# עבודת גמר חלק 1 - הסתברות

על ידי : מיכאל טרושקין

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**.1**

א. בערך 1/125 מהלידות זה תאומים לא זהים ו-1/300 מהלידות זה תאומים זהים. לאלביס היה את תאום שמת בלידה. מה ההסתברות שאלביס היה תאום זהה? (ניתן להניח שההסתברות להולדת בן ובת שווה ל-1/2).

first of all lets calculate the probability of having any kind of twins:

$$1/300 + 1/125 = \frac{12}{1500} + \frac{5}{1500} = \frac{17}{1500}$$

Lets also note the probabilities here:

$$E - having\ a\ twin$$
$$H - Having\ a\ similar\ twin$$
$$P(E \mid H) = 1$$
$$P(E) = 17/1500$$
$$P(H) = 1/300$$

now lets calculate the probability of having a similar twin given that there is a twin boy.

$$P(H \mid E) = P(E \mid H) * P(H) \ / \ P(E) = \frac{\frac{1}{300}}{\frac{17}{1500}} = \frac{1500}{300 \cdot 17} = \frac{5}{17}$$

if it is unkown if the twin was sister or brother then we need to multiply this by $0.5$, and the probability is then $5/34$. ( because well it can't be a sister and similar ).

ב. יש שתי קערות של עוגיות. בקערה 1 יש 10 עוגיות שקדים ו-30 עוגיות שוקולד. בקערה 2 יש 20 עוגיות שקדים ו-20 עוגיות שוקולד. אריק בחר קערה **באקראי** ובחר ממנה עוגיה **באקראי**. העוגיה שנבחרה היא שוקולד. מה ההסתברות שאריק בחר את קערה 1?

we know that arik choose a choclate cockie,
the probability of choosing choclate cockie from plat 1 is exactly 30/50
because there is 30 cockies at plate 1, and 50 total.

**another way of doing that:**

note that choosing choclate cockie from plate 1, is the same
as choosing plate 1, and then choosing choclate cockie from that plate.
in the word if plate 1.

so we can write:

$$E \ - \ choclate\ cookie$$
$$H_1 = \ plate\ 1$$

$$P(H_1 \mid E) = \frac{P(E \mid H_1) \ \cdot \ P(H_1)}{P(E)} = \frac{\frac{30}{40} \cdot \frac{1}{2}}{\frac{50}{80}} = \frac{30}{50} = \frac{3}{5}$$

בשנת 1995 חברת M&M הוסיפה את הצבע כחול. לפני השנה הזו, התפלגות הצבעים בשקית M&M נראית כך:

30% Brown, 20% Yellow, 20% Red, 10% Green, 10% Orange, 10% Tan

החל משנת 1995, ההתפלגות נראית כך:

24% Blue , 20% Green, 16% Orange, 14% Yellow, 13% Red, 13% Brown.

לחבר שלכם יש 2 שקיות M&M, אחת משנת 1994 ואחת משנת 1996 והוא לא מוכן לגלות לכם איזו שקית שייכת לאיזו שנה. אבל הוא נותן לכם סוכריה אחת מכל שקית. סוכריה אחת היא צהובה ואחת היא ירוקה. מה הסיכוי שהסוכריה הצהובה הגיעה מהשקית של 1994?

first of all lets calculate the probability of getting a yellow candy from the 1994 edition.
note that given that we choose a yellow candy the probability of choosing it from Y94 or Y96 is 100% so then we conclude that:

$$P(Y94) = \frac{0.2}{0.34} = 0.588$$

we also given that there was a second candy from the other edition, and we need to consider that in! ( for instance if green from 94 was 0% then g from 96 is 100% and y94 with g96 must be 100%).

so now we can denote, the probability of getting yellow candy from 1994, and the green from 1996.
would be the probability of gettining Yellow from 94, and green from 96.
from all the other options, that is Y94, G96, and Y96, G94.

so we can write:

$$H - Y94 \; and \; G96$$
$$E - Y \; candy \; and \; G \; candy \; are \; from \; seperate \; editions.$$
$$P(H \mid E) = \frac{P(Y94) \cdot P(G96)}{P(Y94) \cdot P(G96) + P(Y96) \cdot P(G94)}$$

