Height $2(p-1)$

Note Title 7/16/2008

(All joint w/ Hopkins & Ravenel)

<u>Outline</u>

I. Dramatis personae

II. Legerdemain: differentials for nothin', extns for free

III. Hopes for the future

I. E_n, $\mathbb{S}_n$ and formal groups.

Formal groups over k, char $k>0$, (k alg closed) are determined by their height.

Let F_n be one such, and $\mathbb{S}_n$ its automorphism group.

<u>Prop</u> $\mathbb{S}_n$ is a p-adic Lie group of <u>virtual cohom. dimension</u> n^2.

ignore finite s.g

Given a perfect field k and a fgl F of ht n over k, there is a deformation theory of F worked out by Lubin & Tate.

<u>Thm</u> (Hopkins-Miller) There is a functor

$$\{(k,F),\ \text{isos}\} \longrightarrow E_\infty\text{-ring spectra}$$
$$(k,F) \longmapsto E(k,F)$$

If $F=$ Honda f.g.l. and $k=\mathbb{F}_{p^n}$, then write E_n for $E(k,F)$. = "Lubin-Tate spectrum"

This gets an action of $\mathbb{S}_n$ $(\rtimes \mathrm{Gal}(k/\mathbb{F}_p))$.

<u>Thm</u> (Morava) The Adams-Novikov E_2 term for $L_{K(n)}S^0$ is

$$H^*(\mathbb{S}_n; E_{n*}).$$

<u>Thm</u> (Devinatz & Hopkins) As E_∞ ring spectra

$$L_{K(n)}S^0 = E_n^{h\mathbb{S}_n} \text{ "=" } F_{\mathbb{S}_n}(E\mathbb{S}_{n+}, E_n)$$

Hard to compute! Idea: maybe finite s.g. see enough.

If $G \le \mathbb{S}_n$, then $EO_n(G) = E_n^{hG}$

If $(p-1)\nmid n$, then $p \nmid |G|$ for all $G \le \mathbb{S}_n$. (related to fact that $\dim_{\mathbb{Q}_p} \mathbb{Q}_p[\zeta_{p^n}] = p^{n-1}(p-1)$)

So we will restrict attention to $n = f\cdot(p-1)$

Also restrict to $G = \mathbb{Z}/p$ or $\mathbb{Z}/p \rtimes \mathbb{Z}/(p-1)(p^f-1)$.

"Thm" As a $\mathbb{Z}/p$ module,

$$E_{n*} = \mathbb{Z}_{p^n}[\![u_1, \dots, u_{n-1}]\!][u^{\pm 1}] \cong S(f\cdot\bar{\rho})[\Delta^{-1}]^{\wedge}_{I},$$

where $\bar{\rho} = \mathbb{Z}_{p^n}[\mathbb{Z}/p] / (1+g+\dots+g^{p-1})$, and $\Delta = 1\cdot g\cdot \dots \cdot g^{p-1} \in S^p(f\cdot\bar{\rho})$.

This lets us compute the AN E_2-term (modulo stuff in H^0) Same Δ

<u>Cor</u> The ANSS E_2-page for $\pi_* EO_n(\mathbb{Z}/p)$ is

$$\underbrace{E(h_{1,0}, \dots, h_{f,0})}_{\substack{\text{correspond in the bar} \\ \text{complex to}}} \otimes P(\underset{H^2(\mathbb{Z}/p;\mathbb{Z})}{\beta}) \otimes \mathbb{Z}_{p^n}[\![\delta_1, \dots, \delta_{f-1}]\!][\Delta^{\pm 1}]$$

corresponding classes for other summands / Δ

$$\underset{\text{via BP}}{t_i} \longleftrightarrow \underset{\text{via } H\mathbb{F}_p}{\bar{\xi}_i}$$

II. How do we get differentials?

1. Compare w/ s.s. we know. often leads to
2. Pray for wisdom

We'll compare w/ 2 kinds of SS:

1. X^{hG}, X finite
2. $(E_n \wedge T)^{hG}$, T choosen well.

"Thm" There are differentials

- $d_{2p^i-1}(\Delta^{p^{i-1}}) = h_{i,0}\beta^{p^i-1}\Delta^{p^{i-1}}$
- (Toda diff)
- $d_{2p^i-1}(\beta^{p^{i-1}}) = h_{i,0}\beta^{p^i+p^{i-1}-1}$
- $d_{2p-1}(h_{i,0}) = h_{1,0}\beta^{p-1}h_{i,0}$

