Economics 144
Economic Forecasting

Lecture 2
Modeling and Forecasting Trend
(Part I)

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Today's Class

- Loss Function
- Modeling Trend
  - Linear
  - Quadratic
  - Log-linear
  - Exponential
- Model Selection
- R Demo

Loss Function

- Good forecasts lead to good decisions!

  → Strong link between forecasts and decisions.

- Example: You started a firm and need to decide (now) how much inventory to hold going into the next sales period.

Strategy

Demand is high → build inventory
Demand is low → reduce inventory

Loss Function

Loss	Demand High	Demand Low
Build Inventory	0	$10,000
Reduce Inventory	$10,000	0

Symmetric Loss Structure: Both bad outcomes have the same loss.

Loss	Demand High	Demand Low
Build Inventory	0	$10,000
Reduce Inventory	$20,000	0

Asymmetric Loss Structure: Outcomes have different losses.

Loss Function 3 of 4

- For every decision-making problem, there is an associated loss structure; for each decision/outcome pair, there is an associated loss.

$$\text{Loss} \begin{cases} 0 \rightarrow \text{Correct Decision} \\ >0 \rightarrow \text{Incorrect Decision} \end{cases}$$

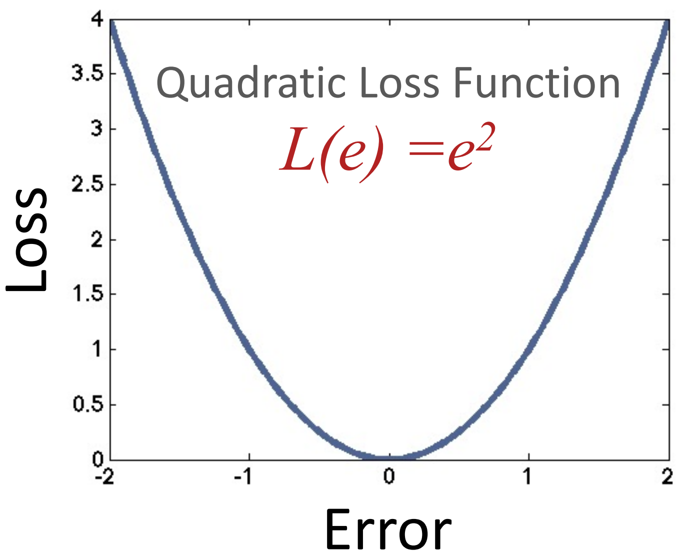
We could also forecast the sales!

Loss	High Actual Sales	Low Actual Sales
High Forecasted Sales	0	$10,000
Low Forecasted Sales	$10,000	0

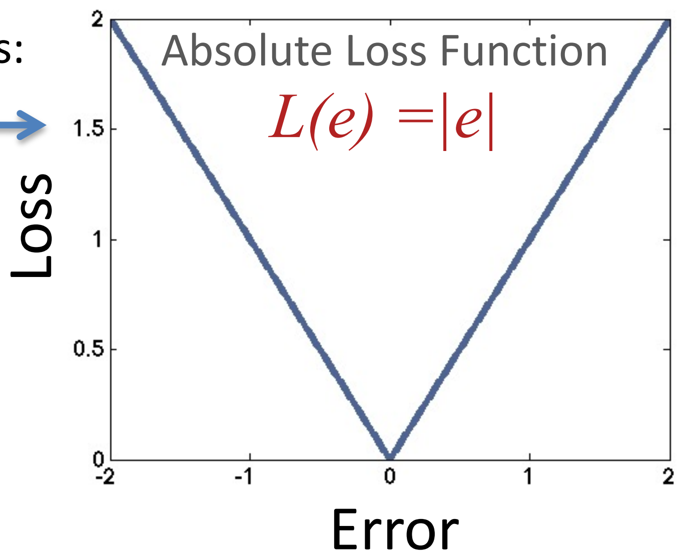
Forecasting with Symmetric Loss: Both bad forecasts have the same loss.

Loss Function

- Forecast Error (e): Difference between the realization (y) and the previously made forecast($\hat{y}$) → $e = y - \hat{y}$
- Loss Function ($L(e)$): Loss associated with a forecast. Must satisfy: (1) $L(0)=0$,(2) $L(e)$ is continuous, and (3) $L(e)$ is increasing on each side of the origin.



