CSC 211: Computer Programming

Sorting Algorithms, Binary Search, Big-O

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THINK BIG WE DO℠

Basic Sorting Algorithms

Sorting and Searching

- Two fundamental problems in CS
  - ✓ Sorting and Searching

Sorting

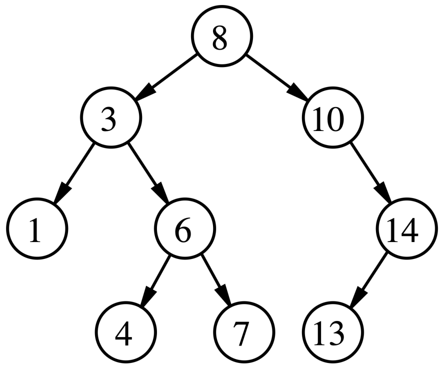
- Given an input sequence of **n** elements that can be compared to each other according to a **total order** relation
  - ✓ we want to rearrange them in non-increasing/non-decreasing order

- Example (sorting in non-decreasing order):
  - **input**: array $A = [k_0, k_1, \ldots, k_{n-1}]$
  - **output**: array B (permutation of A), s.t. $B[0] \leq \ldots \leq B[n-1]$

`Central problem in computer science`

Searching

- Given some data structure **S**, determine if some key **K** exists



Efficiency

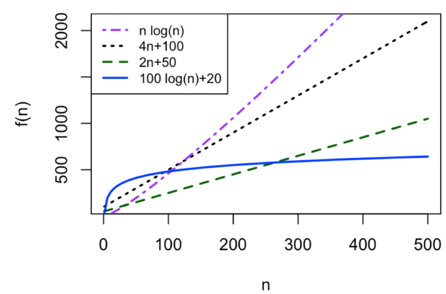
- Efficiency = Time = Money
- We'll talk about efficiency in CSC 212. Why do we care?
- Computers are fast, but they can still take time to do complex actions. And people don't like to wait. A faster algorithm can lead to a company succeeding where others fail.
- A major goal of computer scientists is not just to make algorithms that work, but algorithms that work efficiently.

Function Families

- When we count the actions taken by algorithms, we don't really care about one-off operations; **we care about actions that are related to the size of the input.**
- In math, a **function family** is a set of equations that all grow at the same rate as their inputs grow. (linearly v.s quadratically)
- When determining which equation family represents the actions taken by an algorithm, we say that n is the size of the input.

Function Families

- As n grows, the two linear functions become larger than the log(n) function, and then the n log(n) function becomes larger than both linear functions, regardless of the constants.



Big-O Notation

- When we determine an equation's function family, we ignore constant factors and smaller terms. All that matters is the dominant term (the highest power of n). That is the idea of Big-O notation.

f(n)	Big-O
n	O(n)
32n + 23	O(n)
$5n^2 + 6n - 8$	$O(n^2)$
18 log(n)	O(log n)

Big-O Notation

- Unless specified otherwise, the Big-O of an algorithm refers to its worst case run time
- "Asymptotic Complexity" of algorithm

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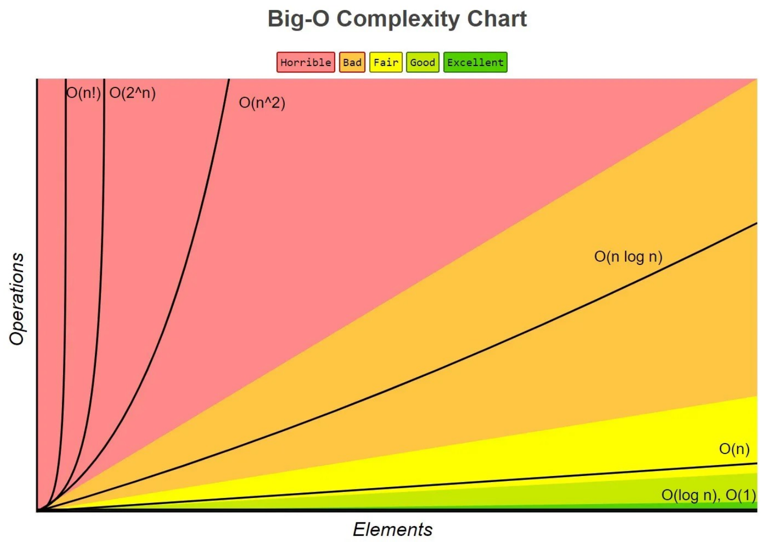
Big-O Notation



