Summary of Model Forms

Table : List of growth model equations, error distributions, and parameters

		Equation: *Deterministic*		Equation: *Stochastic*	Error Distribution	Parameters
Mean	$y_t =$	$\overline{y}$	$+$	ϵ_t	$\epsilon_t \sim N(0,\sigma^2)$	$\{\overline{y},\sigma\}$
Linear Change Model	$y_t =$	$\beta_0+\beta_1(\text{time})_t$	$+$	ϵ_t	$\epsilon_t \sim N(0,\sigma^2)$	$\{\beta_0,\beta_1,\sigma\}$
"Fixed-Effects" Model (see notes)	$y_{ti} =$	$\beta_0+\beta_1(\text{time})_{ti}+\delta_i$	$+$	ϵ_t	$\epsilon_t \sim N(0,\sigma^2)$	$\{\beta_0,\beta_1,\delta_i,\sigma\}$
Random Intercepts Model	$y_{ti} =$	$\gamma_{00}+\gamma_{10}(\text{time})_{ti}$	$+$	$\rho_{0i}+\epsilon_{ti}$	$\epsilon_{ti} \sim N(0,\sigma^2)$ $\rho_{0i} \sim N(0,\tau_{00})$	$\{\gamma_{00},\gamma_{10},\tau_{00},\sigma\}$
Random Slopes Model	$y_{ti} =$	$\gamma_{00}+\gamma_{10}(\text{time})_{ti}$	$+$	$\rho_{1i}(\text{time})_t+\epsilon_{ti}$	$\epsilon_{ti} \sim N(0,\sigma^2)$ $\rho_{1i} \sim N(0,\tau_{11})$	$\{\gamma_{00},\gamma_{10},\tau_{11},\sigma\}$
Random Intercepts and Slopes Model	$y_{ti} =$	$\gamma_{00}+\gamma_{10}(\text{time})_{ti}$	$+$	$\rho_{0i}+\rho_{1i}(\text{time})_t+\epsilon_{ti}$	$\epsilon_{ti} \sim N(0,\sigma^2)$ $\rho \sim \text{MVN}\left(0,\begin{bmatrix}\tau_{00} & \\ \tau_{10} & \tau_{11}\end{bmatrix}\right)$	$\{\gamma_{00},\gamma_{10},\tau_{00},\tau_{11},\tau_{10},\sigma\}$
Latent Class Growth Analysis (LCGA) Model (see notes)	$y_{ti\|c=k} =$	$\gamma_{0k}+\gamma_{1k}(\text{time})_{ti}$	$+$	$\epsilon_{ti},\pi_{i(c=k)}$	$\epsilon_{ti} \sim N(0,\sigma^2)$ $\pi_{i(c=k)} \sim \frac{\exp\{\eta_{ci}\}}{\sum_{k=1}^{K}\exp\{\eta_{ik}\}}$	$\{\sigma\}$ and $\{\gamma_{0k},\gamma_{1k},\pi_{i(c=k)}\}$ for each k
Growth Mixture Model	$y_{ti\|c=k} =$	$\gamma_{00k}+\gamma_{10k}(\text{time})_{ti}$	$+$	$\rho_{0ik}+\rho_{1ik}(\text{time})_t+\epsilon_{ti},\pi_{i(c=k)}$	$\epsilon_{ti} \sim N(0,\sigma^2)$ $\mathbf{T} \sim \text{MVN}\left(0,\begin{bmatrix}\tau_{00} & \\ \tau_{10} & \tau_{11}\end{bmatrix}\right)$ $\pi_{i(c=k)} \sim \frac{\exp\{\eta_{ci}\}}{\sum_{k=1}^{K}\exp\{\eta_{ik}\}}$	$\{\sigma\}$ and $\{\gamma_{0k},\ \gamma_{1k},\ \tau_{00},\ \tau_{11},\ \tau_{01},\ \pi_{i(c=k)}\}$ for each k

Notes: The "Fixed-Effects" Model refers to the econometric description of a model that accounts for variation across higher-level entities by including an indicator for $N-1$ entities represented in the equation by δ_i (therefore, δ_i has $N-1$ parameters); it *does not* refer to the "fixed" portion (i.e., deterministic component) of the growth models above. In the Latent Class Growth Analysis Model and Growth Mixture Model, η_{ik} represents the log-odds of membership in class k being drawn using the convention that η_{iK}, the log-odds of the reference category K, equal 0.

