Skolkovo Institute of Science and Technology

Application period. Collage

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1 Formulas

Let $X = \{x_i\}_{i=1}^{W \times H}$ be a collage of K images we are looking for, where each variable $x_i \in \{1, \dots, K\}$ is equal to the number of image we took i_{th} pixel from.

α-expansion

α-expansion is an iterative process where we firstly initialize X randomly or set all $x_i = 0$ and on each step select
$alpha$ and perform expansion step as was discussed on the lectures.

Let us introduce new variable y_i such that

$$y_i = \begin{cases} 1 & x_i^{new} = \alpha \\ 0 & x_i^{new} = \alpha \text{ and } x_i^{old} \neq \alpha \end{cases}$$

Let's look at the pair of vetrices (Fig.1.1). Here we put ∞ as $\phi_j(0)$ in case $x_j^{old} = \alpha$, because case $y_j = 0$ should not take place.

To construct a graph we will need to reduce the structure to one where there are only two crossing edges with equal values. We will use Fig.1.2 to simplify the notation. Here $*$ means any value and capital letters show values on edges.

Now we perform two steps to achieve our goal (shown in Fig.1.3)

Below we give final formulas to construct the graph. All $\phi_i(y_i)$ and $\phi_{ij}(y_i, y_j)$ are defined in Fig.1.1.

Edge capacity between two variables in lattice:

$$\hat{\phi}_{ij}(0,1) = \hat{C} = \frac{B' + C'}{2} = \frac{B - A + C - D}{2} = \frac{\phi_{ij}(0,1) - \phi_{ij}(0,0) + \phi_{ij}(1,0) - \phi_{ij}(1,1)}{2}$$

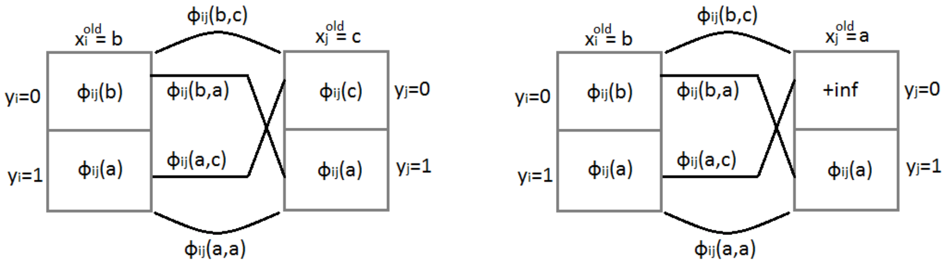

Figure 1.1: Initial structure

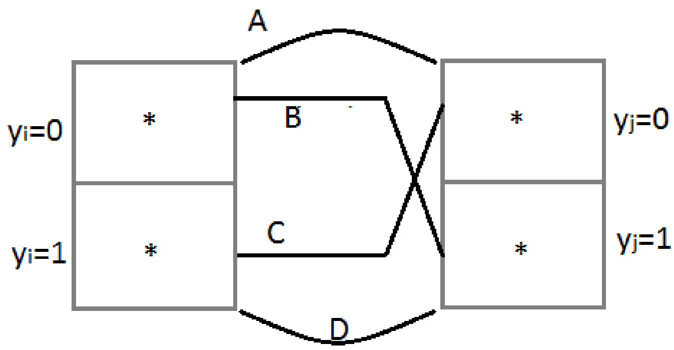

Figure 1.2: Notation

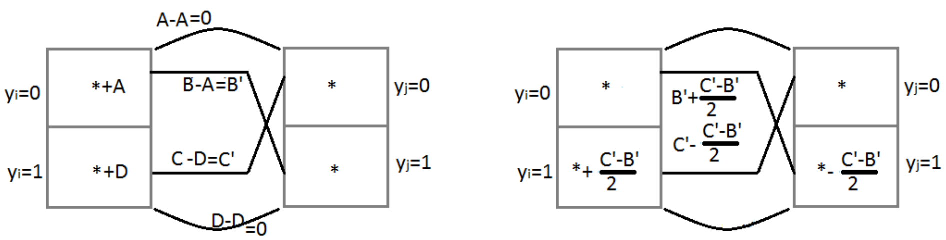

Figure 1.3: Reduction

Edge capacities from source to lattice variable and from lattice variable to sink ares initialized with $\phi_i(0)$ and $\phi_i(1)$ and when looking at edge ij are changed according to the rules:

$$\hat{\phi}_i(0) = \hat{\phi}_i(0) + \phi_{ij}(0,0)$$

$$\hat{\phi}_i(1) = \hat{\phi}_i(1) + \phi_{ij}(1,1) + \frac{\phi_{ij}(1,0) - \phi_{ij}(1,1) - \phi_{ij}(0,1) + \phi_{ij}(0,0)}{2}$$

$$\hat{\phi}_j(1) = \hat{\phi}_j(1) + \frac{\phi_{ij}(0,1) - \phi_{ij}(0,0) - \phi_{ij}(1,0) + \phi_{ij}(1,1)}{2}$$

$\alpha - \beta$-SWAP

$\alpha - \beta$-swap is an alternative iterative process, which chooses α and β labels on each step. Here we introduce a graph of size $|\{i : x_i = \alpha \text{ or } x_i = \beta\}|$ and binary variables

$$y_i = \begin{cases} 0 & x_i^{new} = \alpha \\ 1 & x_i^{old} = \beta \end{cases}.$$

Again, y_i are introduces only for those x_i, where $x_i = \alpha$ or $x_i = \beta$.

We will not need to do any transformation of the graph, because it already satisfies

$$\hat{\phi}_{ij}(0,0) = \phi_{ij}(\beta,\beta) = 0$$

$$\hat{\phi}_{ij}(0,1) = \phi_{ij}(\beta,\alpha) = c_{ij}d(\beta,\alpha) = \phi_{ij}(\alpha,\beta) = \hat{\phi}_{ij}(1,0)$$

$$\hat{\phi}_{ij}(1,1) = \phi_{ij}(\alpha,\alpha) = 0$$

The only thing, which needs to be handled accurately, is construction of capacities from source to lattice and from lattice to sink:

$$\hat{\phi}_i(0) = \phi_i(\beta) + \sum_{k: x_k \neq \alpha \text{ and } x_k \neq \beta} \phi_{ik}(\beta, x_k)$$

$$\hat{\phi}_i(1) = \phi_i(\alpha) + \sum_{k: x_k \neq \alpha \text{ and } x_k \neq \beta} \phi_{ik}(\alpha, x_k)$$