Lets write all the variable we need:

$$P(Y94) = \frac{20}{34}, \; P(Y96) = \frac{14}{34}$$
$$P(G96) = \frac{2}{3}, \; P(G94) = \frac{1}{3}$$

now lets plug in the values.

$$P(H \mid E) = \frac{\frac{20}{34} \cdot \frac{2}{3}}{\frac{20}{34} \cdot \frac{2}{3} + \frac{14}{34} \cdot \frac{1}{3}} = \frac{\frac{20}{51}}{\frac{27}{51}} = \frac{20}{27} = 0.74074$$

so we got that the probability of yellow from 1994 and green from 1996 is about 0.74074 and we can also agree with that probability as the probability of green from 1996 is higher, then green from 94, and for yellow its more likely to be from 1994.

3. הלכת לדוקטור בעקבות ציפורן חודרנית. הדוקטור בחר בך **באקראי** לבצע בדיקת דם הבודקת שפעת חזירים. ידוע סטטיסטית ששפעת זו פוגעת ב-1 מתוך 10,000 אנשים באוכלוסייה. הבדיקה מדויקת ב-99 אחוז במובן שההסתברות ל false positive היא 1%. הווה אומר שהבדיקה סיווגה בטעות אדם בריא כאדם חולה היא 1 אחוז. ההסתברות ל- false negative היא 0 – אין סיכוי שהבדיקה תגיד על אדם החולה בשפעת חזירים שהוא בריא. בבדיקה יצאת חיובי (יש לך שפעת).

א. מה ההסתברות שיש לך שפעת חזירים?

ב. נניח שחזרת מתאילנד לאחרונה ואתה יודע ש-1 מתוך 200 אנשים שחזרו לאחרונה מתאילנד, חזרו עם שפעת חזירים. בהינתן אותה סיטואציה כמו בשאלה א, מה ההסתברות (המתוקנת) שיש לך שפעת חזירים?

First lets write the variables:
$H-\ sick,\ E-positive\ test$, and we need to compute $P(H|E)$.

$$P(H \mid E) = \frac{P(E \mid H)\cdot P(H)}{P(E)} = \frac{P(E \mid H)\cdot P(H)}{P(E \mid H)\cdot P(H) + P(E \mid !H)\cdot P(!H)}$$

plug in the given values:

$$P(E \mid H) = 1$$
$$P(E \mid !H) = 0.01$$
$$P(H) = \frac{1}{10000},\ P(!H) = \frac{9999}{10000}$$

plug in the values into the formula from above:

$$P(H \mid E) = \frac{\frac{1}{10000}}{\frac{1}{10000} + \frac{1}{100}\cdot\frac{9999}{10000}} = \frac{1}{1+\frac{9999}{100}} = \frac{100}{10099} \approx 0.0099 \approx 0.99\%$$

hence we got that the probability of a random person being sick is only about $1\%$ even when the test is positive.

b. now we know that the probability H is different, because we are no longer "random".

$$P(H) = \frac{1}{200},\ P(!H) = \frac{199}{200}$$

compute

$$P(H \mid E) = \frac{\frac{1}{200}}{\frac{1}{200} + \frac{1}{100}\cdot\frac{199}{200}} = \frac{100}{100+199} = \frac{100}{299} \approx 33\%$$

that is if the likelyhood someone is sick is 1 in 200, and he got possitive test, then the probability that he is sick is about 33%.

# Random Variables

1. Roi is playing a dice game with Yael.

Roi will roll 2 six-sided dice, and if the sum of the dice is divisible by 3, he will win 6$. If the sum is not divisible by 3, he will lose 3$.

