Homework 6

Due by 4:30pm on October 21, 2020

Viewings and Readings

[Links to the slides, recordings, and quiz are on the course web site.]
Review the week 6 lecture slides.
View the week 6 lecture recordings and the discussion session recording.
Complete the week 6 quiz.
Read sections 3.7-9 and 4.1-2 of the textbook. (Section 3.10 is optional.)

Problems

Please write precise and concise answers. Except where explicitly indicated, your algorithm descriptions should use either clear, concise, and precise plain English or clear, concise, and precise pseudo-code that uses a style similar to the pseudo-code in your textbook. Submit your solutions to problems **1-5(a)** via D2L as a Word or PDF file or as scans/photos of legible handwritten notes. Submit your solutions to problems **5(b)** and 6 via Kattis.

1. Problem 5(a) page 125.

This exercise asks you to develop efficient algorithms to find optimal subsequences of various kinds. A subsequence is anything obtained from a sequence by extracting a subset of elements, but keeping them in the same order; the elements of the subsequence need not be contiguous in the original sequence. For example, the strings C, DAMN, YAIOAI, and DYNAMICPROGRAMMING are all subsequences of the string DYNAMICPROGRAMMING. [Hint: Exactly one of these problems can be solved in O(n) time using a greedy algorithm.]

(a) Let A[1 .. m] and B[1 .. n] be two arbitrary arrays. A common subsequence of A and B is another sequence that is a subsequence of both A and B. Describe an efficient algorithm to compute the length of the longest common subsequence of A and B.

```
We came up with a recursive solution to this problem in an earlier homework

lcs(A[i .. n], B[j .. m]):
  if length A == 0 OR length B == 0
    return 0
  if A[1] == B[1] {
    return lcs(A[i + 1 .. n], B[j + 1 .. n]) + 1
  } else {
     return max(lcs(A[i + 1 .. n], B[j .. n]), lcs(A[i .. n], B[j + 1 .. n]))
  }

  The subproblems are lcs(i to n+1, j to n+1) where when i = n + 1 or j = m + 1 then the lcs is 0, else the lcs(i, j) is the max of lcs(i + 1, j), lc

  We can store the memozation solutions in a 2d array [i to n+1, j to m+1]

  Since lcs(i, j) = max lcs(i + 1, j), lcs(i, j + 1), we must calculate lcs from n+1->1 so we have those dependencies calulated already

  fastLcs(A[1 ... n], B[1 ... m]) {
    lcsTable
    for x = 1 in range(n + 1)
      lcsTable[x, n + 1] = 0
    for x = 1 in range(m + 1)
      lcsTable[m+1, x] = 0

    for i = length(A) + 1 to 1 {
      for j = length(B) + 1 to 1 {
        if A[i] == B[i] {
          lcsTable[i, j] = lcsTable[i + 1, j + 1]
        } else {
          lcsTable[i, j] = max(lcsTable[i + 1, j], lcsTable[i, j + 1])
        }
      }
    }

    return lcsTable[1, 1]
  }
```

2. Problem 9(a) page 128.

A palindrome is any string that is exactly the same as its reversal, like I, or DEED, or RACECAR, or AMANAPLANACATACANALPANAMA.

Describe and analyze an algorithm to find the length of the longest subsequence of a given string that is also a palindrome. For example, the longest palindrome subsequence of the string MAHDYNAMICPROGRAMZLETMESHOWYOUTHEM is MHYMRORMYHM; thus, given that string as input, your algorithm should return 11.

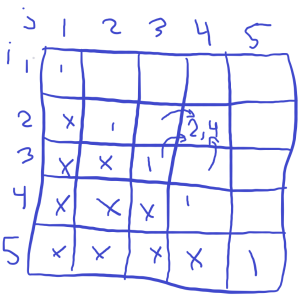
```
i, j start at the ends of a string S then recursively increment/decrement their way until the base case of them equalling each other
palindrome(i, j) {
  // in case the return palindrome(i + 1, j - 1) call causes i to be greater than j
  if i > j
    // this means theres 2 characters and they are equal so return 2
    if S[i] == S[j]
      return 2
    else
      reuturn 0
```

```
  // one character is a palindrome of length 1
  if i == j
    return 1

  // if characters are equal, increment sum by 2 for the 2 matching character then move i and j closer together by 1 each
  if S[i] == S[j]
    return palindrome(i + 1, j - 1) + 1
  else
    // if there not equal try both the next characters for i and j to see if they are equal
    return max(palindrome(i + 1, j), palindrome(i, j - 1))
}
```

```
i > j && S[i] == S[j]            0
i > j && S[i] != S[j]            2
i == j                           1
S[i] == S[j]                     palindrome(i + 1, j - 1) + 1
else                             max(palindrome(i + 1, j), palindrome(i, j - 1))
```

