RESEARCH SUMMARY: MICHAEL HILL

My research focuses on computational algebraic topology. I am especially interested in the use of group actions to isolate infinite phenomina in the stable homotopy groups of spheres.

1. Past Research

Shimura Curves and Homotopy. Algebraic topology has increasingly used algebraic geometry to produce interesting cohomology theories. Building on foundational work of Lurie and using techniques that Behrens-Lawson use to generalize the Goerss-Hopkins-Miller spectrum tmf, Lawson and I studied topological automorphic forms spectra associated to Shimura curves of small discriminant. This yielded several results: the computation of several rings of automorphic forms, a proof that a version of the truncated Brown-Peterson spectrum $BP\langle 2\rangle$ is an E_∞ ring spectrum at the prime 3, and an unexpected connection between the topological automorphic forms spectrum associated to discriminant 10 and tmf.

Kervaire Invariant One Problem. Hopkins, Ravenel, and I recently showed that there are no smooth manifolds of Kervaire invariant one of dimension larger than 126, settling one of the oldest and most significant outstanding problem in algebraic topology. Using Browder's reformulation as the survival of certain classes in the Adams spectral sequence, we translated the problem into one in equivariant homotopy theory, reframing it in terms of certain equivariant homotopy groups of a spectrum $MU^{(G)}$ which is constructed out of MU by tensoring up.

We developed a new spectral sequence, the Slice Spectral Sequence, that computes the homotopy groups of the fixed points for an equivariant spectrum. This generalizes work of Dugger and emulates the Morel-Hopkins and Voevodsky constructions in the motivic setting. For spectra built out of $MU^{(G)}$ by localizing or completing, the E_2-term of the slice spectral sequence is very easy to compute and has several extremely nice properties. In particular, it is immediate that various homotopy groups vanish for these spectra.

Our solution included several results of independent interest. We determined a basic family of differentials in in the slice spectral sequence for $MU^{(G)}$. This gave rise to a series of periodicity results in a family of localizations of this spectrum. We also showed that for this same family of localizations, the fixed points and homotopy fixed points agree. In particular, the slice spectral sequence and the differentials discovered apply in these cases to compute the homotopy fixed points.

This past year was spent working on several related facets of this. Showing many of the expected and intuitive relationships between classical equivariant constructions and our norm functor (which gives rise to $MU^{(G)}$) necessitated some careful analysis and retoolings of the material in the literature. This newfound understanding gave very clean proofs of several general results about slices and lead to a greater ability to comprehend what happens at odd primes. This has resulted in several empirically supported conjectures about slice differentials for odd primes, and systematic patterns in the differentials at all primes. In particular, our conjectures perfectly reproduce several homotopical computations I had done in the work with Lawson.

2. Immediate Research Plans

Homotopy Groups of Spheres and Higher Real K-theories. I plan to compute the 2-local homotopy groups of the higher real K-theory spectra $EO_n(G)$ for G cyclic. The $K(n)$-local sphere is the homotopy fixed points of the action of the Morava stabilizer group on the Lubin-Tate spectrum E_n. To understand the homotopy, we must first understand the action of this p-adic Lie group on a power-series ring. The Hopkins-Miller higher real K-theory spectra $EO_n(G)$ are finite approximations in which we replace the Morava stabilizer group with a finite subgroup G. As the order of G increases, the size of the Hurewicz image in the homotopy of $EO_n(G)$ grows, and thus a systematic understanding of these homotopy groups provides a way to study the stable homotopy groups of spheres. For $p = 2$, the Kervaire solution shows that these homotopy fixed point spectra are essentially the same as the fixed points, and I intend to use the slice spectral sequence to compute the homotopy groups. Additionally, the periodicity results found apply here, giving an upper bound on the complexity of the homotopy fixed point spectral sequence.

Identifying the Hurewicz image of the homotopy groups of spheres is a related and more difficult problem. Building on work from this past year, we see that the filtration of equivariant homotopy groups by the image of the transfer from subgroups puts very harsh restrictions on the kinds of differentials and extensions that can occur. In particular, it seems unlikely that the homotopy element η^3 in $\pi_3 S^0$ ever shows up in the homotopy groups of $EO_n(G)$. Understanding why elements like this never show up is an interesting project.