**What is Roi's expected value of playing this game?**

the expected value of getting a sum devisible by 3, is getting the sum: 3, 6, 9, 12
hence the expected value is: Denote $D - sum\ devisible,\ !D - sum\ not\ devisible$
would be $6 \cdot D3 + !D3 \cdot (-3)$

of course $!D3 = 1 - D3$, so lets calculate D3.
D3 is the sum of the probabilities getting 3, 6, 9, 12.

$$P(3) = \{1:2,\ 2:1\} = \frac{2}{36},\ P(6) = \{1:5,\ 2:4,\ 3:3,\ 4:2,\ 5:1\} = \frac{5}{36}$$
$$P(9) = \{3:6,\ 4:5,\ 5:4,\ 6:3\} = \frac{4}{36},\ P(12) = \{6:6\} = \frac{1}{36}$$

$$P(D3) = \frac{12}{36}$$

hence the expected value is : $6 \cdot \frac{12}{36} - 3 \cdot \frac{24}{36} = 0$

2. Sharon has challenged Alex to a round of Marker Mixup. Marker Mixup is a game where there is a bag of $5$ red markers numbered $1$ through $5$, and another bag with $5$ green markers numbered $6$ through $10$.

Alex will grab $1$ marker from each bag, and if the $2$ markers add up to more than $12$, he will win $5\$$, 5. If the sum is exactly $12$, he will break even, and If the sum is less than $12$, he will lose $6\$$.

the expected value of alex playing marker mixup:

is the probability of getting more then 12 times 5, plus the probability of getting less then 12 times 6.

note that the probability of getting less then 12 is 1 minus the probability of getting 12 and above.

the probability of getting 12 and above.
lets calculate it by calculating by saying he got X from bag A and what is the probability getting Y+.

lets calculate it by summing the number of way's to get 12+.
Bag 1: 1 → 0, 2 → 0, 3 → 1, 4 → 2, 5 → 3.
The total number of possibilities is : $25$.
hence the probability of getting $12+$ is : $\frac{6}{25}$.

the probability of getting exactly 12. is when we get : $2:10,\ 3:9,\ 4:8,\ 5:7$
hence the probability is: $\frac{4}{25}$

then the expected value is:

$$\frac{6}{25}\cdot 5+\frac{15}{25}\cdot(-6) = \frac{6}{5}-\frac{18}{5} = -2.4$$

3. A division of a company has 200 employees, 40%, percent of which are male. Each month, the company randomly selects 8 of these employees to have lunch with the CEO.

**What are the mean and standard deviation of the number of males selected each month?**

there is 80 males in the company.
each month, the company selects 8 random employees, selection 8 random,
from a poll of 200 people when 40% are males would mean that 40% of those 8 are males.

hense on everate there would be 3.2 selected each month, of couse we can't select 3.2 males.
so we would assume that each 5 month 4 males selected and every other month 3 selected, on average.

assuming the 8 candidates are random we can assume,
that on average every 4 month 40% of the choosen are males,
hence from 40 people choosen every 5 month 16 would be males,
with 3-4 males being choosen every month we would get the following set:

$$\begin{array}{lcccccc} Month & 1 & 2 & 3 & 4 & 5 & Total \\ Males & 3 & 3 & 3 & 3 & 4 & 16 \end{array}$$

with leads us to 3.2 mean.
now lets calculate the standart deviation:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2}$$
$$\mu = 3.2,\ N = 5$$

$$\sigma = \sqrt{\frac{0.16 + 0.64}{4}} \approx 0.4472$$

again so the mean males participating is 3.2 but we cannot take 3.2 males, so we would have to pick between 3 to 4 males each month, a resonable sample of what would happen is the sample from above.

we can also assume this is actually what's happening and we will get STD:

$$\sigma = \sqrt{\frac{0.16 + 0.64}{5}} = 0.4$$

Different dealers may sell the same car for different prices. The sale prices for a particular car are normally distributed with a mean and standard deviation of 26,000$ and 2,000$, respectively. Suppose we select one of these cars at random. Let X = the sale price (in thousands of dollars) for the selected car.

