Economics 144
Economic Forecasting

Lecture 16
State Space Models

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State Space Models (SSM)
(e.g., Kalman Filter)

- Q: Why would you use Filtering techniques instead of e.g., ARIMA?
- A: Smoothing techniques (such as filtering and spectral analysis) are often used to *filter* out random noise.
- All previous time-series techniques discussed, relied consisted of regressions of present on past observations →Time Domain

in time domain data can be too noisy

- Now we consider regressions of the present on periodic sines and cosines →Frequency Domain

Abandon time domain and move to frequency domain
- clean up data and looking for signals.

State Space Models (e.g., Kalman Filter)

- Main goals of filtering methods:
  - Identify the dominant frequencies in a series.
  - Find an explanation of the system from which the measurements were derived. This ‘explanation’ usually consists of only a few oscillations, therefore, its simpler and more physically meaningful.

Example: Deterministic Signal

A f

$$x_1 = 2\cos(2\pi t\ 6/100) + 3\sin(2\pi t\ 6/100)$$

$$x_2 = 4\cos(2\pi t\ 10/100) + 5\sin(2\pi t\ 10/100)$$

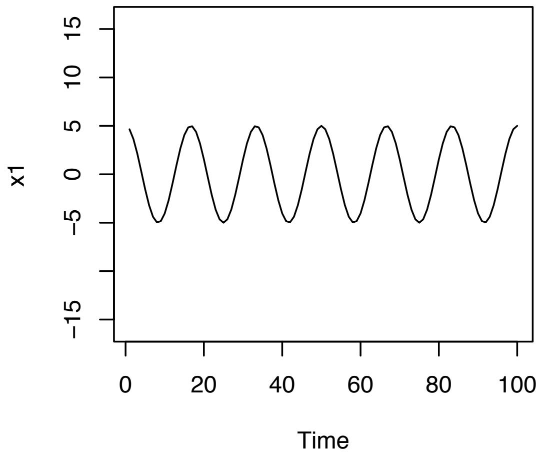
$$+\ x_3 = 6\cos(2\pi t\ 40/100) + 7\sin(2\pi t\ 40/100)$$

$$x = x_1 + x_2 + x_3$$

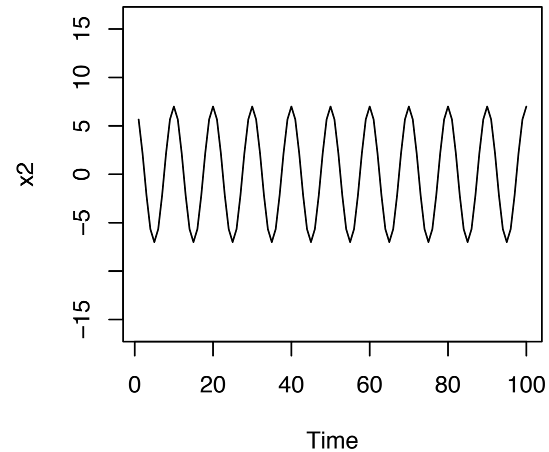
- Summary: x is constructed from 3 signals with respective (f, A^2) given by (0.06, 13), (0.10, 41), and (0.4, 85)

