Economics 144
Economic Forecasting

Lecture 5
Modeling and Forecasting Seasonality

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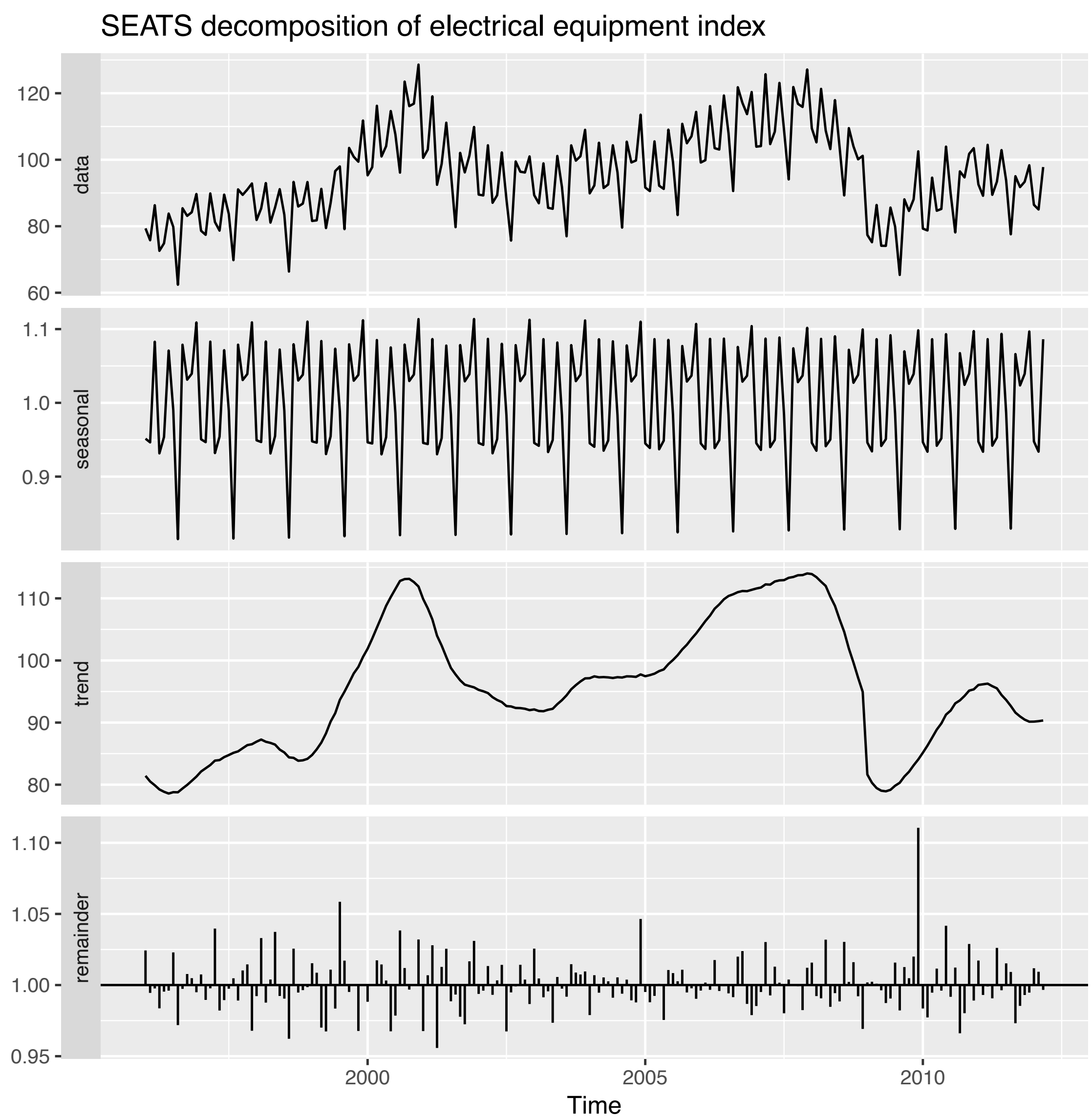
Today's Class