Examples of loss functions:



Modeling Trend

- Trend: Is slow , long-run evolution in the variables that we want to model and forecast.

- Deterministic Trend: Trend evolves in a perfectly predictable way.

- To characterize a particular trend, we need a model. For example, in the case of linear regression, the model is: $y_t = \beta_0 + \beta_1 x_t$

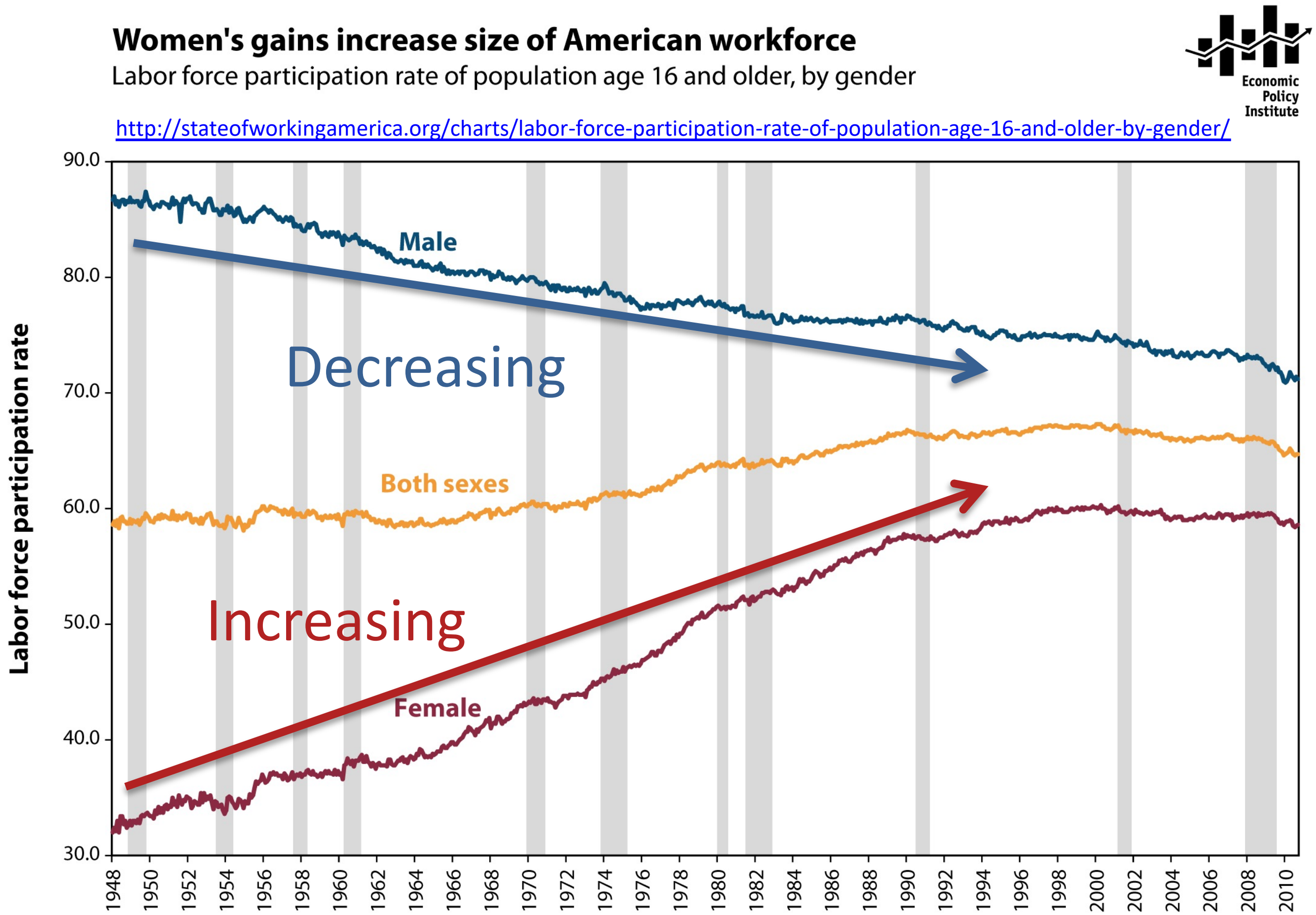
- Often, given the broad range of time scales encountered in time-series, it is convenient to adopt one common time variable (*time dummy or time trend*) such that: $TIME* = (1, 2, \ldots, T)$ where TIME=1 is the first period of the sample, and so on.