Example: Deterministic Signal

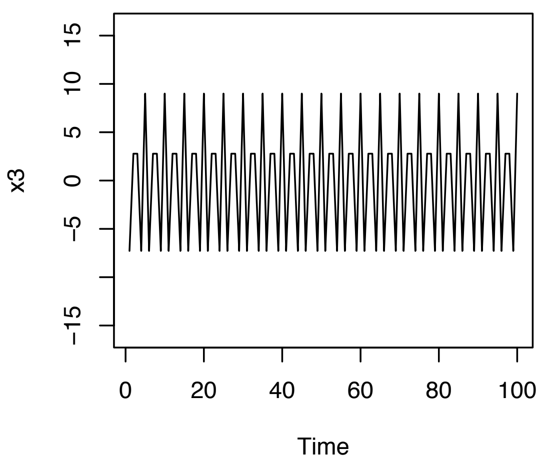
f=0.06, A^2 = 13



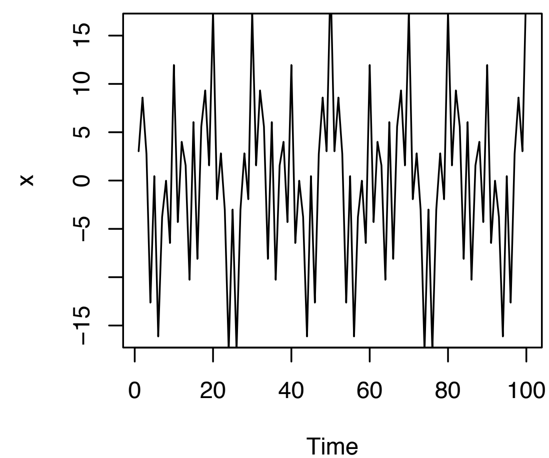
f=0.10, A^2 = 41



f=0.40, A^2 = 85



Sum

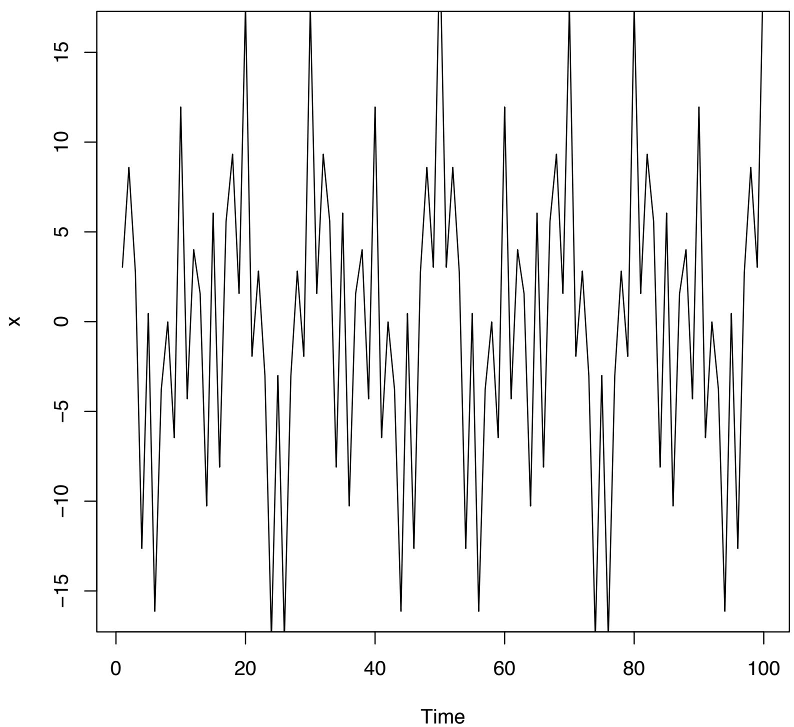


Quite noisy

reverse-engineer the components

Example: Deterministic Signal

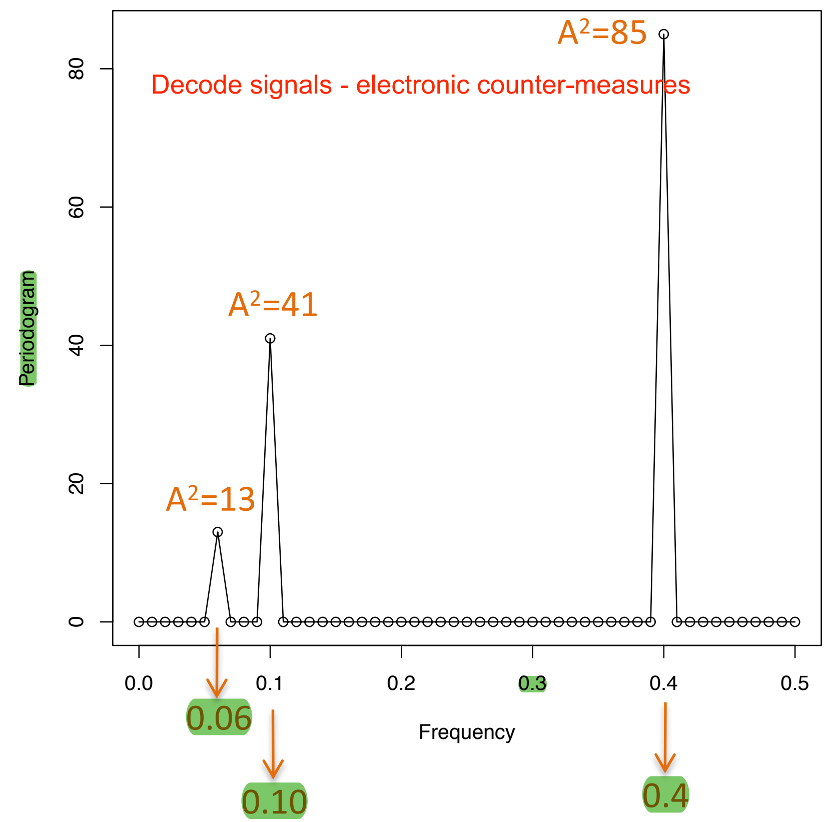
Q: If we only observe the final signal, can we infer its main 'components'?