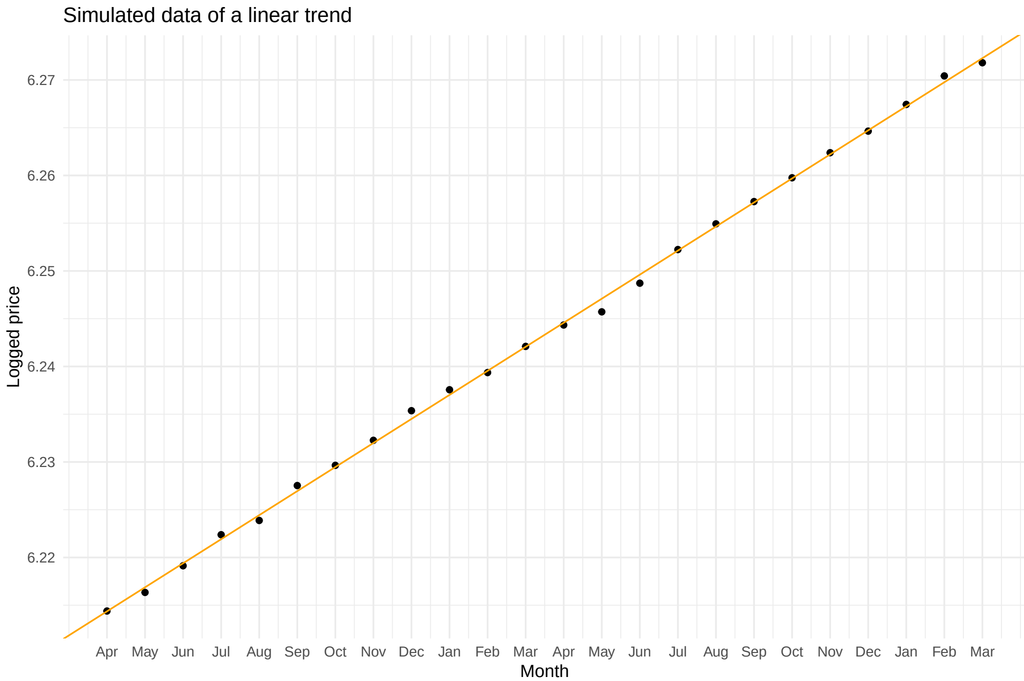
Linear Change Model

Description

The linear change model provides the basis for all other models. It measures a linear trend over time for a single entity (e.g., participant), accounting for the random variation that occurs at each point in time. The linear change model can be generalized by adding additional parameters of time to account for polynomial (e.g., squared or cubic) patterns or non-parametric patterns (e.g., splines).

The model assumes that the errors at every time t, σ_t are normally distributed around a mean of zero with variance σ^2.

Descriptive plot



Example files

01b_lineartrend_simulation.R
01b_lineartrend_analysis.R

Equation

$$\underbrace{y_t}_{\text{outcome}} = \beta_0 + \beta_1(\text{time})_t + \epsilon_t$$
$$e_t \sim N(0, \sigma^2)$$