Bubble-Sort

- Basic sorting algorithm
  - yet too slow in practice

- Scan the input sequence from left-to-right
  - compare all adjacent elements and swap them if they are in the wrong order
- Repeat the scan until the list is sorted

```
After every pass (iteration), the smaller/larger
 element bubbles up to the end of the sequence
```

Bubble Sort in 2

Implement Bubble Algorithm

```
BUBBLESORT(A)
for i = 1 to A.length − 1
    for j = A.length downto i + 1
        if A[j] < A[j − 1]
            exchange A[j] with A[j − 1]
```

```
void swap(int& v1, int& v2) {
        int temp = v1;
        v1 = v2;
        v2 = temp;
    }
```



```
void swap(int& v1, int& v2) {
    int temp = v1;
    v1 = v2;
    v2 = temp;
}

void bubble(int A[], int n_elem) {
    bool sorted = false;
    while (! sorted) {
        sorted = true;
        for (int i = 0 ; i < (n_elem-1) ; i++) {
            if (A[i] > A[i+1]) {
                sorted = false;
                swap(A[i], A[i+1]);
            }
        }
    }
}

int main() {
    int array[] = {15, 12, 13, 24, 5};
    bubble(array, 5);
}
```


Selection-Sort

- Basic sorting algorithm
  - yet too slow in practice

- Keep two parts: **left part** is already sorted and **right part** is to be sorted
  - initially, the sorted part is empty and the unsorted part is the input sequence
- At every iteration, find the smallest (or largest) element in the unsorted part and swap it with the leftmost unsorted element
  - then move the boundary between parts one element to the right

`At every iteration we select the minimum/maximum`

Selection Sort in 3

Implement Selection Algorithm

```
function SELECTION-SORT(A, n)
    for i = 1 to n-1 do
        min ← i
        for j = i + 1 to n do
            if A[j] < A[min] then
                min ← j
            end if
        end for
        swap A[i], A[min]
    end for
end function
```



```cpp
void swap(int& v1, int& v2) {
    int temp = v1;
    v1 = v2;
    v2 = temp;
}

int find_min(int A[], int start, int last) {
    int min = start;
    for (int i = start + 1 ; i < last ; i++) {
        if (A[i] < A[min]) {
            min = i;
        }
    }
    return min;
}

void selection(int A[], int n_elem) {
    for (int i = 0, j ; i < (n_elem-1) ; i ++) {
        j = find_min(A, i, n_elem);
        swap(A[i], A[j]);
    }
}

int main() {
    int array[] = {15, 12, 13, 24, 5};
    selection(array, 5);
}
```


Insertion-Sort

- Basic sorting algorithm
  - slightly faster than bubble-sort and selection-sort

- Keep two parts: **left part** is already sorted and **right part** is to be sorted
  - initially, the sorted part contains the first element in the array and the unsorted part is the remaining elements
- At every iteration, the first element of the unsorted part is selected, and the algorithm finds the location it belongs within the sorted part, and inserts it there
  - then move the boundary between parts one element to the right
  - repeat until no elements remain in the unsorted part

Implement Insertion Algorithm

ALGORITHM *InsertionSort*$(A[0..n-1])$
//Sorts a given array by insertion sort
//Input: An array $A[0..n-1]$ of n orderable elements
//Output: Array $A[0..n-1]$ sorted in nondecreasing order
for $i \leftarrow 1$ **to** $n-1$ **do**
    $v \leftarrow A[i]$
    $j \leftarrow i-1$
    while $j \geq 0$ **and** $A[j] > v$ **do**
        $A[j+1] \leftarrow A[j]$
        $j \leftarrow j-1$
    $A[j+1] \leftarrow v$

Binary Search

Linear search

- Two fundamental problems in CS
  - ✓ Sorting and Searching
- **Linear (sequential) search** is a method for finding a **value** within a sequence
- A naive solution works by sequentially checking each element until a match is found
  - ✓ it also stops when there are no more elements to check
  - ✓ performs at most **n** comparisons for sequences of length **n**
  - ✓ considered slow for finding elements in collections of data

