CSC 211: Computer Programming
Recursion

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THINK BIG WE DO℠

Recursion

Recursion

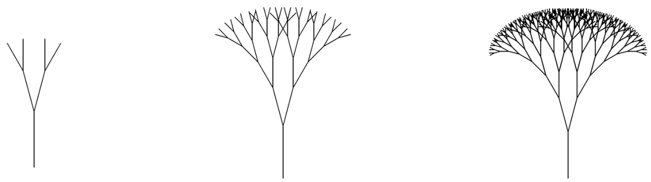
- Problem solving technique in which we solve a task by reducing it to smaller tasks (of the same kind)
  - then use same approach to solve the smaller tasks
- Technically, a recursive function is one that **calls itself**
- General form:
  - **base case**
    - solution for a **trivial case**
    - it can be used to stop the recursion (prevents "*stack overflow*")
    - every recursive algorithm needs at least one base case
  - **recursive call(s)**
    - divide problem into **smaller instance(s)** of the **same structure**

General form

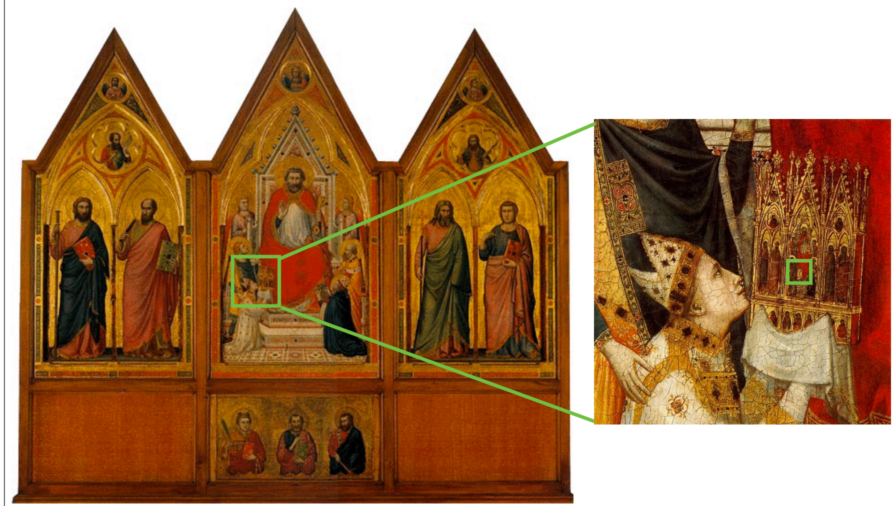
```
function() {

    if (this is the base case) {

        calculate trivial solution

    } else {

        break task into subtasks
        solve each task recursively
        combine solutions if necessary

    }
}
```

Why recursion?

- Can we live without it?
  - ✓ yes, you can write “any program” with arrays, loops, and conditionals
- However …
  - ✓ some formulas are explicitly recursive
  - ✓ some problems exhibit a natural recursive solution



https://courses.cs.washington.edu/courses/cse120/17sp/labs/11/tree.html



The Stefaneschi Altarpiece is a triptych by the Italian medieval painter Giotto, commissioned by Cardinal Giacomo Stefaneschi to serve as an altarpiece for one of the altars of Old St. Peter's Basilica in Rome. It is now at the Pinacoteca Vaticana, Rome. Circa 1320.

https://en.wikipedia.org/wiki/Stefaneschi_Triptych

Example: factorial

$$n! = 1 \cdot 2 \cdot \ldots \cdot n = \prod_{k=1}^{n} k$$

Piecewise function

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ \boxed{(n-1)! \cdot n} & \text{if } n > 0 \end{cases}$$

Recurrence Relation

Example: factorial

- Apply the recursive definition of factorial to calculate:

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ (n-1)! \cdot n & \text{if } n > 0 \end{cases}$$

- 3 !

General form

```
function() {

    if (this is the base case) {

        calculate trivial solution

    } else {

        break task into subtasks
        solve each task recursively
        combine solutions if necessary

    }
}
```


Example: factorial

```
int fact(int n) {

    // base case
    if (n == 0) {
        return 1;
    }

    // recursive call
    return fact(n-1) * n;
}
```


Example: factorial

```
int fact(int n) {

    // base case
    if (n < 2) {
        return 1;
    }

    // recursive call
    return fact(n-1) * n;
}
```


Recursion call stack

```
fact(3) == 6
```

```
int fact(int n) {
    if (n < 2) {
        return 1;
    }
    return fact(n-1) * n;
}
```

fact (1)
n = 1
return 1

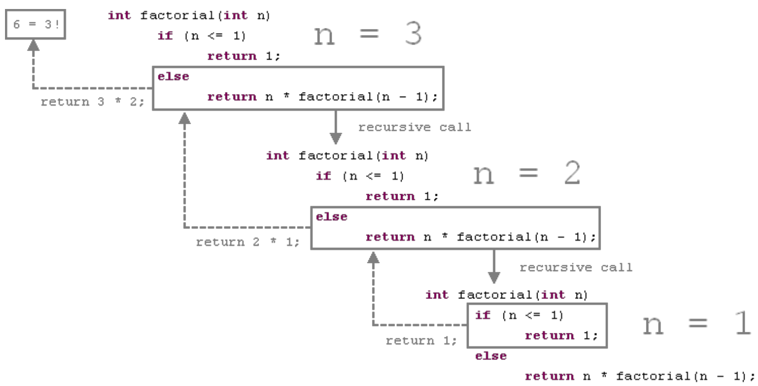
fact (2)
n = 2
return ~~fact(1)~~ 1 * 2 = 2

