Economics 144
Economic Forecasting

Lecture 4
White Noise

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White Noise

- Time Series Process: Let y denote the observed series of interest.

$$y_t = \varepsilon_t$$

Where ε_t ("shock") is uncorrelated over time.

$$\varepsilon_t \sim (0, \sigma^2)$$

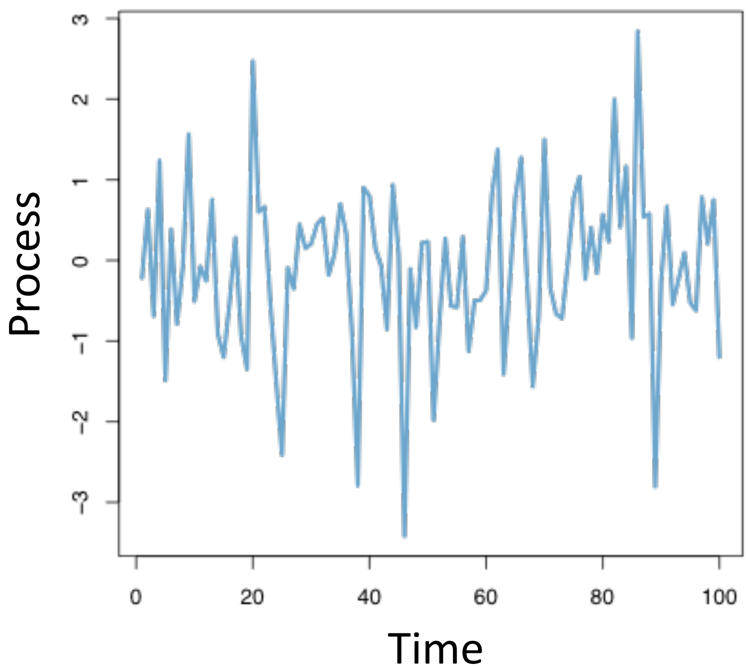
Therefore, y_t and ε_t are serially uncorrelated.

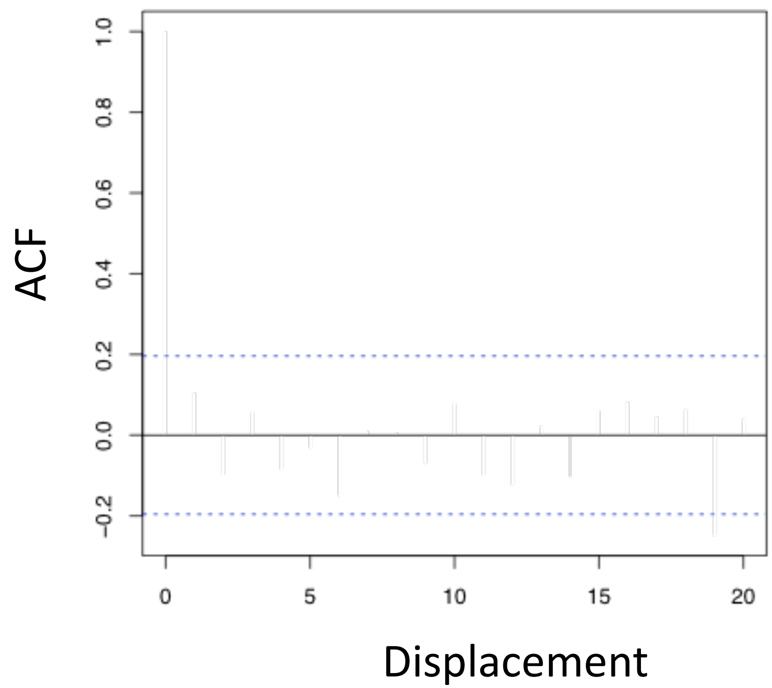
- White Noise: Time series process with zero mean, constant variance, and no serial correlation.

$$\boxed{\varepsilon_t \sim WN(0, \sigma^2) \quad \text{and} \quad y_t \sim WN(0, \sigma^2)}$$