added components

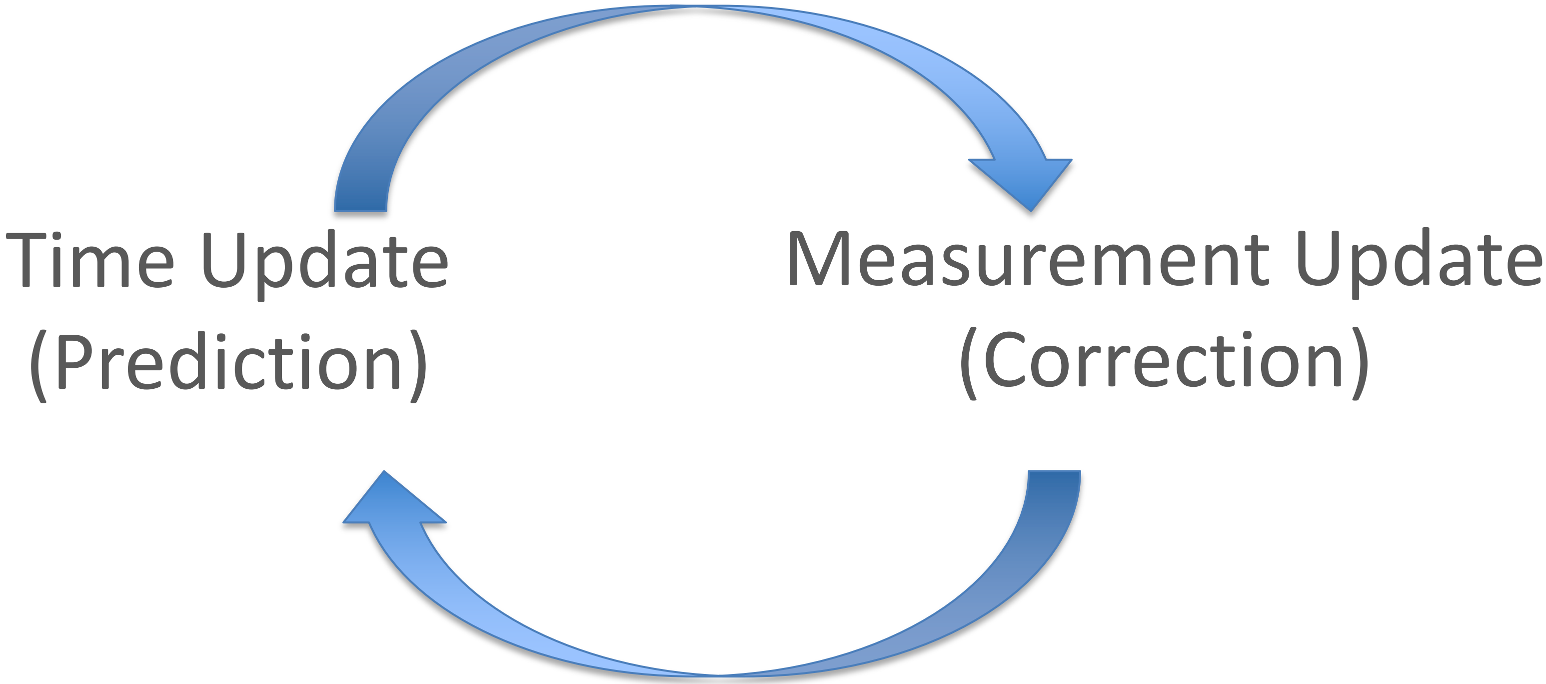
amplitude square vs. omega

Yes! We look at the periodogram's (FFT) frequencies and amplitudes.



matches the three series that made up the signal

Example: Kalman Filter



The time update projects the current state estimate ahead in time.
The measurement update adjusts the projected estimate by an actual measurement at that time.

Example: Kalman Filter

action based on all the information coming in - at the heart - take a ton of other diffusion and fuse it

- The KF 'data fusion algorithm' consists of updating the means and (co)variances of process modeled.

Control Input Matrix

non-constant acceleration

Effect of acceleration → position and velocity

comprised of kinematic variables

$$\vec{x}_t = \underbrace{\mathbf{F}_t}_{\text{Transition Matrix}} \vec{x}_{t-1} + \overbrace{\mathbf{B}_t}^{\text{Control Input Matrix}} \vec{u}_t + \vec{e}_t$$

State vector (position, velocity,...)

can also have acceleration

Transition Matrix

Position at t-1 → position at t

same equation as V = V0+at

Velocity at t-1 → velocity at t

state vector

some room for error

$$\vec{z}_t = \underbrace{\mathbf{H}_t}_{\text{Transformation Matrix}} \vec{x}_t + \vec{v}_t$$

Vector of measurements

explained by decompostion

Transformation Matrix

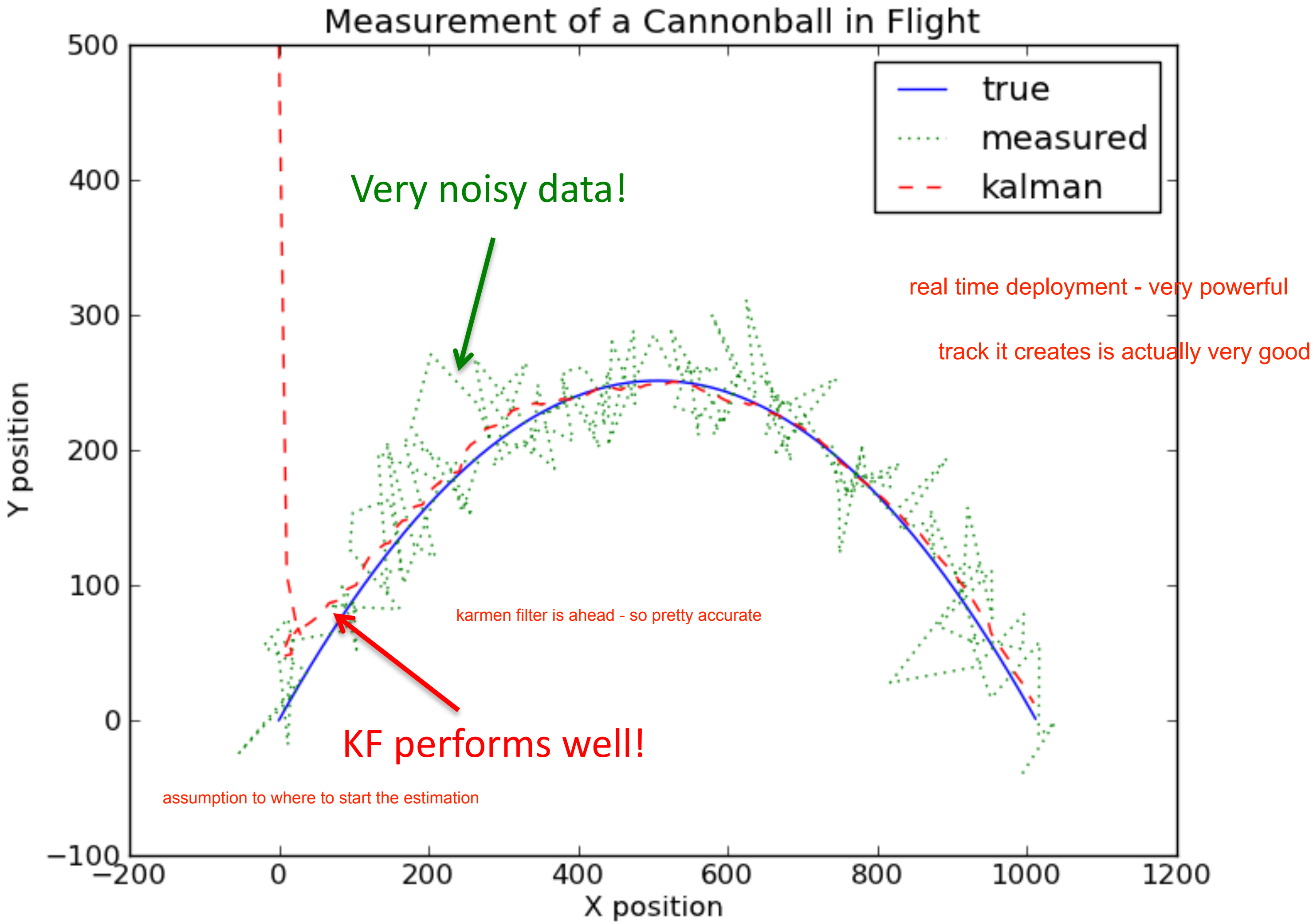
Maps the state vector parameters into the measurement domain

