

$$d = 942074$$

1.  $4G =$

(103388573995635080359749164254216598308788835304023601477803095234286494993683,  
37057141145242123013015316630864329550140216928701153669873286428255828810018)

2.  $5G =$

(21505829891763648114329055987619236494102133314575206970830385799158076338148,  
98003708678762621233683240503080860129026887322874138805529884920309963580118)

3.  $Q = dG = 942074G =$

(105071373288702886554749698371318794802666861735086494955172518052502509427025,  
73435797439995586110931057112850462637487255569759910087762908567935386067993)

4. First,  $d = 942074 = (1110010111111111010)_2$  in binary representation.

For every *zero* in  $d$ , a double is required; and for every *one* in  $d$ , a double and an addition are required (excluding the first 1, which is the initial setting). Since there are 15 *ones* and 5 *zeros* in  $d$ , a total of  $(15 - 1) + 5 = 19$  **doubles** and  $(15 - 1) = 14$  **additions** are required.

5. When observing multiple consecutive *ones* in the binary representation of a number (with a preceding *zero*), we can view them as multiple consecutive *zeros* (with a preceding *one*) subtracted by 1. For example,

$$(\cdots 0111)_2 = (\cdots 1000)_2 - 1.$$

Thus, when there are  $n$  consecutive *ones*, we can calculate the result using  $(n + 1)$  doubles and 2 additions (one for the leading *one* and one for adding the inverse) instead of  $(n + 1)$  doubles and  $(n + 1)$  additions.

For the case when  $d = 942074 = (1110010111111111010)_2$  where the consecutive *ones* are highlighted in red:

- **111**: 2 doubles, 2 additions,
- **001**: 3 doubles, 1 addition,
- **01111111111**: 11 doubles, 2 additions,
- **010**: 3 doubles, 1 addition.

The result can be computed using a total of **19 doubles** and **6 additions** (one of which is an inverse addition).

## 6. Input

```
1  # Definition of secp256k1
2  F = FiniteField(0xFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFC2F)
3  CURVE = EllipticCurve([F(0), F(7)])
4  N = FiniteField(CURVE.order())
5
6  # Base point
7  GX = 0x79BE667EF9DCBBAC55A06295CE870B07029BFCDB2DCE28D959F2815B16F81798
8  GY = 0x483ADA7726A3C4655DA4FBFC0E1108A8FD17B448A68554199C47D08FFB10D4B8
9  G = CURVE(GX, GY)
10
11 d = int(942074) # Private key
12 Q = d * G      # Public key
13
14 # Random transaction from blockchain
15 z = int(0x3217F8EF32F55DCED1C50F4AB0C35D551C23D2D293264AFDBBB436D8E09CA0E7)
16
17 # ECDSA Signing
18 k = N.random_element()
19 kG = int(k) * G
20 x1 = kG.xy()[0]
21 r = N(x1)
22 s = (1 / k) * (z + r * d)
23
24 # ECDSA Verifying
25 w = N(1 / N(s))
26 u1 = N(z * w)
27 u2 = N(r * w)
28 x1 = (int(u1) * G + int(u2) * Q).xy()[0]
29 print("r == x1:", int(r) == int(x1))
```

## Output

```
1  ('r == x1:', True)
```