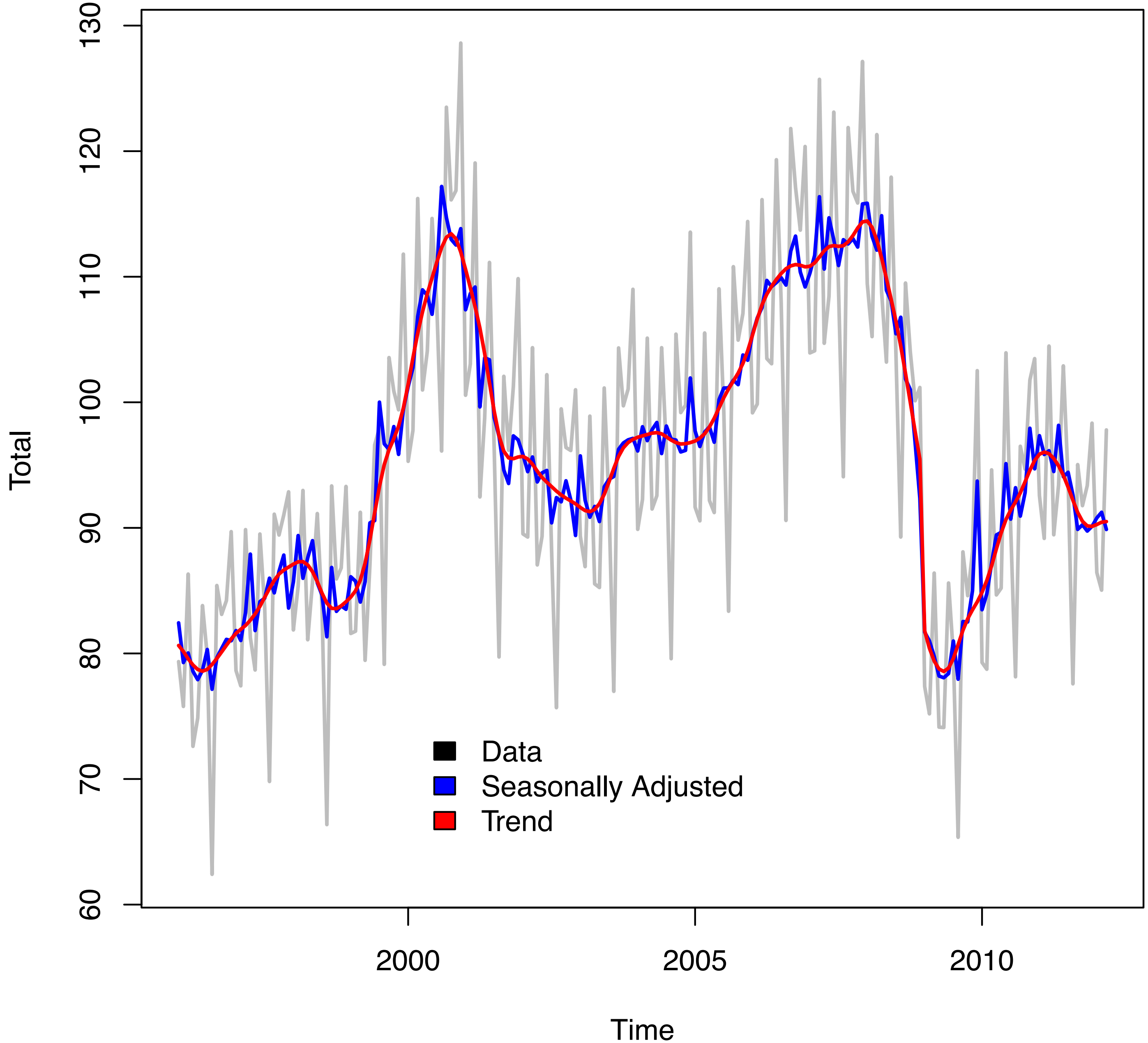
- Seasonality Characteristics
- Modeling Seasonality
- Forecasting Seasonality
- Forecasting Performance
- Example: Forecasting Housing Starts
- R Example