Example: Kalman Filter

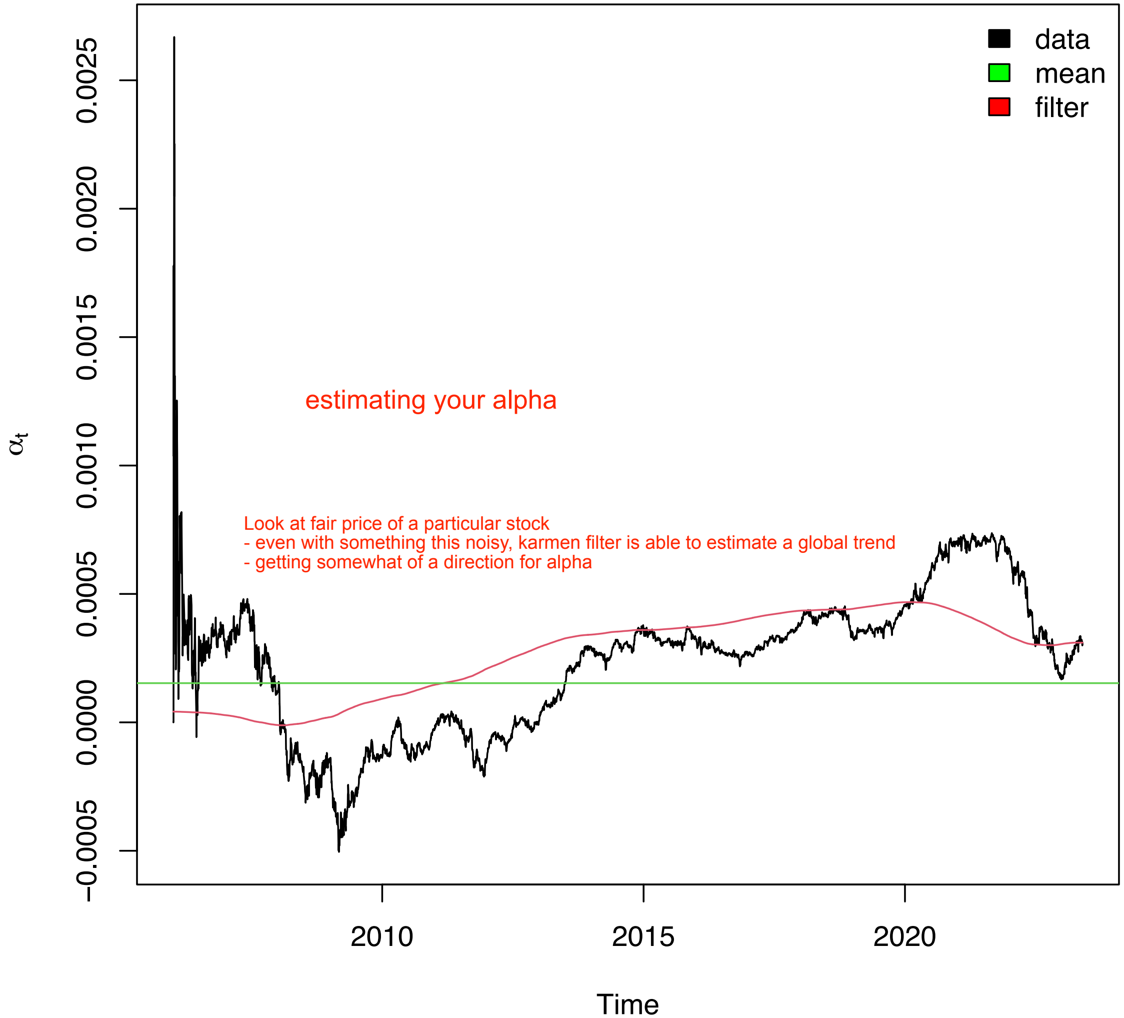
$$\left.\begin{aligned}\underbrace{x_t}_{\text{position}} &= x_{t-1} + \dot{x}\Delta t + \frac{1}{2}\underbrace{\left(\frac{f_t}{m}\right)}_{\text{acceleration}}(\Delta t)^2 \\ \underbrace{\dot{x}_t}_{\text{velocity}} &= \dot{x}_{t-1} + \left(\frac{f_t}{m}\right)\Delta t\end{aligned}\right\} \text{Kinematic Equations of Motion}$$

