NGPF COHORT 23 PROJECT

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During the second cohort of the NGPF certification classes, I participated in the Advanced Investing course. One of the key ideas discussed in the class was interest rates and their effects on bond funds and various types of other loans. For the project for this course, I designed a set of problems that can be used in precalculus classes covering the topics of amortization of loans as well as how to compute interest rates on these loans. These problems can be covered in two 50 minute periods. A prerequisite to doing these lessons would be knowledge of annuities and present value as well as familiarity with geometric sequences and their properties.

Financing/Amortization

1. An annuity pays $10,000 quarterly. If the present value of the annuity is $105,753.41, assuming 8% APR compounded quarterly, for how many years does the annuity pay?

$\frac{8\%}{4} = 2\%$

$$105{,}753.41 = 10{,}000\left(\frac{1-1.02^{-n}}{.02}\right)$$

$$\frac{105{,}753.41(.02)}{10{,}000} = 1 - 1.02^{-n} \Rightarrow 1.02^{-n} = 1 - \frac{105{,}753.41(.02)}{10{,}000}$$

$$-n = \frac{\ln\left(1 - \frac{105{,}753.41(.02)}{10{,}000}\right)}{\ln(1.02)} = -11.99 = -12$$

$n = 12$

so $\frac{12}{4} = \boxed{3 \text{ yrs}}$

2. Stacey finances a new car by putting no money down (meaning she pays nothing at the time of purchase) and making monthly payments of $415.15 for 5 years. Assuming 5% APR compounded monthly, what is the price of the car?

$$A_p = 415.15\ \frac{1-\left(1+\frac{.05}{12}\right)^{-60}}{\frac{.05}{12}} = 21{,}999.09 \approx \boxed{\$21{,}999}$$

3. In order to buy a house you take out a 30 year mortgage of $500,000 with an interest rate of 4%.

a) What will your monthly payments be?

$\frac{4\%}{12} = \frac{.04}{12} = \frac{.01}{3}$

$$P = \frac{iA_p}{1-(1+i)^{-n}} = \frac{\frac{.04}{12}(500{,}000)}{1-\left(1+\frac{.04}{12}\right)^{-360}} = \boxed{\$2387.08}$$

764

b) How much do you wind up paying over the life of the loan?

$\$2387.08 \times 360 = \boxed{\$859{,}348.80}$ * technically off by a few dollars due to rounding - last payment less

c) In order to borrow $500,000 how much interest did you have to pay?

$859{,}348.80 - 500{,}000 = \boxed{\$359{,}348.80}$

d) If you instead get a 15 year mortgage at the same interest rate, what are your monthly payments?

$$P = \frac{\frac{.04}{12}(500{,}000)}{1-\left(1+\frac{.04}{12}\right)^{-180}} = \boxed{\$3698.44}$$

396

e) How much interest do you pay?

$180(3698.44) = \$665{,}719.20$

$\$665{,}719.20 - 500{,}000 = \boxed{\$165{,}719.20}$

* technically a bit less since last payment a tiny bit less due to rounding of payments

4. Mike is looking to buy a house. The interest rates for a 30 year mortgage are 3.75%. Mike figures he can afford monthly payments of $2500. About How big of a loan can Mike afford? (Round to nearest thousand dollar)

$\frac{3.75\%}{12} = .3125\%$

$A_p = 2500 \frac{1-(1+\frac{.0375}{12})^{-360}}{\frac{.0375}{12}} = 539{,}822.03$

S: $\boxed{\text{about } \$540{,}000}$ ← $2500.62 monthly payment

5. The Smiths want to borrow $725,000 to purchase a home. A bank is offering a 30 year mortgage at 5% or a 15 year mortgage at 3.5%

a) How much more is the monthly payment for the 15 year loan than the 30 year?

30 yr: $\frac{\frac{.05}{12}(725{,}000)}{1-(1+\frac{.05}{12})^{-360}} = 3891.96$

15 yr $\frac{\frac{.035}{12}(725{,}000)}{1-(1+\frac{.035}{12})^{-180}} = 5182.90$

$5182.90 - 3891.96 = 1290.94$

$\boxed{\$1290.94}$

b) How much less will the total payment be for the 15 year loan than the 30 year loan?