Time Series Components

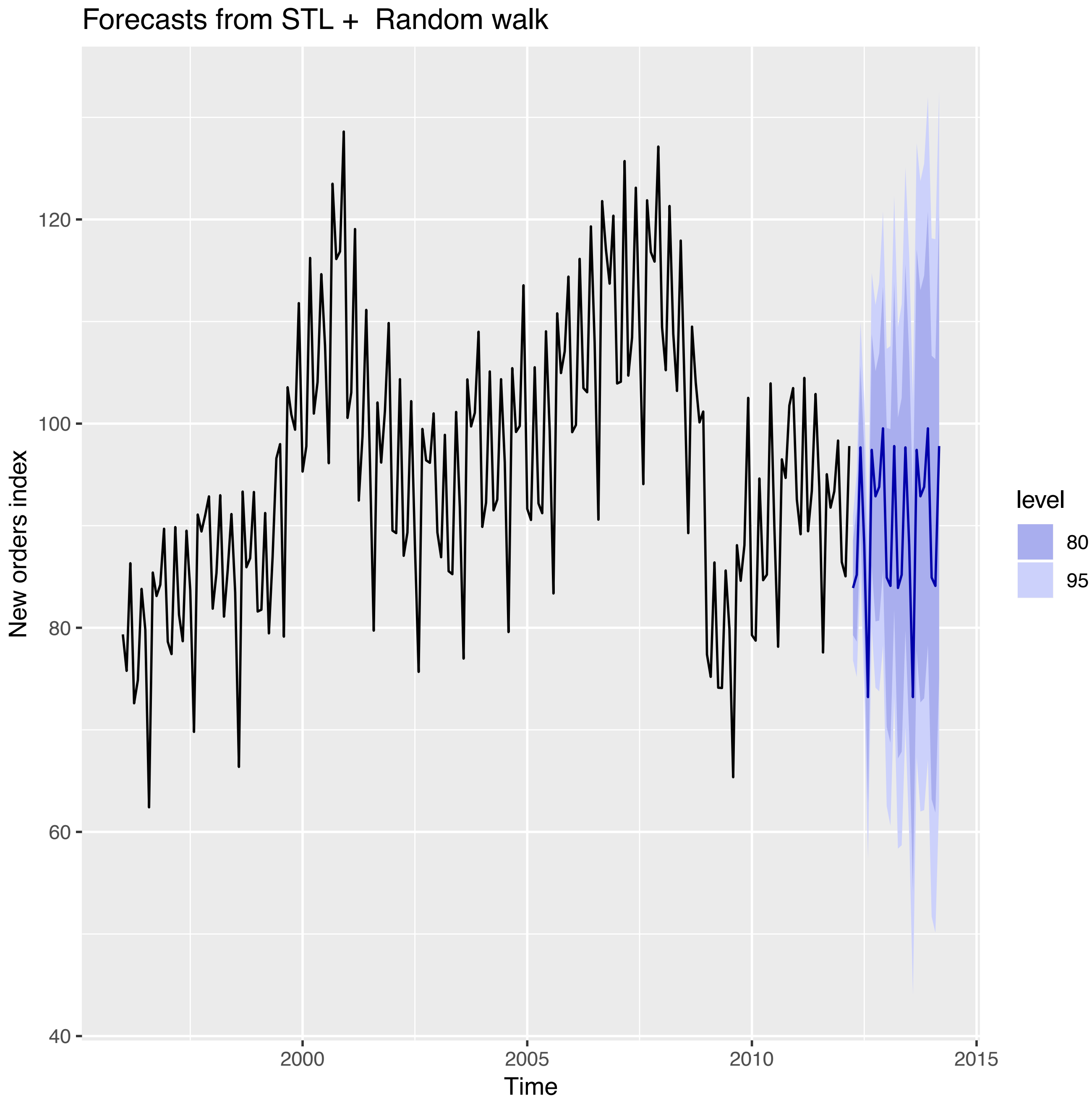
- Additive: $y_t = S_t + T_t + R_t$
  - Good when the seasonal fluctuations do not vary much with time.
  - Seasonally adjusted Series: $y_{adjusted} = y_t - S_t$
- Multiplicative: $y_t = S_t \times T_t \times R_t$
  - Good when the seasonal fluctuations vary with time.
  - Seasonally adjusted Series: $y_{adjusted} = y_t / S_t$