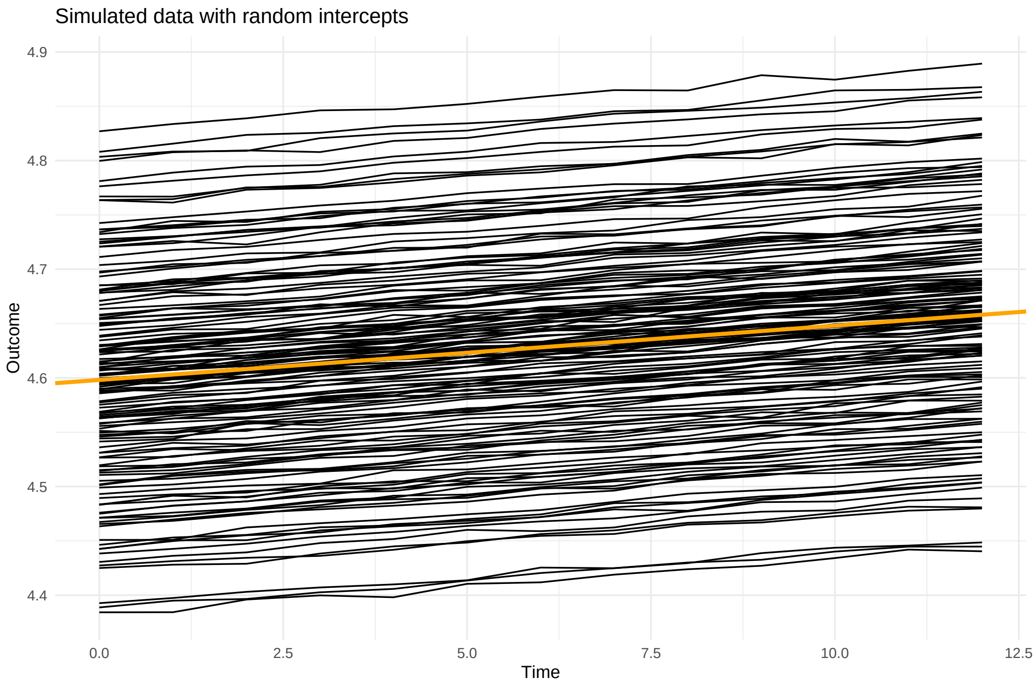
Random Intercepts Model

Description

Use the random intercepts model when you believe that entities (e.g., participants) start at different values of the outcome from one another, but change at similar rates to one another.

Assumes that the trajectories that entities follow vary according to a single trend where the errors of the intercepts (ρ_{0i}) has a mean of zero and are *continuously* distributed according to a normal distribution with a mean of zero and variance τ_{00}.

Descriptive plot



Example files

01e_randomintercepts_simulation.R
01e_randomintercepts_analysis.R

Equation

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{10}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\rho_{0i} + \epsilon_{ti}}_{\text{stochastic}}$$

$$\begin{aligned} \epsilon_{ti} &\sim N(0, \sigma^2) \\ \rho_{0i} &\sim N(0, \tau_{00}) \end{aligned}$$

