ECE 361: Digital Communications

Lecture 1: Discrete Nature of information

Introduction

The currency of today's information age is digital: *bits*. Digital communication is reliable transmission of this currency over an unreliable physical medium. It is an interesting question to ask why very different types of information sources such as voice and video are represented using a common currency, but we will not pursue this question in this course. It is quite a loaded question and the final word on the matter is not yet said; answers to this question in certain communication contexts are provided in a separate field of study known as information theory. The block diagram in Figure 1 shows a high level representation of a typical communication system. The *discrete message source* continuously outputs a stream of bits that represent the information we would like to transmit. Bits are abstract entities that need to be *mapped* to a physical quantity, such as an electromagnetic signal, to be transmitted over a physical medium.
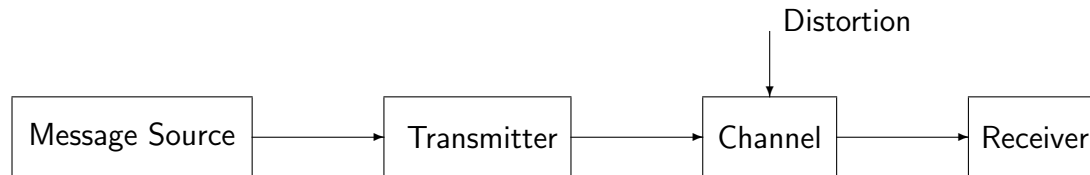

Figure 1: The basic block diagram of a communication system

The behavior of the physical medium is uncertain: what you get is *not a deterministic* function of what you send; this uncertainty is the essence of communication. While the behavior of the channel[1] over one experiment cannot be predicted, the *average* behavior, averaged over many experiments turns out to be well behaved in many physically interesting scenarios. The characterization of the average behavior, or in other words, the *statistical characterization* of the physical medium is crucial to understanding how to communicate the bits reliably to the receiver. A primary component of the a communication engineer's tool-box is robust and reasonable statistical models of important physical channels such as the wireline telephone channel and the wireless channels.

A Simple Noise Model

We will begin with a simple form of a physical medium where we only transmit and receive voltages (real numbers). The received voltage y, is the transmitted voltage x, plus "noise" w:

$$y = x + w \tag{1}$$

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[1]Channel is a term we will use throughout these notes to denote the unreliable physical medium.

The simplest model of the noise is that w is strictly within a certain range, say $\pm\sigma_{\text{th}}$. In other words, we receive a voltage that is within $\pm\sigma_{\text{th}}$ Volts from the voltage we transmitted.

A Simple Communication Scheme

Suppose we want to send a single bit across this channel. We can do this by transmitting a voltage v_0 to transmit an information content of the bit being "zero", and a voltage v_1 when transmitting an information content of the bit being "one". As long as

$$|v_0 - v_1| > 2\sigma_{\text{th}}, \tag{2}$$

we can be certain that our communication of the one bit of information is reliable over this channel. Physically, the voltage transmitted corresponds to some energy being spent: we can say that the energy spent in transmitting a voltage v Volts is (proportional to) v^2 Joules. In this context, a natural question to ask is the following: how many bits can we reliable communicate with an energy constraint of E Joules?

Some thought lets us come up with the following transmission scheme: we choose to transmit one of a collection of *discrete* voltage levels:

$$\left\{ -\sqrt{E}, -\sqrt{E} + 2\sigma_{\text{th}}, \ldots, -\sqrt{E} + 2k\sigma_{\text{th}}, \ldots, +\sqrt{E} \right\}, \tag{3}$$

where we have assumed for simplicity that $\sqrt{E}$ is divisible by σ_{th}. So, we can communicate one of

$$1 + \frac{\sqrt{E}}{\sigma_{\text{th}}} \tag{4}$$

discrete voltage levels entirely *reliably* to the receiver. This corresponds to

$$\log_2\left(1 + \frac{\sqrt{E}}{\sigma_{\text{th}}}\right) \tag{5}$$

bits being reliable communicated to the receiver (why?). The diagram in Figure 2 demonstrates one possible mapping between the 4 sequences of 2 bits to the 4 discrete voltage levels being transmitted. Here $\sqrt{E} = 3\sigma_{\text{th}}$ and

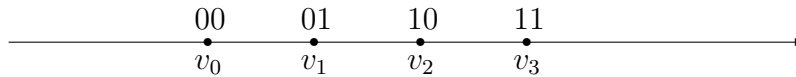$$v_k = -\sqrt{E} + 2k\sigma_{\text{th}}, \quad k = 0, 1, 2, 3. \tag{6}$$

Figure 2: Mapping from bits to voltage levels.

Relation between Energy and Reliable Information Transmitted

For a given energy constraint E, the number of bits we can communicate reliably is, from (5),

$$\log_2\left(1 + \frac{\sqrt{E}}{\sigma_{\text{th}}}\right). \tag{7}$$

A natural sort of question that the communication engineer is interested in is the following: if we want to send an additional bit reliably how much more energy do we need to expend? We can use the above expression to answer this question: the new energy energy $\tilde{E}$ required to send an extra bit of informamtion reliably has to satisfy:

$$\log_2\left(1 + \frac{\sqrt{\tilde{E}}}{\sigma_{\text{th}}}\right) = 1 + \log_2\left(1 + \frac{\sqrt{E}}{\sigma_{\text{th}}}\right), \tag{8}$$

$$1 + \frac{\sqrt{\tilde{E}}}{\sigma_{\text{th}}} = 2\left(1 + \frac{\sqrt{E}}{\sigma_{\text{th}}}\right), \tag{9}$$

$$\sqrt{\tilde{E}} = \sigma_{\text{th}} + 2\sqrt{E}. \tag{10}$$

In other words, we need to more than *quadruple* the energy constraint to send just one extra bit of information reliably.

Another interesting thing to note is that the amount of reliable communication transmitted depends on the *ratio* between the transmit energy budget E and the energy of the noise σ_{th}^2. This ratio, E/σ_{th}^2, is called the *signal to noise* ratio and will feature prominently in the other additive noise models we will see.

Looking Forward

This simple example of a channel model gave us a feel for simple transmission and reception strategies. It also gave us an idea of how a physical resource such as energy is related to the amount of information we can communicate reliably. The deterministic channel model we have used here is rather simplistic; in particular, the choice of σ_{th} might have to be overly conservative if we have ensure that the additive noise has to lie in the range $\pm\sigma_{\text{th}}$ with full certainty. If we are willing to tolerate some error in reliable communication, we can set a lower range σ_{th} in our channel model and thus allowing for a higher rate of reliable communication of information. This is the topic of the next lecture.