**Find P(26<X<30),**

$$\mu = 26,\ \sigma = 2$$

$$Z = \frac{x-\mu}{\sigma} = \frac{30-26}{2} = 2$$

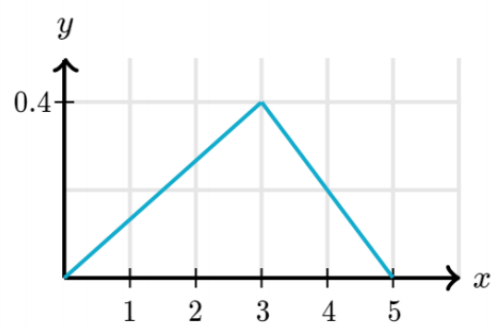
נמיר למצב שבוא הממוצע הוא 0, ונשתמש בנוסחא הבא:

$$\phi(x) = \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

הפונקציה הבאה היא ההסתברות שמישהו במרחק $x$ (לפי סיגמה) מהממוצע.
אצלנו אנחנו מסתכלים על הסתברות להיות בתווח 0 עד 2, בפונקציה.
ולכן על מה שעלינו לעשות זה לחשב את:

$$P(26 < x < 30) = \int_0^2 \phi(x)dx = 0.4772$$

5. Given the following distribution, what is P(x>3)?




נחשב את השטח המשולש שמשמאל ל 3. נשים לב כי גובה המשולש הוא 0.4 והרוחב שלו הוא 2.
ולכן:

$$P(x > 3) = \frac{0.4*2}{2} = 0.4$$

היות וחישבנו את ההסתברות של $P(x \geq 3) = 0.4$ נסיק שההסתברות לממש גדול מ-3, יהיה כמעט שואף ל 0.4.

6. A company has 500 employees, and 60% of them have children. Suppose that we randomly select 4 of these employees.

What is the probability that exactly 3 of the 4 employees selected have children?

the company has 500 emplyees 60% of them have children hence $\frac{300}{500}$ have children,
the probability of having 3 out of 4 randomly choosen have children is then:

$$\frac{300}{500}\cdot\frac{299}{499}\cdot\frac{298}{498}\cdot\frac{200}{497}=0.086$$

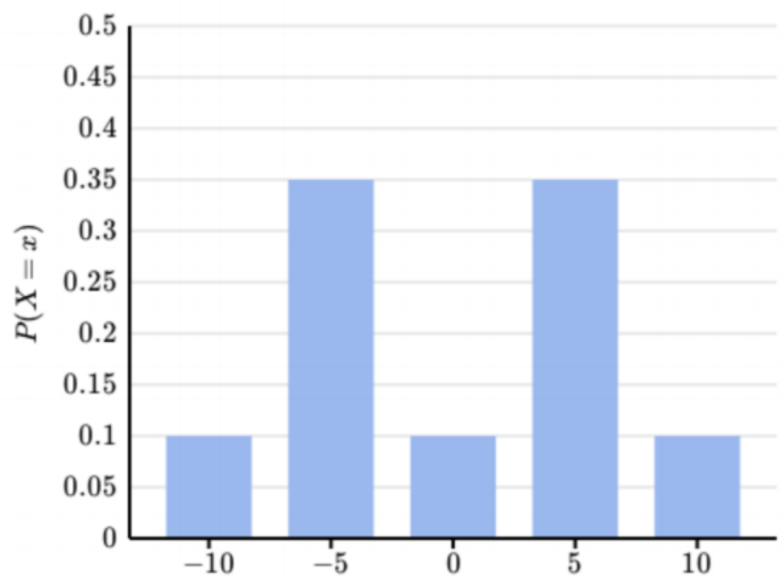
this is the probability of 3 out of 4 being with children but there is 4 ways to get this result.
either the first one without children or the second one, of the third one, or the forth one.

so in reallity we will get this figure:

$$\frac{300}{500}\cdot\frac{299}{499}\cdot\frac{298}{498}\cdot\frac{200}{497}\cdot\binom{4}{1}=0.34629$$

from a simple checkout we can see that by this formula we get a probability of 1,
if we combine 0 out of 4 1 out of 4 etc...
so we can see that this is indeed the correct formula.

## 7. Look at the next Graph. What is the expected value of X?




well the graph is simetrical and the mean is 0, so we can conclude that the expected value is the mean, hence, the expected value of x is 0.