*The notation convention is $TIME_t = t$.

Modeling Trend (Linear) 1 of 4

Women's gains increase size of American workforce
Labor force participation rate of population age 16 and older, by gender

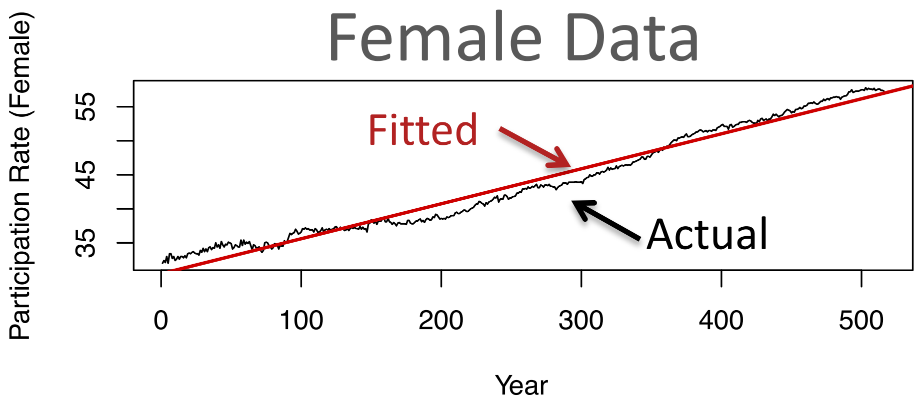
http://stateofworkingamerica.org/charts/labor-force-participation-rate-of-population-age-16-and-older-by-gender/



Note: Shaded areas denote recession.
Source: Bureau of Labor Statistics, Current Population Survey.

Model: $T_t = \beta_0 + \beta_1 \, TIME_t$

Modeling Trend (Linear)



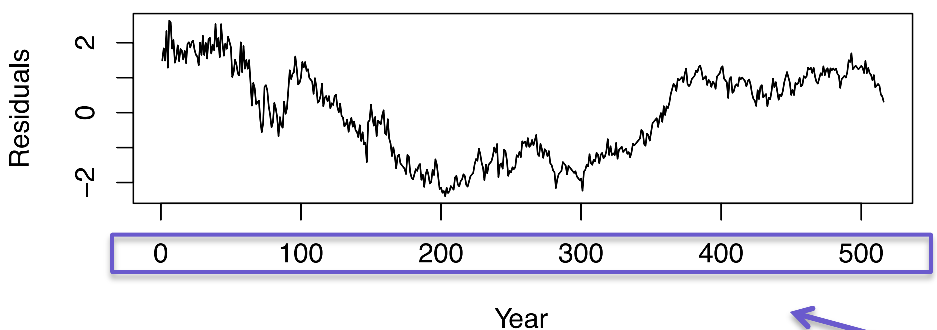

```
Call:
lm(formula = female ~ TIME)

Residuals:
    Min      1Q  Median      3Q     Max
-2.3959 -1.1994  0.2523  1.0177  2.6321

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.046e+01  1.107e-01   275.1   <2e-16 ***
TIME        5.141e-02  3.712e-04   138.5   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.256 on 514 degrees of freedom
Multiple R-squared: 0.9739, Adjusted R-squared: 0.9739
F-statistic: 1.919e+04 on 1 and 514 DF,  p-value: < 2.2e-16
```

$$Fem\_Part\_Rate = 30.46 + 0.051T$$



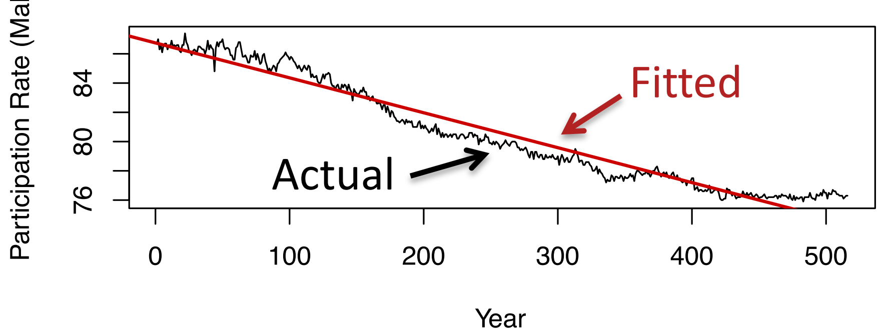
R^2 =0.97→Excellent!