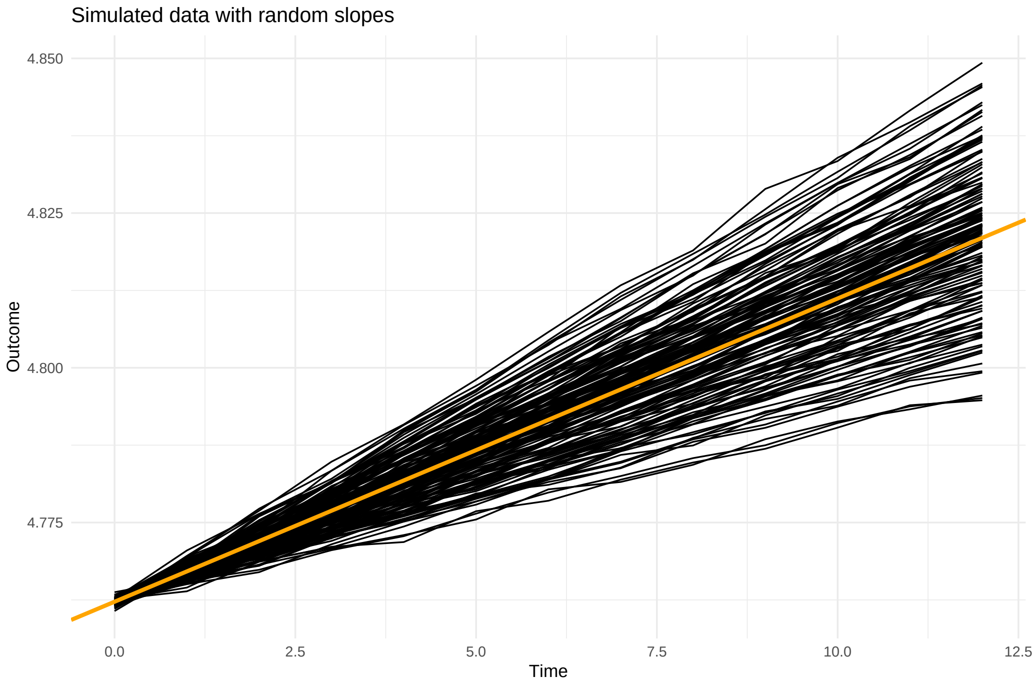
Random Slopes Model

Description

Use the random slopes model when you believe that entities (e.g., participants) start at the same (or very similar) values of the outcome and then change at different rates from one another.

The model assumes that the trajectories that entities follow vary according to a single trend, except that the slopes, ρ_{1i}, are *continuously* distributed according to a normal distribution with a mean of zero and variance τ_{11}.

Descriptive plot



Example files

01e_twocityslope_analysis.R (shows example with two metros)
01e_randomslopes_simulation.R

Equation

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{10}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\rho_{1i}(\text{time})_t + \epsilon_{ti}}_{\text{stochastic}}$$

$$\epsilon_{ti} \sim N(0, \sigma^2)$$
$$\rho_{1i} \sim N(0, \tau_{11})$$

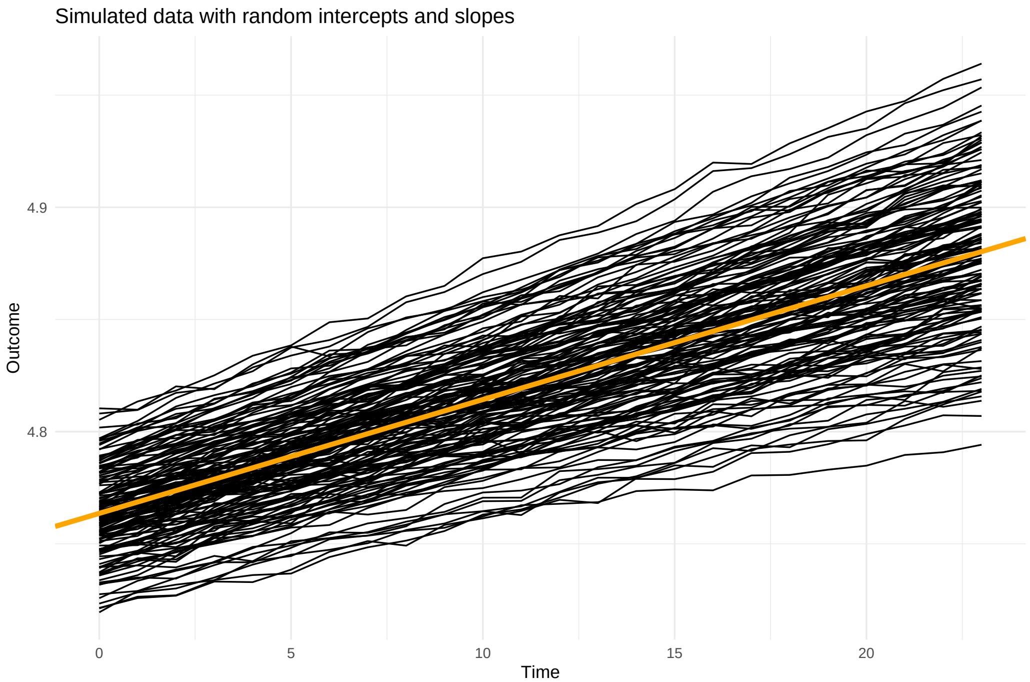
Random Intercepts and Slopes Model

Description

Use the random slopes and intercepts model when you believe that entities (e.g., participants) start at different values of the outcome from one another *and* change at different rates from one another.

Assumes that the trajectories that entities follow vary according to a single trend where the errors for both intercepts (ρ_{0i}) and slopes (ρ_{1i}) have means of zero and are distributed *continuously* with a multivariate normal distribution $\mathbf{T}$ that represents the variance/covariance matrix of ρ_{0i} and ρ_{1i}.