$$\underbrace{\begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix}}_{\mathbf{x}_t} = \underbrace{\begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}}_{\mathbf{F}_t} \underbrace{\begin{bmatrix} x_{t-1} \\ \dot{x}_{t-1} \end{bmatrix}}_{\mathbf{x}_{t-1}} + \underbrace{\begin{bmatrix} \frac{1}{2}(\Delta t)^2 \\ \Delta t \end{bmatrix}}_{\mathbf{B}_t} \underbrace{\left(\frac{f_t}{m}\right)}_{\mathbf{u}_{t-1}}$$

Example: Kalman Filter



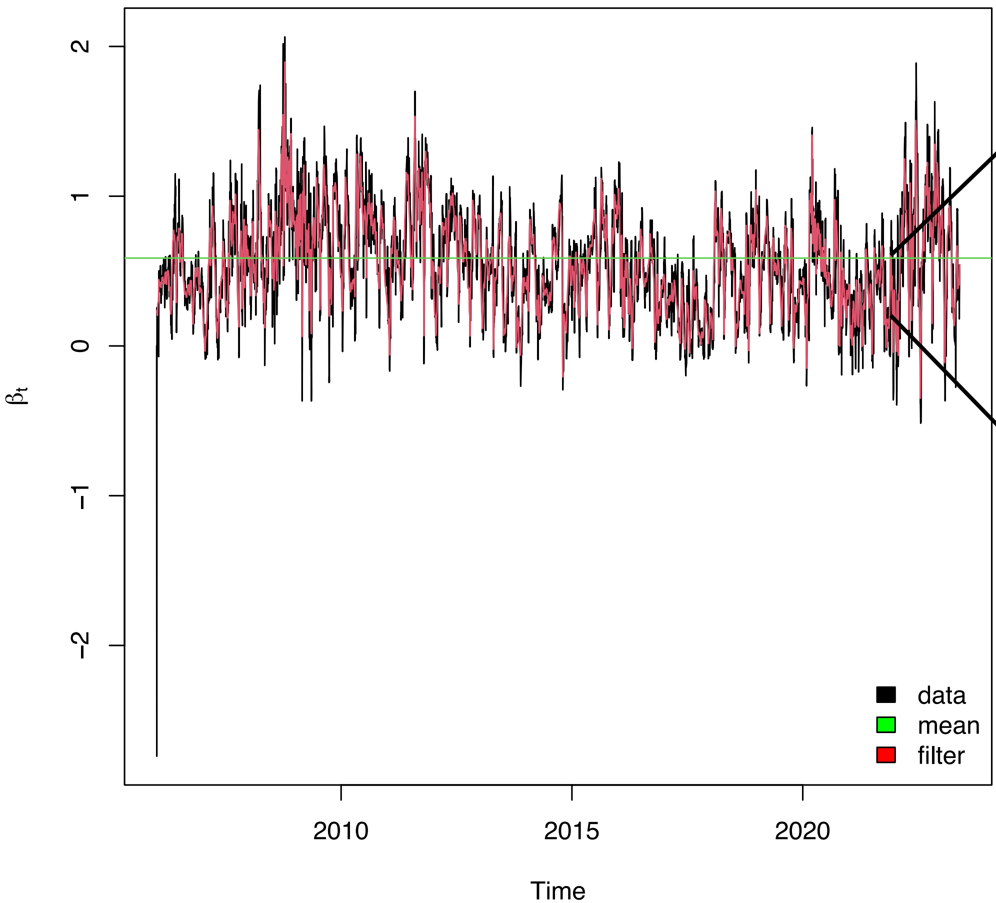
Application: CAPM (alpha estimate)