Example: Number of New Orders of Electrical Equipment





Total
130
120
110
100
90
80
70
60
Data
Seasonally Adjusted
Trend
2000
2005
2010
Time

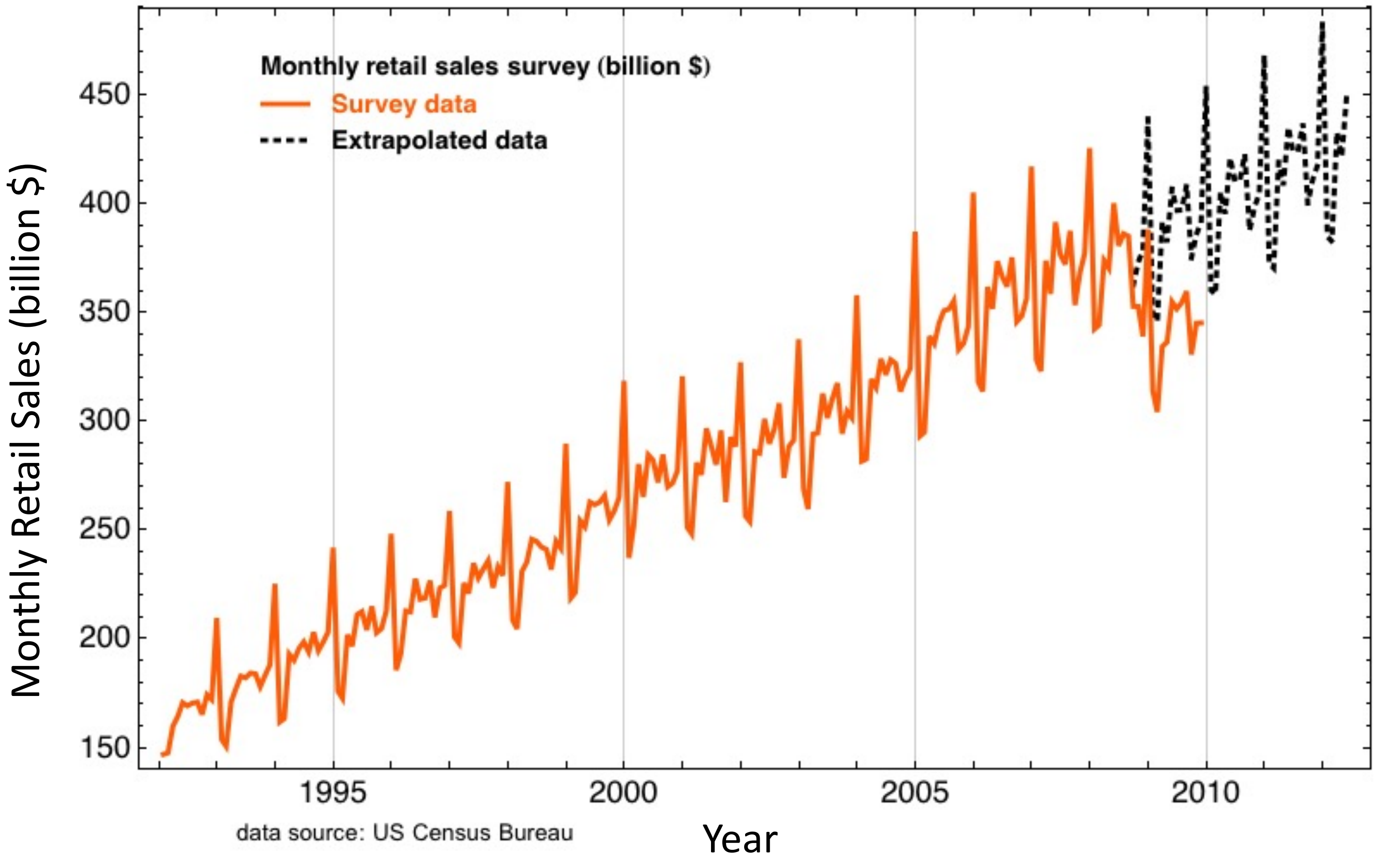

Forecasts from STL + Random walk
New orders index
120
100
80
60
40
2000
2005
2010
2015
Time
level
80
95

