Initialization & Library Loading

Circular Motion Worldline

```
In[ ]:= SetAttributes[r, Constant];
        SetAttributes[ω, Constant];
        $Assumptions = r > 0 && ω > 0 && r ω < 1 && λ ≥ 0;

        CircularMotion[r_, ω_] := t ↦ mkFourVector[t, r Cos[ω t], r Sin[ω t]];

In[ ]:= CircularMotion[r, ω][t]
Out[ ]= FourVector[t, r Cos[t ω], r Sin[t ω], 0]
```

Proper Time

```
In[ ]:= Dtau[CircularMotion[r, ω]][λ]
```

$$\sqrt{1 - r^2 \omega^2}\ \mathrm{Dt}[\lambda]$$

```
In[ ]:= ProperTime[CircularMotion[r, ω]][λ]
```

$$\lambda \sqrt{1 - r^2 \omega^2}$$

```
In[ ]:= ProperTimeToFrameTime[CircularMotion[r, ω]][τ] // Simplify
```

$$\left\{\frac{\tau}{\sqrt{1 - r^2 \omega^2}}\right\}$$

```
In[ ]:= FrameTimeToProperTime[CircularMotion[r, ω]][t] // Simplify
```

$$\left\{t \sqrt{1 - r^2 \omega^2}\right\}$$

```
In[ ]:= ProperTimeParametrization[CircularMotion[r, ω]][τ] // Simplify
```

$$\left\{\mathrm{FourVector}\left[\frac{\tau}{\sqrt{1 - r^2 \omega^2}}, r \cos\left[\frac{\tau\, \omega}{\sqrt{1 - r^2 \omega^2}}\right], r \sin\left[\frac{\tau\, \omega}{\sqrt{1 - r^2 \omega^2}}\right], 0\right]\right\}$$

Frame Properties

```
In[ ]:= FrameVelocity[CircularMotion[r, ω]][λ]
Out[ ]= {-r ω Sin[λ ω], r ω Cos[λ ω], 0}

In[ ]:= FrameAcceleration[CircularMotion[r, ω]][λ] // Simplify
Out[ ]= {-r ω^2 Cos[λ ω], -r ω^2 Sin[λ ω], 0}
```

Velocity and Acceleration

In[ ]:= **FourVelocity[CircularMotion[r, ω]][λ]**

Out[ ]= $\text{FourVector}\left[\frac{1}{\sqrt{1-r^2\,\omega^2}}, -\frac{r\,\omega\,\text{Sin}[\lambda\,\omega]}{\sqrt{1-r^2\,\omega^2}}, \frac{r\,\omega\,\text{Cos}[\lambda\,\omega]}{\sqrt{1-r^2\,\omega^2}}, 0\right]$

In[ ]:= **ProperVelocity[CircularMotion[r, ω]][λ] // Simplify**

Out[ ]= 1

In[ ]:= **FourAcceleration[CircularMotion[r, ω]][λ]**

Out[ ]= $\text{FourVector}\left[0, \frac{r\,\omega^2\,\text{Cos}[\lambda\,\omega]}{-1+r^2\,\omega^2}, \frac{r\,\omega^2\,\text{Sin}[\lambda\,\omega]}{-1+r^2\,\omega^2}, 0\right]$

In[ ]:= **ProperAcceleration[CircularMotion[r, ω]][λ] // Simplify**

Out[ ]= $-\frac{i\,r\,\omega^2}{-1+r^2\,\omega^2}$