fact (3)
n = 3
return ~~fact(2)~~ 2 * 3 = 6 → 6

Stack

Example

- Factorial



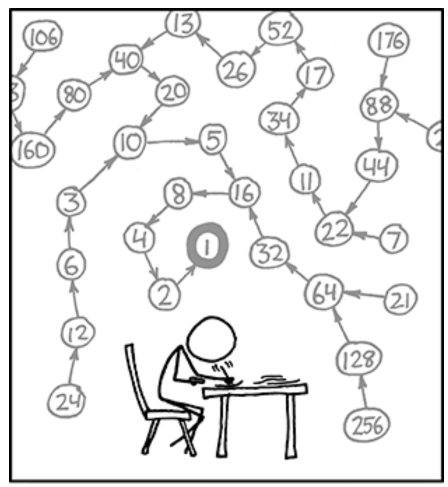
Question

- Given $f(n) = f(n-1) + 2n - 1$, what is the value of $f(3)$?

`Must have base case and make progress towards base case`

Rules of the game

- Your code must have **at least one base case** for a trivial solution
  - that is, for a non-recursive solution
- Recursive calls should **make progress** towards the base case
- Your code must break a larger problem into smaller problems
  - each smaller problem should be of the same '**nature**' as the larger problem



THE COLLATZ CONJECTURE STATES THAT IF YOU PICK A NUMBER, AND IF IT'S EVEN DIVIDE IT BY TWO AND IF IT'S ODD MULTIPLY IT BY THREE AND ADD ONE, AND YOU REPEAT THIS PROCEDURE LONG ENOUGH, EVENTUALLY YOUR FRIENDS WILL STOP CALLING TO SEE IF YOU WANT TO HANG OUT.

No one knows whether or not this function terminates for all values of N

Example: power of a number

$$b^n = \underbrace{b \cdot b \cdot \ldots \cdot b}_{n \text{ times}}$$

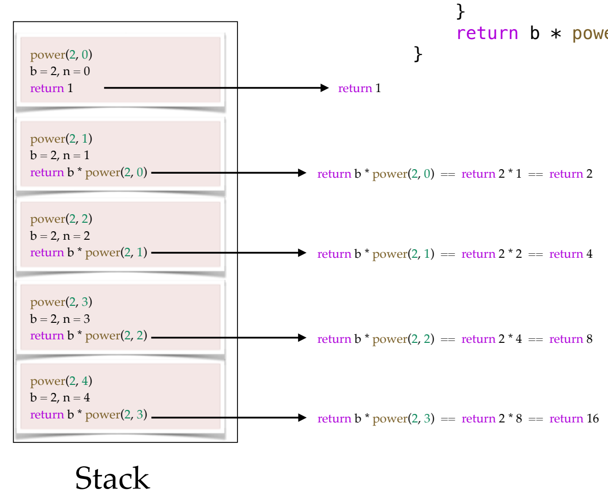
base case?

recursive case?

Recursion call stack

```
power(2, 4)
```

```
double power(double b, int n) {
    if (n == 0) {
        return 1;
    }
    return b * power(b, n-1);
}
```



Recursion call tree (tracing recursion)

```
double power(double b, int n) {
    if (n == 0) {
        return 1;
    }
    return b * power(b, n-1);
}
```


What is the output of `foo(1234)`?

```
int foo(int n) {
    if (n < 10) {
        return n;
    }
    int b = n % 10;
    return b + foo(n/10);
}
```

What is the output of `mystery(7)`?

```
void mystery(unsigned int n) {
    if (n < 2) {
        std::cout << n;
    } else {
        mystery(n/2);
        std::cout << n % 2;
    }
}
```


Indirect Recursion

```
void f2(int n);

void f1(int n) {
    if (n > 1) {
        std::cout << "1";
        f2(n - 1);
    }
}

void f2(int n) {
    std::cout << "0";
    f1(n - 1);
}
```

f1(1) ?

f1(2) ?

f1(4) ?

f1(7) ?

f1(10) ?

Some considerations and summary

- Recursion is a powerful technique that solves problems by breaking them down into smaller subproblems of the same form, and applying the same strategy to solve the subproblems
- One can always write an iterative solution to a problem solved recursively
  - ✓ recursive code is often simpler to read, write, and maintain
- Not always an efficient solution (iterative counterparts are faster)
  - ✓ why not?
  - ✓ overhead

Overhead is any combination of excess or indirect computation time, memory, bandwidth, or other resources that are required to perform a specific task.

Lets Try It - Fibonacci sequence

$$F_0 = 0$$
$$F_1 = 1$$
$$F_n = F_{n-1} + F_{n-2}$$

0 1 1 2 3 5 8 13 21 34 …

The **Fibonacci sequence** first appears in the book **Liber Abaci** (1202) by Fibonacci, using it to calculate the growth of rabbit populations. The sequence had been described by Indian mathematicians as early as the **sixth century**.

from: wikipedia

Fib we've seen before

- Write a program to print the first 50 terms of the Fibonacci sequence (pick your favorite loop)

$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$

```
int num = 50;
int x = 0, y = 1, z = 0;
for (int i = 0; i < num; i++) {
    std::cout << x << " ";
    z = x + y;
    x = y;
    y = z;
}
```

0 1 1 2 3 5 8 13 21 34 …

Recursion call tree (tracing recursion)

fib(4) =

$F_0 = 0$
$F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$

```
int num = 4;
int x = 0, y = 1, z = 0;
for (int i = 0; i < num; i++) {
    std::cout << x << " ";
    z = x + y;
    x = y;
    y = z;
}
```

```
int fib(int n) {
    if (n <= 1) {
        return 1;
    }
    return fib(n-1) + fib(n-2);
}
```