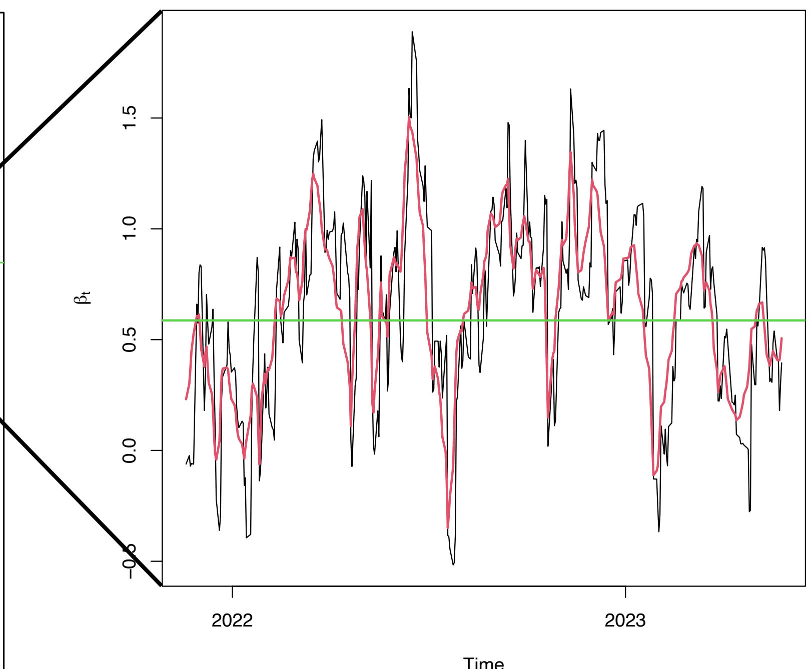


karman filter - caputure compilation

mean - green one

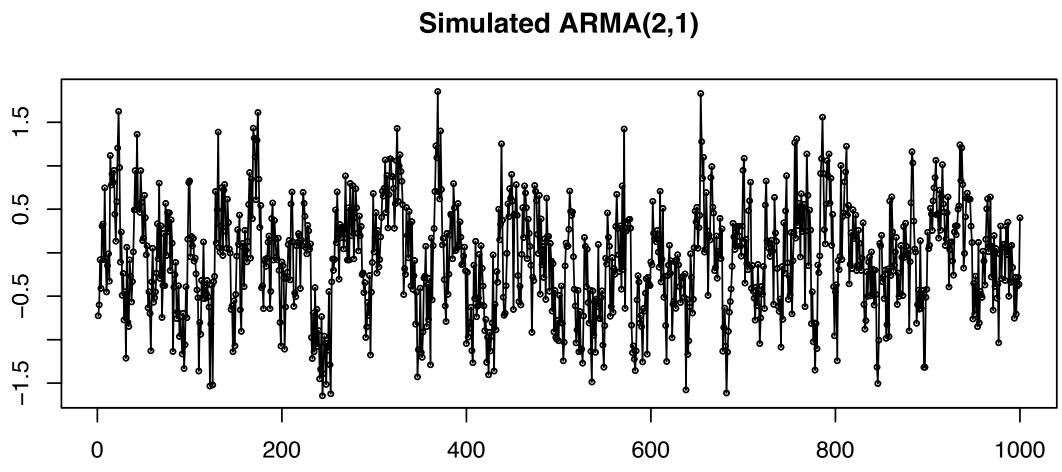
Application: CAPM (beta estimate)

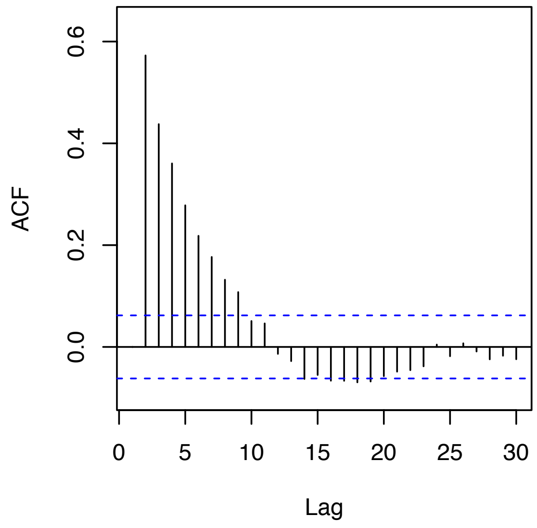


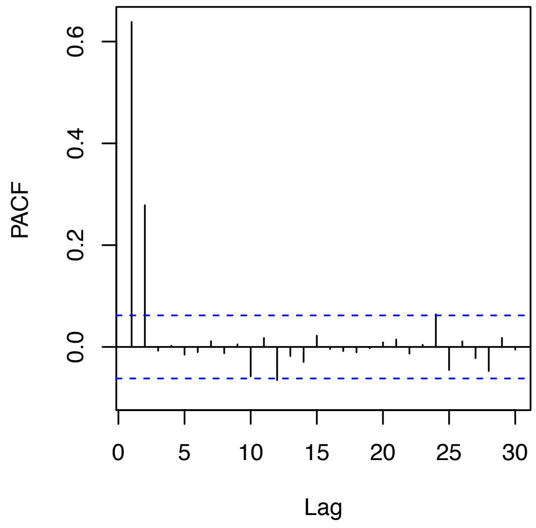


really noisy, karmen filter is ahead - impressive
- green - one size fits all - red - more robust and rigorous