Seasonality Characteristics

- Seasonal Pattern: Is a pattern that repeats itself every year.
- Deterministic Seasonality: When the annual repetition is exact.
- Stochastic Seasonality: When the annual repetition is approximate.
- Sources of Seasonality: links to the calendar, technologies, preferences, institutions, etc.

Seasonality Characteristics

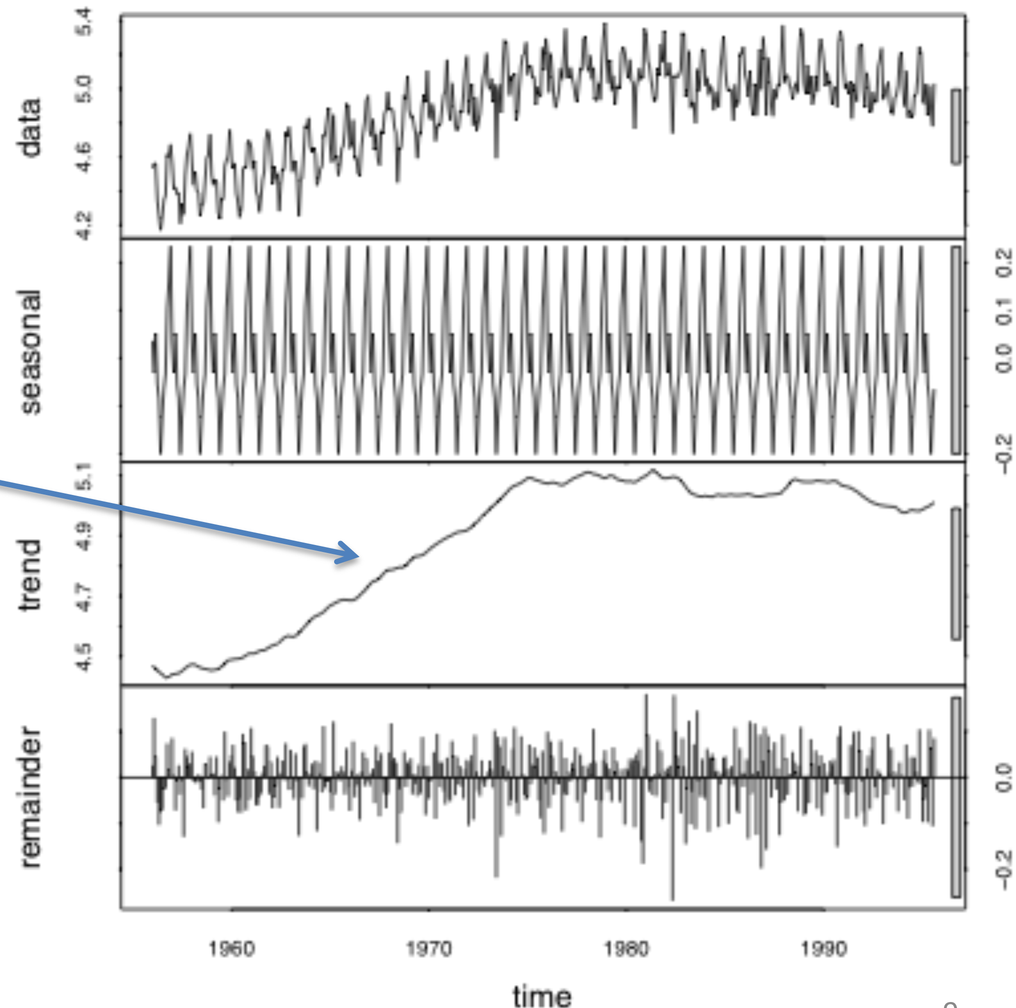
US Census Bureau Data



Seasonality Characteristics

- **Seasonally Adjustment:**
  Removal of seasonality.