} Power operations on $\Delta \mapsto h_{1,0}\beta^{p-1}\Delta$ + (2)

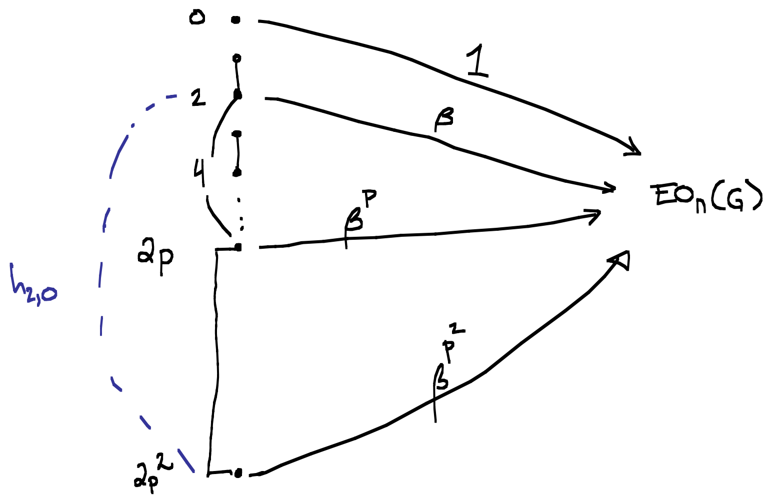
(!!) } (1)

For differentials on β^k, use unit map:

$S^0 \longrightarrow E_n$ is G-equivariant $\Rightarrow$

$(S^0)^{hG} \longrightarrow EO_n(G)$

$F_G(EG, S^0) = D(BG) =$



$\Rightarrow \quad d_*(\beta) = h_{1,0}\beta^p + h_{2,0}\beta^{p^2} + \dots + h_{i,0}\beta^{p^i}$

For differentials on $h_{i,0}$: $\quad (S[\mathbb{Z}/p]/S) \xrightarrow{\mathbb{Z}/p} E_n$ & same argument.

Height $2(p-1)$

$$E(h_{1,0}, h_{2,0}) \otimes P(\beta) \otimes \mathbb{Z}_{p^n}[[S_1]][\Delta] \qquad \langle h_{1,0}, \dots, h_{1,0} \rangle = \beta \Delta^{p-1} \cdot \delta_1$$

$$(\Delta, \beta, h_{2,0}) \longmapsto h_{1,0}\beta^{p-1}(\Delta, \beta, h_{2,0}) \qquad \langle h_{2,0}, \dots, h_{2,0} \rangle = \beta \Delta^{p^2-1}$$

The classes $h_{2,0}$ & β are not cycles but we can multiply by big enough powers of Δ to make a cycle!

$$\left.\begin{aligned} b &= \beta \Delta^k \\ a_i &= h_{i,0}\Delta^{e_i} \end{aligned}\right\} \text{ These give us systems that we can use to find } k, e_i$$

$$\Delta^p \longmapsto \underbrace{(a_2 \Delta^{-e_2})(b\Delta^{-k})^{p^2-1}\Delta^p}_{a_2 b^{p^2-1} \underbrace{\Delta^{-e_2+p+k-kp^2}}_{\text{cycle}}}$$

$$a_2(\Delta^p)^{p-1} \longmapsto \langle a_2, a_2 b^{p^2-1}\Delta^{-e_2+p+k-kp^2}, \dots, a_2 b^{p^2-1}\Delta^{-e_2+p+k-kp^2} \rangle$$
$$= (b\Delta^{-k}) \cdot (\Delta^{-pe_2}) \cdot b^{(p^2-1)(p-1)} \Delta^{(p-1)(-e_2+p+k-kp^2)}$$

$\Delta \longmapsto h_{1,0}(b\Delta^{-k})^{p-1}\Delta$ and the Toda bracket on this gives another system

This leaves us with classes

classes			b torsion
$[a_1 \Delta^{i+pj}]$	$0 \le j \le p-1$	$0 \le i \le p-2$	b^{p-1}
$\delta_1 \Delta^{pj}$			$b^{(p-1)^2+1}$
$[a_2 \Delta^{pi}]$	$0 \le i \le p-2$		b^{p^2-1}
1			$b^{(p-1)(p^2-1)+1}$