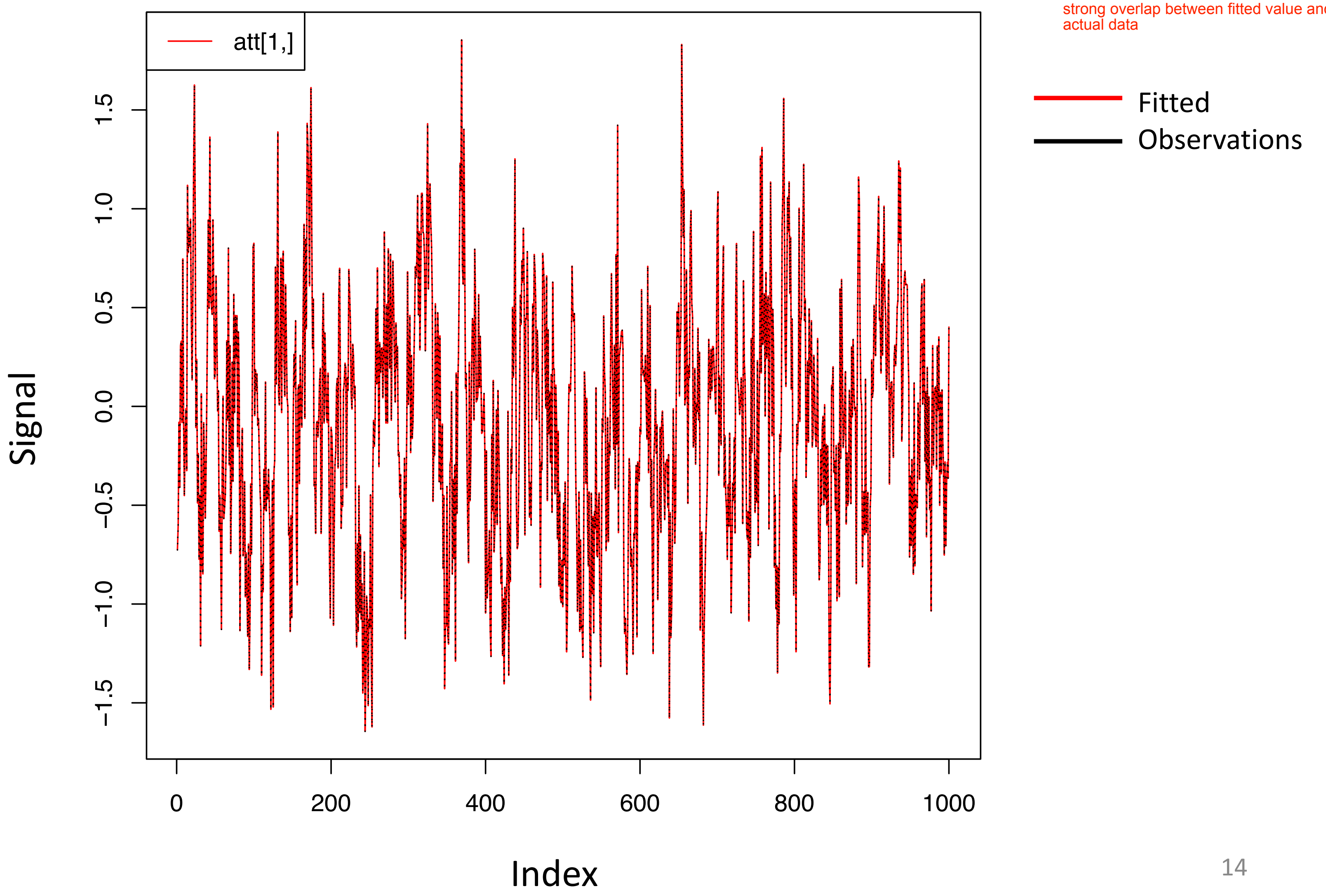
Filters vs. ARIMA







Filters vs. ARIMA



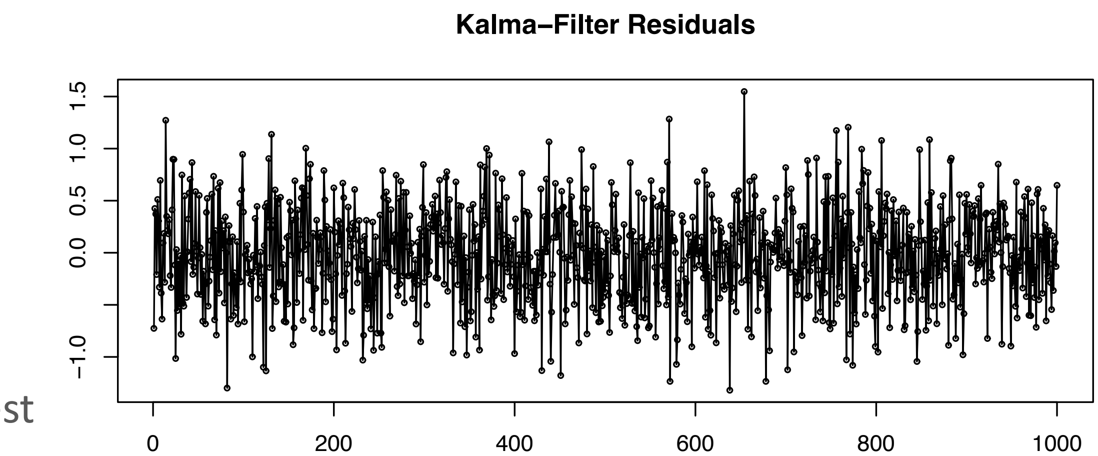
strong overlap between fitted value and actual data

Results from the Kalman Filter

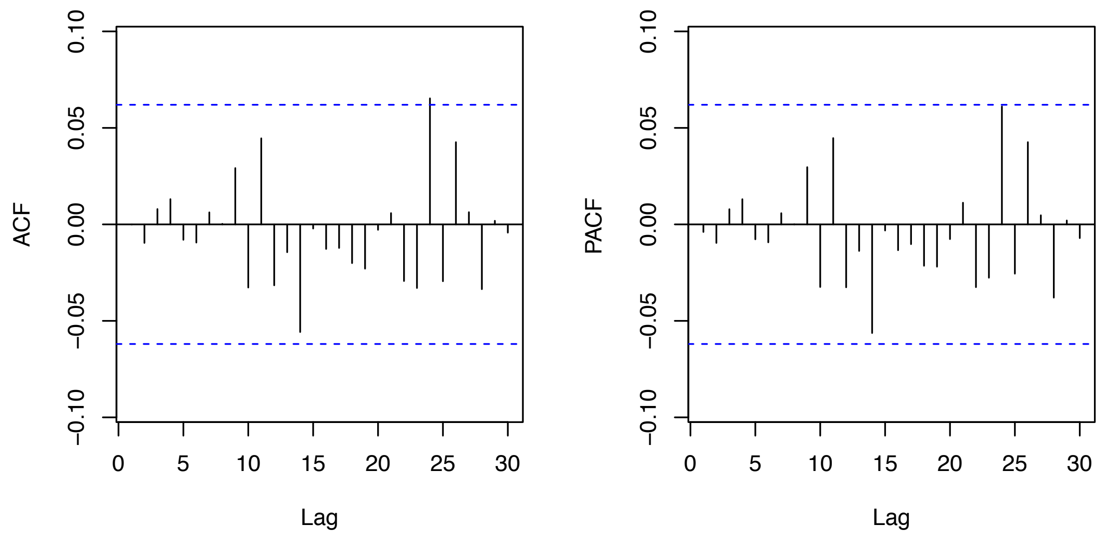
Test sample, look at overall fit, use RMSE(accuracy).