- **Nonseasonal Fluctuations:**
  Fluctuations left in the seasonally adjusted time series.



Modeling Seasonality

- Preliminary Definitions:
  - s: Number of observations on a series in each year (e.g., quarterly data (s=4), monthly (s=12), weekly (s=52),…,etc.).
  - Seasonal Dummy Variables (D_i): Indicate which season we are in. For example, in the case of four seasons, we have 4 quarters:

    D_1 = (1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0,…)
    D_2 = (0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0,…)
    D_3 = (0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0,…)
    D_4 = (0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1,…)
  - Seasonal Factors (γ_i): Summarize the seasonal pattern over the year.

Modeling Seasonality

- Pure Seasonal Dummy Model:

$$y_t = \sum_{i=1}^{s} \gamma_i D_{it} + \varepsilon_t$$

- Seasonal Dummy Model including Linear Trend:

$$y_t = \beta_1 TIME_t + \sum_{i=1}^{s} \gamma_i D_{it} + \varepsilon_t$$

Modeling Seasonality 3 of 3

- Seasonal Dummy Model including Linear Trend and Holiday Variation: Holidays' dates change over time.

$$y_t = \beta_1 TIME_t + \sum_{i=1}^{s} \gamma_i D_{it} + \sum_{i=1}^{\nu_1} \delta_i^{HD} HDV_{it} + \varepsilon_t$$

=1 if the month contains e.g., Easter, and =0 otherw

- Seasonal Dummy Model including Linear Trend, Holiday Variation, and Trading-day Variation: Different months contain different numbers of trading days, or business days.

$$y_t = \beta_1 TIME_t + \sum_{i=1}^{s} \gamma_i D_{it} + \sum_{i=1}^{\nu_1} \delta_i^{HD} HDV_{it} + \sum_{i=1}^{\nu_2} \delta_i^{TD} TDV_{it} + \varepsilon_t$$

Forecasting Seasonality

- Example (Point Forecast): Initially at T, and want to use a seasonal model to forecast the *h-step-ahead* value.
- Assume a full seasonal model:

$$y_t = \beta_1 TIME_t + \sum_{i=1}^{s} \gamma_i D_{it} + \sum_{i=1}^{\nu_1} \delta_i^{HD} HDV_{it} + \sum_{i=1}^{\nu_2} \delta_i^{TD} TDV_{it} + \varepsilon_t$$

- At time $T+h$:

$$y_{T+h} = \beta_1 TIME_{T+h} + \sum_{i=1}^{s} \gamma_i D_{i,T+h} + \sum_{i=1}^{\nu_1} \delta_i^{HD} HDV_{i,T+h} + \sum_{i=1}^{\nu_2} \delta_i^{TD} TDV_{i,T+h} + \varepsilon_{T+h}$$

- Point Forecast: Project the right side of the equation on Ω_T .

$$y_{T+h,T} = \beta_1 TIME_{T+h} + \sum_{i=1}^{s} \gamma_i D_{i,T+h} + \sum_{i=1}^{\nu_1} \delta_i^{HD} HDV_{i,T+h} + \sum_{i=1}^{\nu_2} \delta_i^{TD} TDV_{i,T+h}$$

- Use Parameter Estimates

$$\boxed{\hat{y}_{T+h,T} = \hat{\beta}_1 TIME_{T+h} + \sum_{i=1}^{s} \hat{\gamma}_i D_{i,T+h} + \sum_{i=1}^{\nu_1} \hat{\delta}_i^{HD} HDV_{i,T+h} + \sum_{i=1}^{\nu_2} \hat{\delta}_i^{TD} TDV_{i,T+h}}$$

Forecasting Performance

Mean Forecast Error (MFE or Bias): Measures average deviation of forecast from actuals.

$$MFE = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)$$

Mean Absolute Deviation (MAD): Measures average absolute deviation of forecast from actuals.