Descriptive plot



Example files

01f_randominterceptslope_simulation.R
01f_randominterceptslope_analysis.R

Equation

$$\underbrace{y_{ti}}_{\text{outcome}} = \underbrace{\gamma_{00} + \gamma_{10}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\rho_{0i} + \rho_{1i}(\text{time})_{t} + \epsilon_{ti}}_{\text{stochastic}}$$

$$\begin{aligned} \epsilon_{ti} &\sim N(0, \sigma^2) \\ \rho &\sim \text{MVN}\left(0, \begin{bmatrix} \tau_{00} & \\ \tau_{10} & \tau_{11} \end{bmatrix}\right) \end{aligned}$$

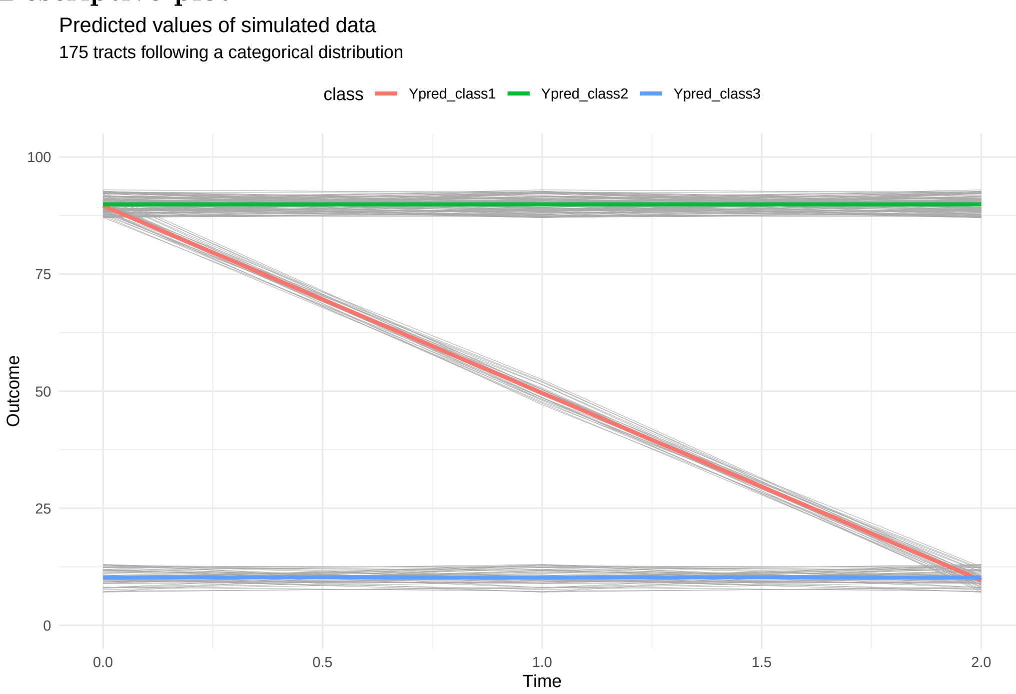
Latent Class Growth Analysis (LCGA) Model

Description

Use the latent class growth analysis model when you believe that change over time among the population of entities (e.g., participants) can best be described *categorically*. Rather than deviating from a single trend, entities follow one of a discrete number of trajectories.

The model assumes that differences between entities (other than small variations from one time to the next) can best be described as a multinomial logistic distribution where an entity has a probability $\pi_{i(c=k)}$ of being in class c where c is one of K different classes and $\sum_{k=1}^{K} \pi_{ik} = 1$.