$3891.96 \times 360 = 1{,}401{,}105.60$

$5182.90 \times 180 = 932{,}922$

$1{,}401{,}105.60 - 932{,}922 = \boxed{\$468{,}183.60}$

c) How much ~~more in~~ interest is paid when the loan is amortized over 30 years? ~~rather than~~ Over 15 years?

$1{,}401{,}105.60 - 725{,}000$ = 30 yr interest: $676,105.60

$932{,}922 - 725{,}000$ = 15 yr interest $207,922

Difference obviously

6. Susan remodels her kitchen at a cost of $50,000. The company allows her to finance the cost by paying $1,521.10 per month. Assuming 6% APR, how many months will it take Susan to pay for the remodeling job?

$\frac{6\%}{12} = .5\%$

$50{,}000 = 1{,}521.10 \frac{(1-1.005^{-n})}{.005}$

$1 - \frac{50{,}000}{1{,}521.10}(.005) = 1.005^{-n}$

$n = -\frac{\ln\left(1 - \frac{50{,}000}{1{,}521.10}(.005)\right)}{\ln(1.005)} \approx 35.999$

$\boxed{36 \text{ months}}$

Interest Rates

1. A couple agrees to purchase a new home for \$700,000. They can put 20% down but must finance the rest. Bank A is offering a 30 year mortgage at 3.5% while Bank B is offering 4.0%.

a) What is the difference in the monthly payments between the 2 banks?

80% of 700,000 = \$560,000

BANK A: $\dfrac{560{,}000\left(\frac{.035}{12}\right)}{1-\left(1+\frac{.035}{12}\right)^{-360}} = \$2514.65$

BANK B: $\dfrac{560{,}000\left(\frac{.04}{12}\right)}{1-\left(1+\frac{.04}{12}\right)^{-360}} = \$2673.53$

$2673.53 - 2514.65 = \boxed{\$158.88}$ ~~x 360 = \$57,196.80~~

b) What is the difference in the amounts of interest paid over the life of the loans?

$158.88 \times 360 = \boxed{\$57{,}196.80}$

OR FIND TOTAL PAID FOR EACH AND SUBTRACT

c) What interest rate would the couple need to get in order to have monthly payments of \$2000?

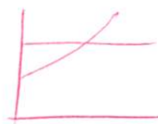
$\dfrac{560{,}000\,i}{1-(1+i)^{-360}} = 2000$

Then multiply i by 12

OR

$\dfrac{560{,}000\left(\frac{x}{12}\right)}{1-\left(1+\frac{x}{12}\right)^{-360}} = 2000$

F2 Solve Doesn't work!
Solve Graphically



[0, .005] x [0, 2500]
↑ 1/10 of 5%

$x_c = .00145662$

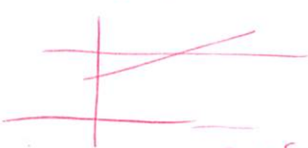
$12 \times .00145662 = .0174795$

≈ 1.75%

If doing 560,000 x/12
[0, .05] x [0, 2500]

2. What APR is needed in order to have \$1800 monthly payments on a 30 year \$400,000 loan?

$\dfrac{400{,}000\,i}{1-(1+i)^{-360}} = 1800$



[0, .005] x [0, 2500]

$x_c = .00293092$

x 12

= .03517104

≈ 3.52%

3. For a \$500,000 loan, what interest rate would be needed on a 15 year mortgage so that the monthly payments would be equivalent to a 30 year mortgage with 6.5% APR ?

$\dfrac{500{,}000\left(\frac{.065}{12}\right)}{1-\left(1+\frac{.065}{12}\right)^{-360}} = \$3160.34$

$3160.34 = \dfrac{500{,}000\,i}{1-(1+i)^{-180}}$ ← This one can be solved using F2 Solve

$i = .00145845509$

$i \times 12 = .01750$

≈ 1.75%

CERTIFICATE
OF COMPLETION

1 hour of professional development presented to

Michael Kumaresan

awarded on

Cohort 23 - Advanced Investing Certification Exam (Passed!)

Jim Lanzetta & Jessica Endlich

Co Founders, Next Gen Personal Finance

COMPLETION OF ngpf COMPLETION