$$MAD = \frac{1}{n}\sum_{t=1}^{n}|y_t - \hat{y}_t|$$

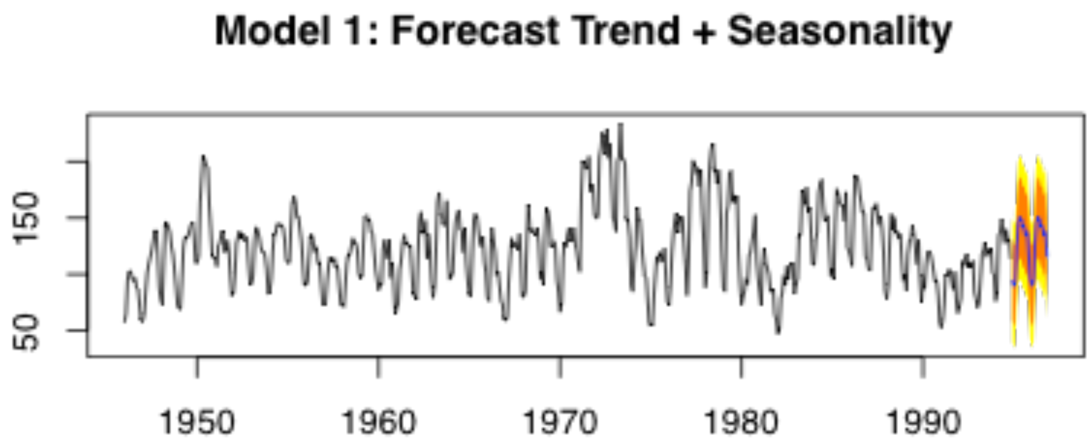
Mean Absolute Percentage Error (MAPE): Measures absolute error as a percentage of the forecast.

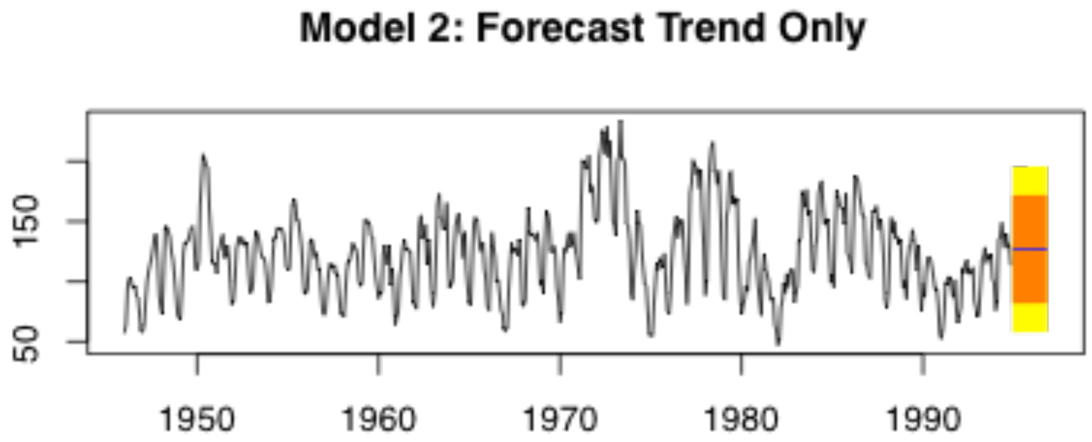
$$MAPE = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{y_t - \hat{y}_t}{y_t}\right|$$

Standard Squared Error (MSE): Measures variance of forecast error.