Do you see anything wrong?

Time : 1948-1990

Modeling Trend (Linear)

Male Data



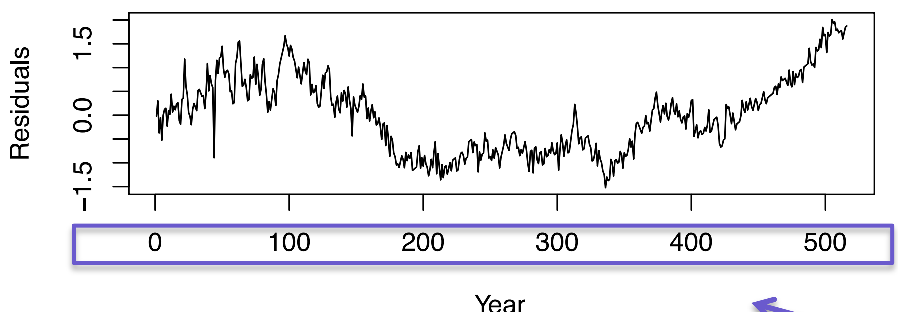

```
Call:
lm(formula = male ~ TIME)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.52321 -0.70824 -0.00946  0.51965  2.01150 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) 86.7448867  0.0699255  1240.5   <2e-16 ***
TIME        -0.0238740  0.0002344  -101.9   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.793 on 514 degrees of freedom
Multiple R-squared: 0.9528,	Adjusted R-squared: 0.9527 
F-statistic: 1.038e+04 on 1 and 514 DF,  p-value: < 2.2e-16
```

$$Male\_Part\_Rate = 86.75 - 0.024T$$

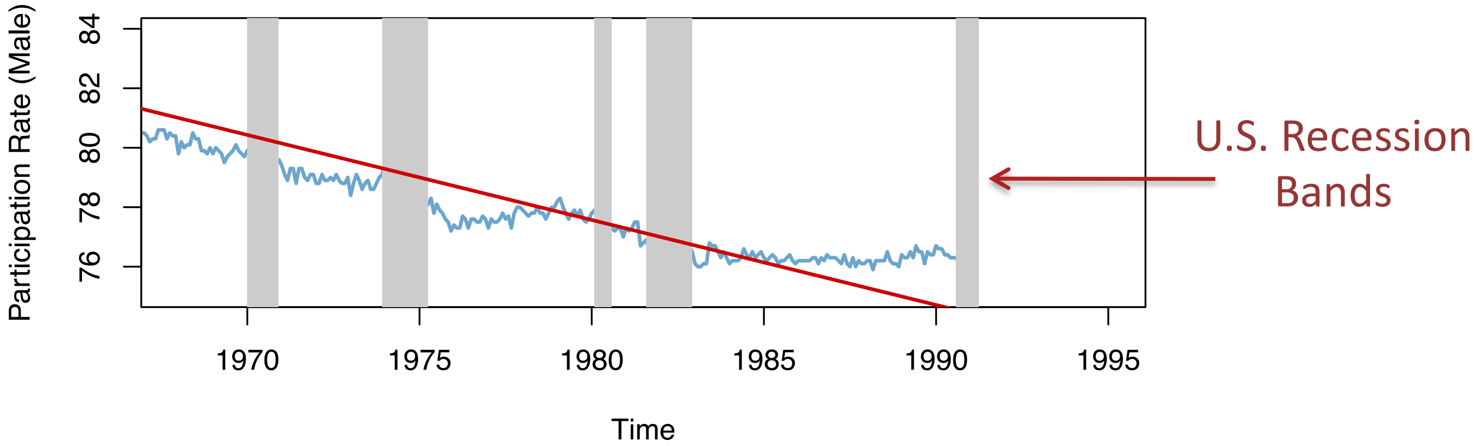


R^2 =0.95→Excellent!