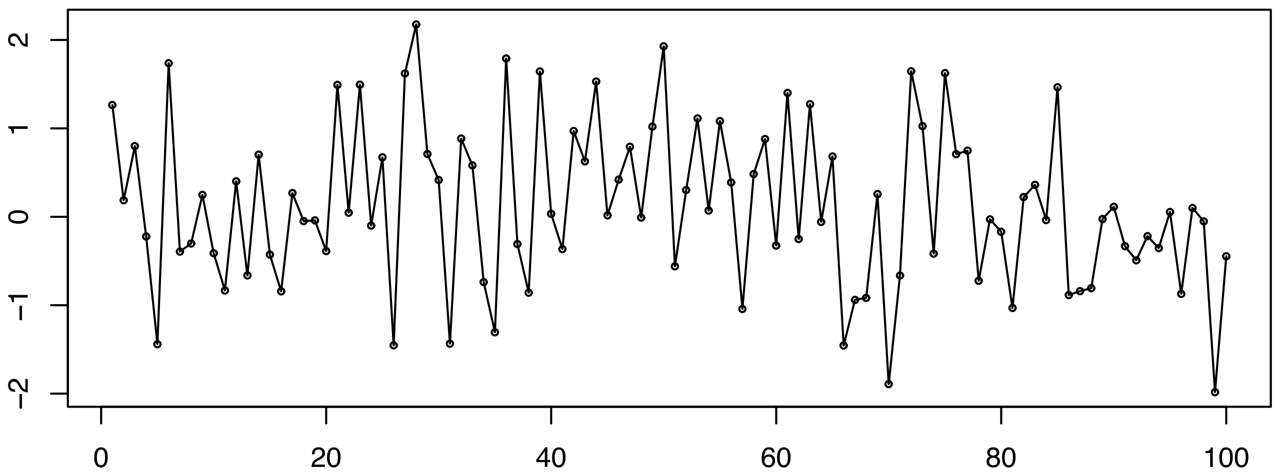
White Noise

- Gaussian White Noise: If y is serially uncorrelated and normally distributed, and thus, serially independent . $\boxed{y_t \stackrel{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2)}$