$$MSE = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2$$

Forecasting Housing Starts





Forecasting Housing Starts

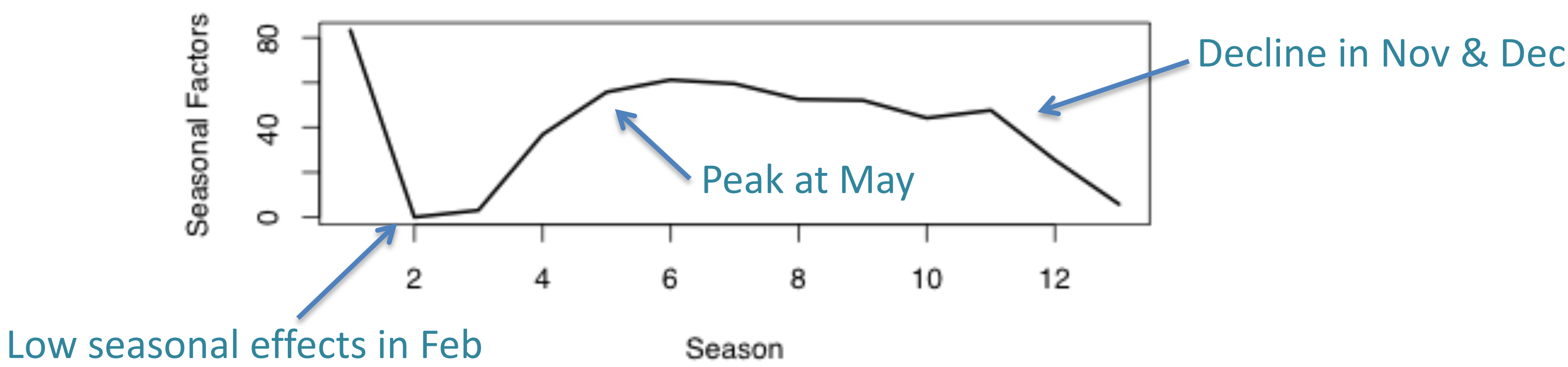
```
Call:
lm(formula = formula, data = "housets", na.action = na.exclude)

Residuals:
    Min      1Q  Median      3Q     Max 
-59.480 -17.125  -1.709  13.937  89.904 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 83.274857   4.400202  18.925  < 2e-16 ***
trend        0.010446   0.006731   1.552    0.121    
season2      3.077309   5.581779   0.551    0.582    
season3     36.801556   5.581792   6.593 9.78e-11 ***
season4     55.750294   5.581812   9.988  < 2e-16 ***
season5     61.190867   5.581840  10.962  < 2e-16 ***
season6     59.455932   5.581877  10.652  < 2e-16 ***
season7     52.525078   5.581921   9.410  < 2e-16 ***
season8     52.077896   5.581974   9.330  < 2e-16 ***
season9     44.175612   5.582035   7.914 1.29e-14 ***
season10    47.632514   5.582104   8.533  < 2e-16 ***
season11    25.466965   5.582181   4.562 6.19e-06 ***
season12     5.812224   5.610873   1.036    0.301    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 27.63 on 574 degrees of freedom
Multiple R-squared: 0.3903, Adjusted R-squared: 0.3775 
F-statistic: 30.62 on 12 and 574 DF,  p-value: < 2.2e-16
```

(γ_i)

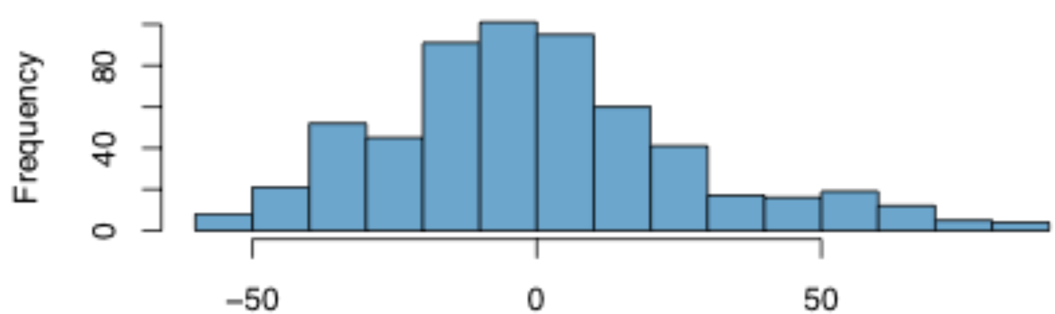
Trend

Forecasting Housing Starts

Plot of Seasonal Factors (γ_i)



Histogram of Residuals



Residuals