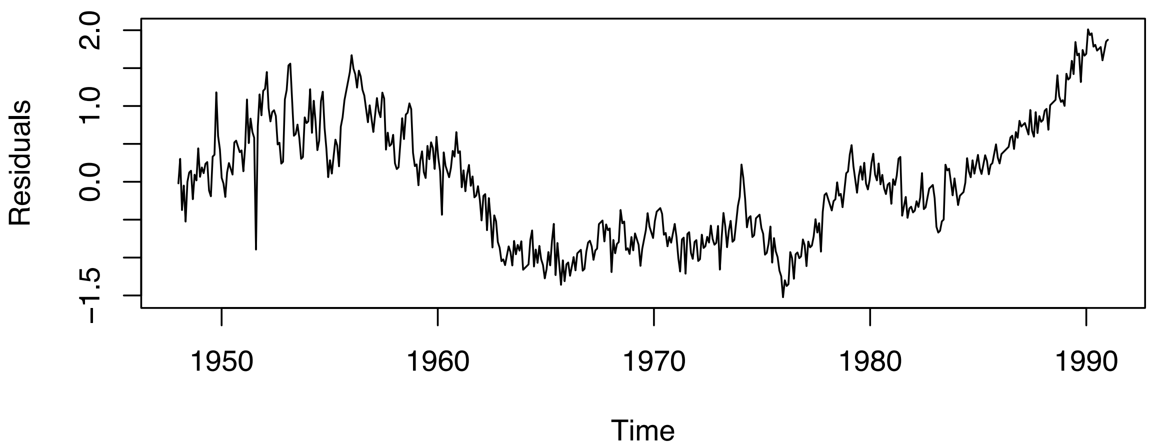
Do you see anything wrong?

Time : 1948-1990

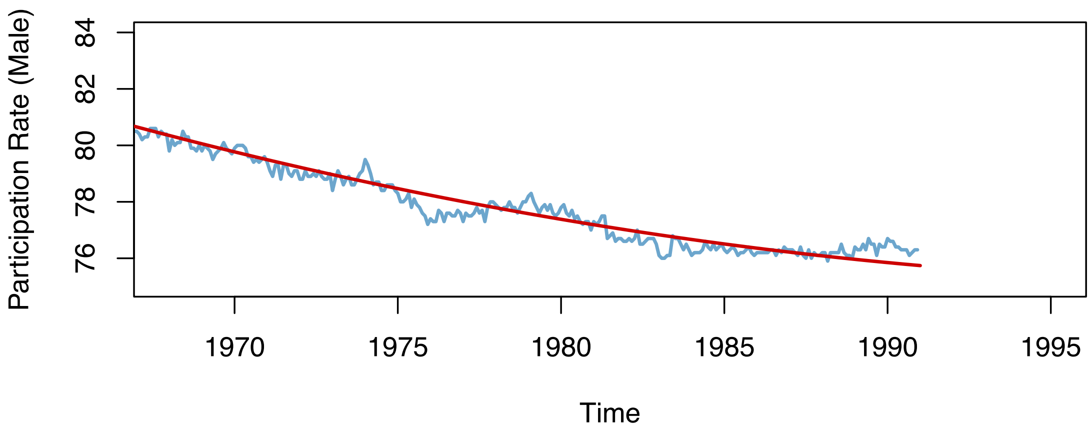
Modeling Trend (Linear)



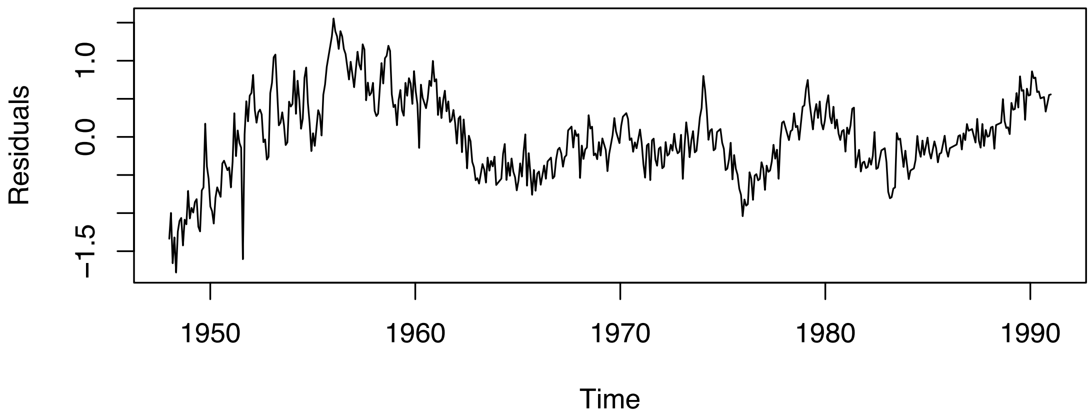
Model: $T_t = \beta_0 + \beta_1 \, TIME_{\,t}$