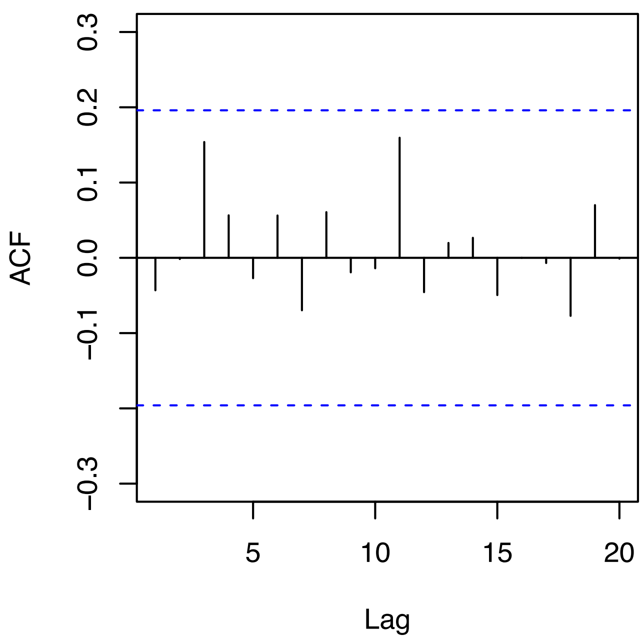




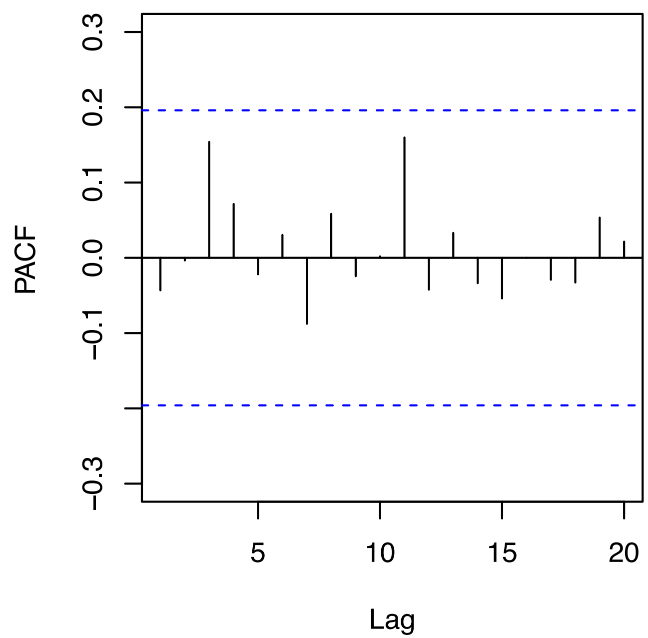
White Noise



2
1
0
−1
−2
0
20
40
60
80
100



ACF
0.3
0.2
0.1
0.0
−0.1
−0.3
5
10
15
20
Lag



PACF
0.3
0.2
0.1
0.0
−0.1
−0.3
5
10
15
20
Lag

White Noise

- Given: $y_t \sim WN(0, \sigma^2)$ → $E(y_t) = 0$ and $var(y_t) = \sigma^2$
- However, recall that $\sigma^2 = \gamma(0)$ →

$$\gamma(k) = \begin{cases} \sigma^2, & k = 0 \\ 0, & k \geq 1 \end{cases} \quad \text{and} \quad \rho(k) = \begin{cases} 1, & k = 0 \\ 0, & k \geq 1 \end{cases}$$

Autocovariance Function

Autocorrelation Function

$$p(k) = \begin{cases} 1, & k = 0 \\ 0, & k \geq 1 \end{cases}$$

Partial Autocovariance Function

Note 1: Please solve problems 3 and 4 from Chapter 7[b].

Note 2: Please review conditional means and conditional variances (see e.g., page 122[b]).

The Lag Operator (Review)

- Recall: Distributed lag of current and past shocks:

$$B(L)\varepsilon_t = b_0\varepsilon_t + b_1\varepsilon_{t-1} + b_2\varepsilon_{t-2} + \cdots = \sum_{i=0}^{\infty} b_i\varepsilon_{t-i}$$

- Example: Use a white-noise process to construct a more complex time series:

$$y_t = \sum_{i=0}^{q} \beta_i\varepsilon_{t-i}, \; \varepsilon_t \sim (0, \sigma^2)$$

White Noise Example (Moving Average)

- Example: Suppose you win \$1 if a fair coin shows heads and loose \$1 if it shows tails.
  - Denote the outcome on toss t by ε_t (i.e., for toss t, ε_t is either +\$1 or -\$1).
  - If you want to keep track of your 'hot streaks', you can e.g., calculate your average winnings on the last four tosses. For each coin toss t, your average payoff on the last four tosses is:

$$= \beta_0 \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \beta_3 \varepsilon_{t-3}$$

White Noise Example (Moving Average)

- Case 1: We can set e.g., $\beta_i = ¼$ for $i \leq 3$.

- Case 2: We can set e.g., $\beta_0 = \beta_1 = 0.5$, and all other $\beta_i = 0$.
- For this case, although the $\{\varepsilon_t\}$ sequence is a white-noise process, the constructed $\{x_t\}$ sequence will not be a white-noise process is two or more of the β_i differ from 0.
- $E[x_t] = E[0.5\varepsilon_t + 0.5\varepsilon_{t-1}] = 0$ and
- $var[x_t] = var[0.5\varepsilon_t + 0.5\varepsilon_{t-1}] = 0.5\sigma^2$

White-noise conditions are satisfied.

- $cov(x_t, x_{t-1}) = 0.5\sigma^2 \neq 0$

→ $\{x_t\}$ is not a white noise process!

Note: Case 2 is known as an 'MA(1)' Process

Estimation and Inference

- Q: How can we assess whether a series is reasonably approximated as white noise?

- A: If the series is white noise, then for large samples:

$$\hat{\rho}(\tau) \sim \mathcal{N}\left(0, \frac{1}{T}\right)$$

The sample autocorrelations are unbiased estimators of the true autocorrelations.

$$\sqrt{T}\hat{\rho}(\tau) \sim \mathcal{N}(0,1) \xrightarrow[\text{Square both sides}]{} \boxed{T\hat{\rho}^2(\tau) \sim \chi^2_1}$$