White noise statistical test suggest the autocorrelation functions are consistent white noise at 1%.

Our model seems to have captured all the dynamics.

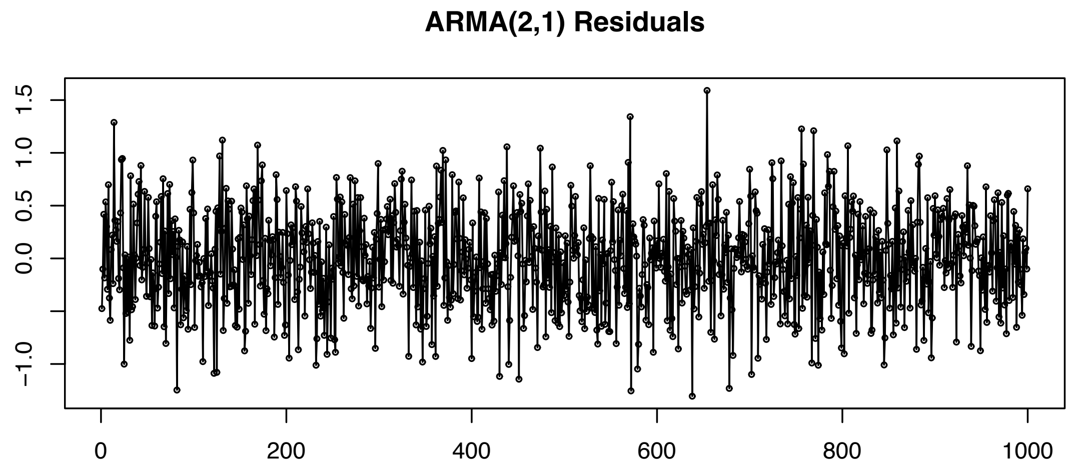


Check residuals - these spikes are within the window - done a pretty good job of wiping out the dynamics
- done a good job in estimating the models.



Results from ARMA

ARMA results are almost identical to the KF results.



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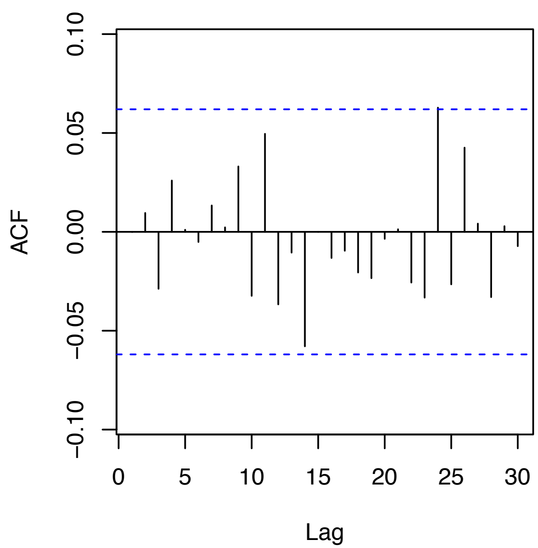

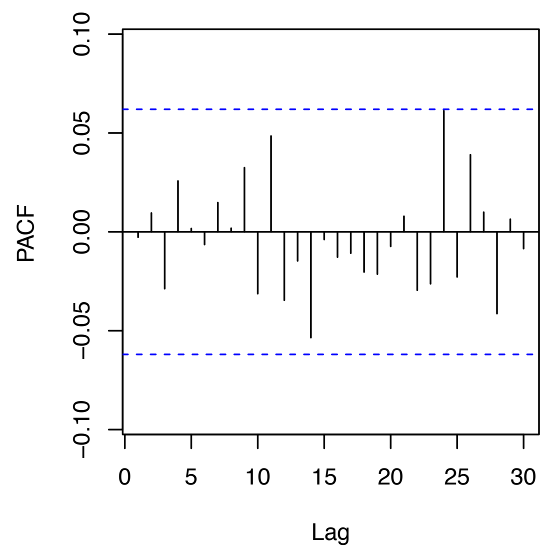

solved problem of unevenly spaced observations
frequency is not a problem

Accurate

popular when u have high dymaic/volatility/real time frequency

don't have classic interpretability - dynamical system

State Space Models

vs. ARIMA, VAR, and others

Some Advantaged of SSM over ARIMA, VAR, …

- Any ARIMA model can be represented in a state-space form, but only simple state-space models can be represented exactly in ARIMA form.
- SSM easily handles structural breaks, shifts, and time-varying parameters of some static model.
- SSM handles missing (and/or irregular) data better than e.g., ARIMA, VAR, …
- SSM allows for changes on-a-fly parameters of the state-space model itself.
- SSM models allows use of data from different sources simultaneously in the same model to estimate one underlying quantity.