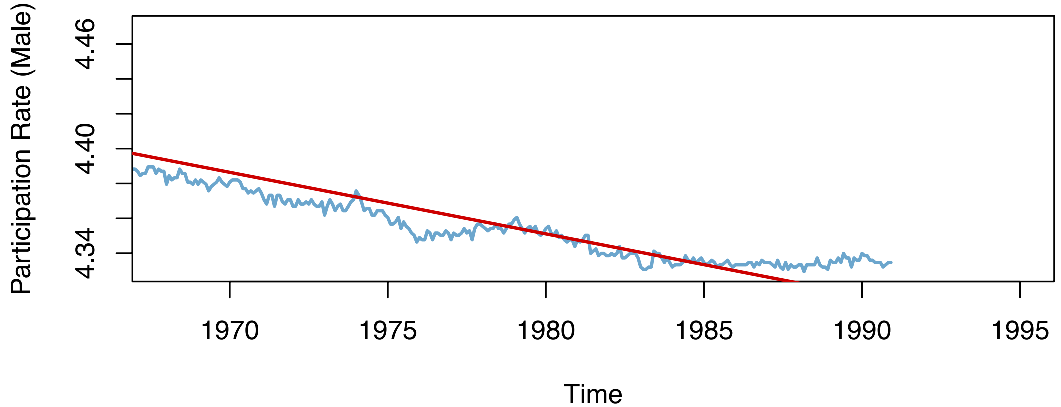
Modeling Trend (Quadratic)



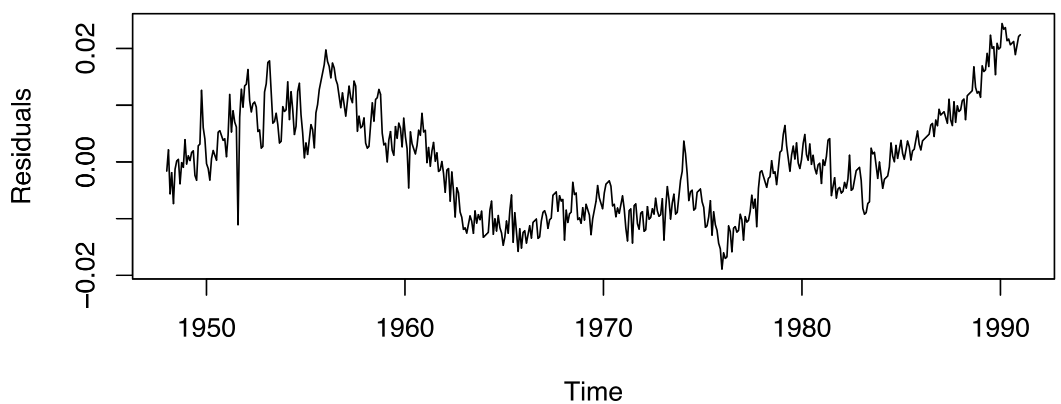
Model: $T_t = \beta_0 + \beta_1\, TIME_{\ t} + \beta_2\, TIME^2$



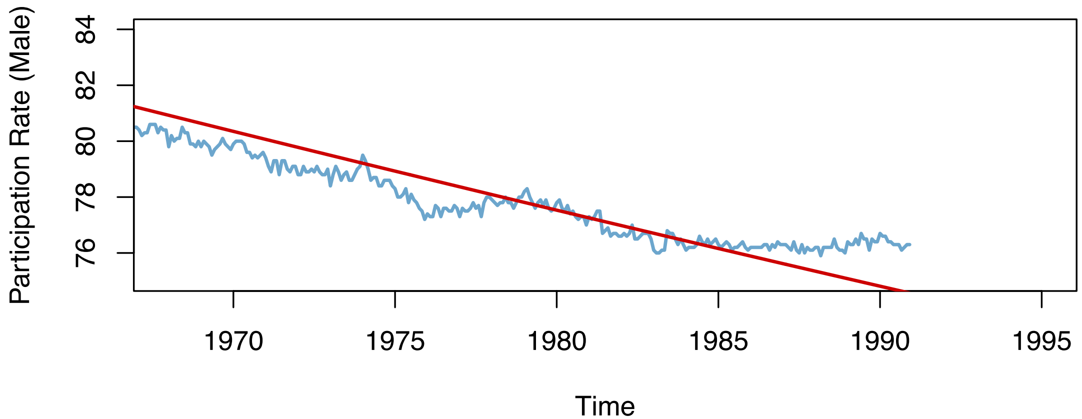
Modeling Trend (Log-Linear)