Descriptive plot



Example files

02a_lcga_simulation.R
02a_lcga_analysis.R

Equation

$$\underbrace{y_{ti|c=k}}_{\text{outcome}} = \underbrace{\gamma_{0k} + \gamma_{1k}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\epsilon_{ti}, \pi_{i(c=k)}}_{\text{stochastic}}$$

$$\begin{aligned} \epsilon_{ti} &\sim N(0, \sigma^2) \\ \pi_{i(c=k)} &\sim \frac{\exp\{\eta_{ci}\}}{\sum_{k=1}^{K} \exp\{\eta_{ik}\}} \end{aligned}$$

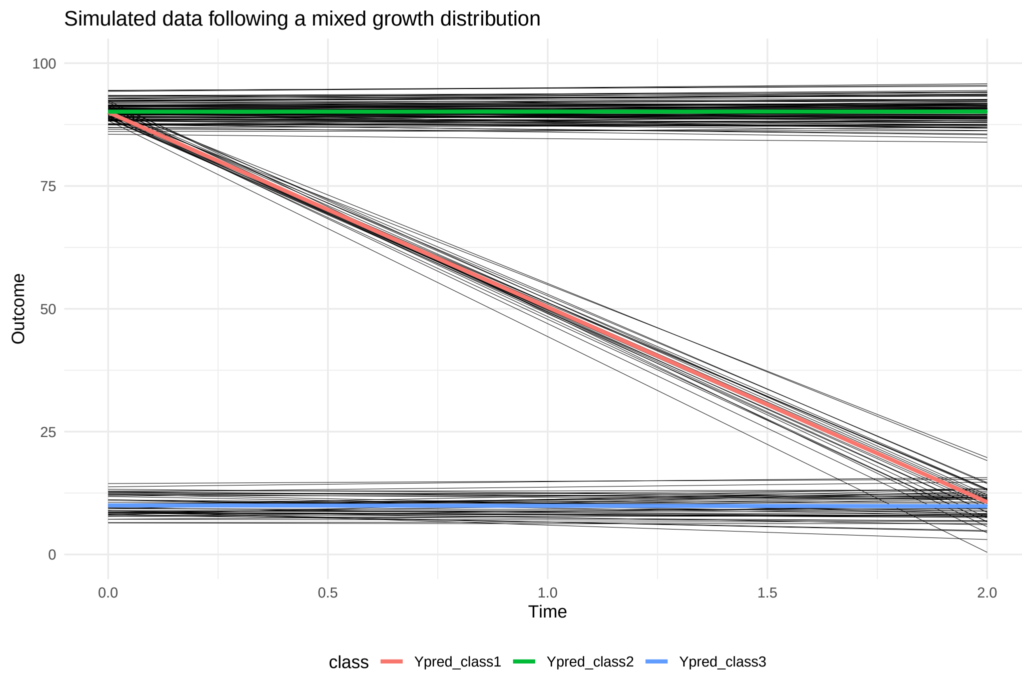
Growth Mixture Model

Description

Use the growth mixture model when you believe that the change over time among the population of entities (e.g., partipants) can best be described *categorically*, ***but*** when you also believe that individual trajectories vary around each of the different trajectories. Entities follow one of a discrete number of trajectories, but also vary *continuously* off of the predicted trend.

The model assumes that differences between entities drawn from a mixture of a multinomial logistic distribution with probability of being in class c, $\pi_{i(c=k)}$ and a continuous, multivariate normal distribution of intercepts, ρ_{0i}, and slopes, ρ_{1i}, with a variance/covariance $\mathbf{T}$.

Descriptive plot



Example files

02b_gmm_simulation.R
02b_gmm_analysis.R

Equation

$$\underbrace{y_{ti|c=k}}_{\text{outcome}} = \underbrace{\gamma_{00k} + \gamma_{10k}(\text{time})_{ti}}_{\text{deterministic}} + \underbrace{\rho_{0ik} + \rho_{1ik}(\text{time})_{t} + \epsilon_{ti}, \pi_{i(c=k)}}_{\text{stochastic}}$$

$$\begin{aligned}
\epsilon_{ti} &\sim N(0, \sigma^2) \\
\mathbf{T} &\sim \text{MVN}\left(0, \begin{bmatrix} \tau_{00} & \\ \tau_{10} & \tau_{11} \end{bmatrix}\right) \\
\pi_{i(c=k)} &\sim \frac{\exp\{\eta_{ci}\}}{\sum_{k=1}^{K} \exp\{\eta_{ik}\}}
\end{aligned}$$