

Start date: Friday, May 20.

Report and code submission deadline: **Wednesday, May 25, 17:59.**

Presentation day: **Thursday, May 26.**

Programming Languages: Python + NumPy



1 Markov random field

Markov random field (MRF) is a graphical model which energy is defined as follows:

$$E(X) = \sum_{i \in \mathcal{V}} \theta_i(x_i) + \sum_{(i,j) \in \mathcal{E}} \theta_{ij}(x_i, x_j), \quad x_i \in \mathcal{P},$$

where \mathcal{V} is the set of variable indices, \mathcal{E} is the lattice, $\theta_i : \mathcal{P} \rightarrow \mathbb{R}$ are the unary potentials, $\theta_{ij} : \mathcal{P} \times \mathcal{P} \rightarrow \mathbb{R}$ are the pairwise potentials. Note that in the sum over the edges $(i, j) \in \mathcal{E}$ each edge in the graph is considered exactly once.

To reduce the number of parameters we parametrize the pairwise potentials as follows:

$$\theta_{ij}(x_i, x_j) = c_{ij} \cdot d(x_i, x_j),$$

where c_{ij} are the coefficients, which corresponds to the edges of the lattice, and $d(x_i, x_j)$ is the distance between the labels of neighboring variables.

Consider a model with the following constraints:

- variables x_p are discrete and take values from the set $\mathcal{P} = \{1, \dots, K\}$, $K \geq 2$,
- the graph \mathcal{E} is a square-lattice.

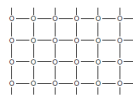


Figure 1: Square-lattice.

Your task is to implement two energy minimization algorithms: α -expansion and $\alpha\beta$ -swap.

2 MRF for automatic collage

Consider the problem of combining photos into a collage. We assume that all the photos are of the same size and aligned. Under this assumption we can make a collage by minimizing an MRF energy. Each pixel of the collage is from the corresponding pixel of one of the original photos, and for each pixel there is a variable which label is the index of the photo for this pixel.

The energy to make a collage should meet the following criteria:

- The variables x_p corresponds to the pixels of the final collage.
- The value of each variable corresponds to the index of the photo from which we will take the color of this pixel.
- The graph is the square-lattice.
- Unary potentials should define for some pixels the photos from which to take this pixels (the so-called seeds).
- Pairwise potentials should encourage 1) short cuts (edges between different photos) and 2) place the cuts in places where the photos are similar, so the cut would as invisible as possible.

You should try different potentials and come up with one that would produce collages that looks high-quality.

3 Problem formulation

1. Derive the formulas for one step of α -expansion and $\alpha\beta$ -swap.
2. Implement α -expansion and $\alpha\beta$ -swap using the code for graph cuts provided with the assignment.
3. Test the algorithms on synthetic data.
4. Implement the procedure to collage two photos using these energy minimization algorithms. Make at least one *good* collage from two photos.
5. Implement procedure to collage arbitrary number of photos. Make at least 1 *good* collage from at least 4 photos.
6. Compare the α -expansion and $\alpha\beta$ -swap algorithms on the task of collaging your photos. Compare the obtained energies, running time and visual quality. Back up your conclusions with evidence such as numbers, pictures, and plots..
7. Come up with an idea of a startup that uses automatic collaging.
8. Make a PDF report with everything you've found out.

Note that each team capture its own photo-content for the collages.

A collage is *good*, if the edges between the photos not worse than on the collage in the beginning of this document. To achieve *good* quality we encourage you to use photo editing software to make the photos similar in colors and lighting.

Sending the full assignment. Send the completed assignment via Canvas. We kindly ask you to send the assignment only once.

The submission should include:

- A PDF report with the list of team members and formulas for α -expansion and $\alpha\beta$ -swap.
- Code that you have used in your experiments.