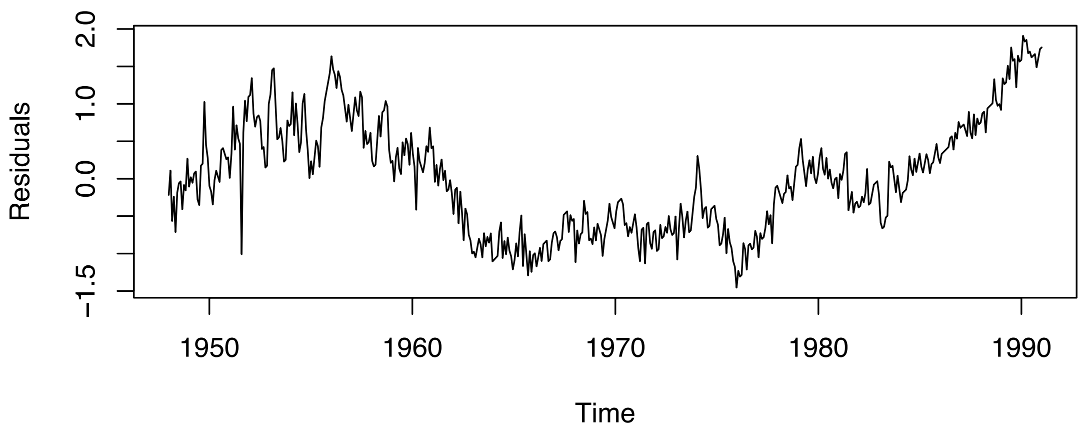
Model: $\log(T_t) = \beta_0 + \beta_1 \, TIME_t$



Modeling Trend (Exponential)



Model: $T_t = \beta_0 e^{\beta_1 TIME_t}$



Model Selection via AIC and BIC

Model	df	AIC	BIC
Linear	3	1229.0482	1241.7865
Quadratic	4	808.3932	825.3776
Log-Linear	3	-3361.0625	-3348.3242
Exponential	3	1160.3758	1173.1141

The smaller the AIC/BIC value, the better the model.

Both AIC and BIC select the quadratic fit as the preferred model.

Forecasting Trend

- Example (Point Forecast): Initially at T, and want to use a trend model to forecast the *h-step-ahead* value.

- Assume a linear trend: $y_t = \beta_0 + \beta_1 TIME_t + \varepsilon_t$

~ 0 (zero-mean random noise)

- At time $T{+}h$: $y_{T+h} = \beta_0 + \beta_1 TIME_{T+h} + \cancel{\varepsilon_{T+h}}$

- Point forecast: $y_{T+h,T} = \beta_0 + \beta_1 TIME_{T+h}$

Forecast is for $t = T{+}h$ but based on $t = T$

- However, β_0 and β_1 are unknown. Solution: replace them with their LS estimates $\hat{\beta}_0$ and $\hat{\beta}_1$.

- Point Forecast: $\boxed{\hat{y}_{T+h,T} = \hat{\beta}_0 + \hat{\beta}_1 TIME_{T+h}}$

Forecasting Trend

- Example (Interval Forecast): Same idea as before. Assume the trend regression disturbance is normally distributed, then:
- Interval Forecast: $y_{T+h,T} \pm z^* \sigma$
- In practice, use: $\hat{y}_{T+h,T} \pm z^* \hat{\sigma}$
- Example (Density Forecast): Same idea, yet again!
- Density Forecast: $\mathcal{N}(y_{T+h,T}, \sigma^2)$
- In practice, use: $\mathcal{N}(\hat{y}_{T+h,T}, \hat{\sigma}^2)$

Forecasting Trend (Example)