on using the Logitech C922 camera. This camera has excellent video clarity, low light performance, sound and field of view. It also has a streaming option at 1080P resolution. I plan to use this setup when doing direct teaching or when doing work that is more formal and requires more of a classroom look. The setup using this equipment is shown in the pictures on the next page.

Non-traditional Assessments. In mathematics courses, one of the biggest challenges in the online environment becomes assessment. It is very difficult to "replace" a traditional test in terms of measuring student progress in understanding content. However, doing traditional tests in the online environment becomes challenging, particularly when trying to mitigate cheating. One way around this problem which David suggested is to design different types of assessments. After our class discussion, I wanted to explore this further, and designed a project that can be given to students in an Algebra II course. The assignment is attached at the end of this document. The project walks students through the derivation of a famous formula for computing the solutions to cubic polynomial equations. The project is designed so that students can collaborate and work through responses together. Though this does not quite do the same job as a traditional test, I think it does allow students to extend their knowledge and apply it to something they have not seen before. It also encourages students to work together, which disincentivizes cheating or looking up answers elsewhere.

FIGURE 0.1. Option #1 Setup with Board and Document Camera

FIGURE 0.2. Option #1 Multiple Monitor Setup

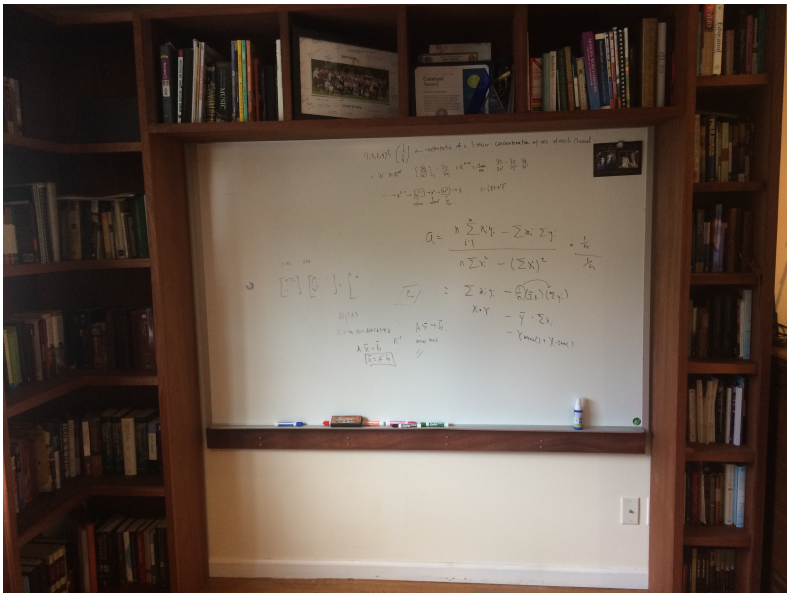

FIGURE 0.3. Option #2 Whiteboard Example

FIGURE 0.4. Option #2 Board and Camera Setup

FIGURE 0.5. Logitech C922 Camera

EXTRA CREDIT PROJECT - DEL FERRO AND CARDAN'S CUBICS

Instructions. Follow the steps below to derive del Ferro's formula for the solution of the depressed cubic $x^3 + px = q$, and then a formula for calculating the solution to a general cubic. Write your answers on a separate sheet of paper and be sure to show all your steps. Good luck!

Problem 1. The cubic solved by del Ferro has the general form $x^3 + px = q$, where p and q are non-negative.

(1) Assume that the solution can be written as the sum of two terms, namely, $x = u + v$. Substitute this into the depressed cubic, expand the terms and then factor to show the following equation:

$$u^3 + v^3 + (3uv + p)(u + v) = q$$

(2) Set $u^3 + v^3 = q$. Then explain why $3uv + p = 0$.

(3) Solve $3uv + p = 0$ for v in terms of p and u.

(4) Substitute your answer from (3) into the equation $u^3 + v^3 = q$ and show that

$$u^6 - qu^3 - \frac{p^3}{27} = 0$$

(5) Substitute $w = u^3$ in your answer to (4) to obtain a quadratic equation in w. Solve the quadratic equation, and substitute back to obtain:

$$u^3 = \frac{q}{2} \pm \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$$

(6) Take the cube root of both sides and keep only the positive cube root. You should get:

$$u = \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

(7) Since $u^3 + v^3 = q$, show that $v^3 = q - u^3$. Then substitute your answer in (6) for $u$and solve for v to get:

$$v = \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

(8) Finally, since $x = u + v$, show that the solution to the depressed cubic is

$$x = \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

Problem 2. Use del Ferro's formula which you derived in Problem 1 to show that:

(1) $x^3 + 6x = 20$ has a solution at $x = 2$

(2) $x^3 = 6x + 20$ has a solution at $x = 3.4377073$

Problem 3. Cardan built on del Ferro's work to find a formula to find the roots of a general cubic. Follow the steps below to see how he did it!

(1) Consider $x^3 + a_1x^2 + a_2x + a_3 = 0$. Substitute $x = y - \frac{1}{3}a_1$ in to the formula. Expand and factor the terms using the powers y. You should get:

$$y^3 + \left(a_2 - \frac{1}{3}a_1^2\right) y = -\frac{2}{27}a_1^3 + \frac{1}{3}a_2a_1 - a_3$$

(2) Show that the answer in (1) is a depressed cubic, and identify the values of p and q.

Problem 4. Consider the cubic $x^3 - 15x^2 + 81x - 175 = 0$. Use Cardan's formula (and the steps you took in Problem 3) to show that the cubic has the solution $x = 7$.