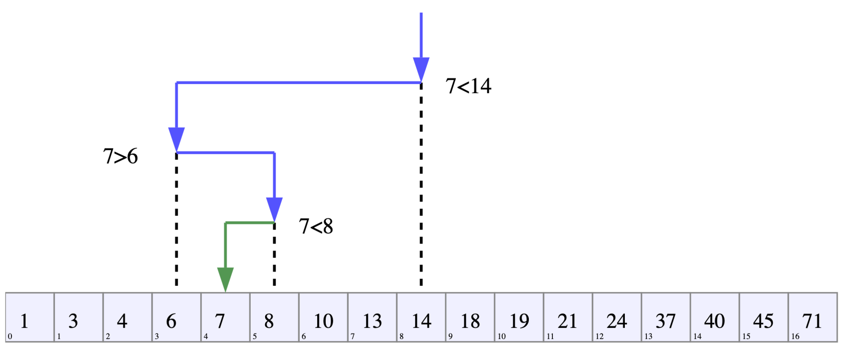
```c
bool lin_search(int *A, int key, int len) {
    for (int i = 0 ; i < len; i++){
        if (A[i] == key){
            return true;
            }
        }
    return false;
}
```


Binary search

- Search algorithm to find a given value within a **sorted array**
- The algorithm compares the value to the middle element
  - ✓ if they are not equal, one of the half is eliminated and the search continues on the other half
  - ✓ repeat until value is found or no more elements are left (value is not in the array)
- Binary search is **faster than linear search**

Binary search

k = 7



Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	5	10	15	20	22	30	35	40	43	48	51

low high

k = 48?

Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	5	10	15	20	22	30	35	40	43	48	51

low mid high

k = 48?

Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
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Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
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Binary Search

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low mid high

k = 48?

Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	5	10	15	20	22	30	35	40	43	48	51

low high

k = 22?

k = 0?

k = 55?

Call stack

0	1	2	3	4	5	6	7	8	9
1	2	5	10	15	20	22	30	35	40

```
#define NOT_FOUND -1

int bsch(int *A, int lo, int hi, int k) {
    if (hi < lo) {
        return NOT_FOUND;
    }
    int mid = lo + ((hi-lo)/2);
    if (A[mid] == k) {
        return mid;
    }
    if (A[mid] < k) {
        return bsch(A, mid+1, hi, k);
    }
    return bsch(A, lo, mid-1, k);
}

int main() {
    int arr[] = {1,2,5,10,15,20,22,30,35,40};
    int idx = bsch(arr, 0, 9, 1);
}
```

https://bit.ly/36cEQWK

Google Research Blog

https://research.googleblog.com/2006/06/extra-extra-read-all-about-it-nearly.html

The latest news from Research at Google

"The version of binary search that I wrote for the JDK (java.util.Arrays) contained the same bug. It was reported to Sun recently when it broke someone's program, after lying in wait for nine years or so."

Extra, Extra - Read All About It: Nearly All Binary Searches and Mergesorts are Broken

Friday, June 02, 2006

Posted by Joshua Bloch, Software Engineer

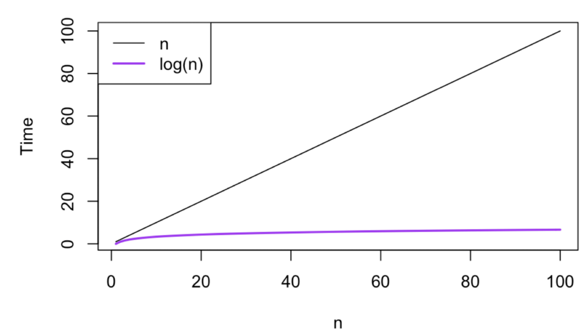
overflow

```
int mid = (low + high) / 2;
```

```
int mid = lo + ((hi-lo) / 2);
```


Big-O of Linear Search v. Binary Search

- Because runtime for linear search is proportional to the length of the list in the worst case, it is O(n). Every time we double the length of the list, binary search does just one more comparison; it is O(log n).



Find peak in unimodal arrays

Unimodal arrays

- An array is (**strongly**) **unimodal** if it can be split into an increasing part followed by a decreasing part

1	2	5	16	20	18	17	16	15	12	10	8	5

- An array is (**weakly**) **unimodal** if it can be split into a nondecreasing part followed by a nonincreasing part

1	2	5	5	15	20	22	22	35	38	13	8	5

Find the peak (strongly unimodal)

1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5

Find the peak (weakly unimodal)

1	2	5	5	15	20	22	22	35	38	13	8	5
1	2	5	5	15	20	22	22	35	38	13	8	5
1	2	5	5	15	20	22	22	35	38	13	8	5

Two recursive calls