Again we can memoize this with a 2d array of [1 to n, 1 to n]



Based on this table we need to calculate 5 to 1 for i and 1 to 5 for j

```
palindrome(S) {
  n = length(S)
  p = []
  for i to n
    p[i, i] = 1

  for i = n to 1
    for j = i+1 to n
      if i + 1 == j && S[i] == S[j]
        p[i, j] = 2
      else if S[i] == S[j]
        p[i, j] = 2 + p[i + 1, j - 1]
      else
        p[i, j] = max(p[i + 1, j], p[i, j - 1])
}
```

3. Follow the suggestion in the footnote on page 162 of you textbook and develop a recursive backtracking algorithm and then a dynamic programming algorithm for the activity selection problem. Do this by going through the steps 1(a), 1(b), 2(a), ... listed at the beginning of the lecture 6 slides. Steps 1(a) and 1(b) and the insight shown in slides 85-89 will lead you to the recursive backtracking algorithm. Steps 2(a) through 2(d) will lead you to the dynamic programming algorithm. Make sure to analyze the running time of your dynamic programming algorithm.

4. Problem 1 page 176.

The GreedySchedule algorithm we described for the class scheduling problem is not the only greedy strategy we could have tried. For each of the following alternative greedy strategies, either prove that the resulting algorithm always constructs an optimal schedule, or describe a small input 176 Exercises example for which the algorithm does not produce an optimal schedule. Assume that all algorithms break ties arbitrarily (that is, in a manner that is completely out of your control). [Hint: Three of these algorithms are actually correct.]

Choose the course x that ends last, discard classes that conflict with x, and recurse.

Choose the course x that starts first, discard all classes that conflict with x, and recurse.

Choose the course x that starts last, discard all classes that conflict with x, and recurse.

Choose the course x with shortest duration, discard all classes that conflict with x, and recurse.

Choose a course x that conflicts with the fewest other courses, discard all classes that conflict with x, and recurse.

If no classes conflict, choose them all. Otherwise, discard the course with longest duration and recurse

If no classes conflict, choose them all. Otherwise, discard a course that conflicts with the most other courses and recurse.

Let x be the class with the earliest start time, and let y be the class with the second earliest start time. - If x and y are disjoint, choose x and recurse on everything but x. - If x completely contains y, discard x and recurse. - Otherwise, discard y and recurse.

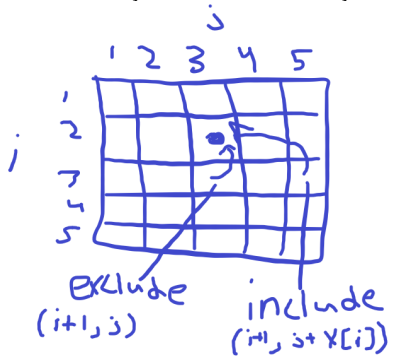
If any course x completely contains another course, discard x and recurse. Otherwise, choose the course y that ends last, discard all classes that conflict with y, and recurse.

5. Week 6 problem *walrusweights* on Kattis.

(a) Develop a dynamic programming algorithm for the problem. Do this by going through the steps 1(a), 1(b), 2(a), ... listed at the beginning of the lecture 6 slides. You should, in particular, describe the recursive solution/algorithm that is the result of 1(b).

```
1a.   Input of index i for an array X[i ... n] and the current sum.  Recursively include and exclude each option in the array and return the one that
1b.   start at input walrus(1, 0) each recuresive call will follow this formula
      walrus(i, sum)
        if sum = 1000             return 1000
        if i = length(X)          return sum
        else                      include = walrus(i + 1, sum + X[i])
                                  exclude = walrus(i + 1, sum)
                                  if absoluteValue(1000 - include) <= absoluteValue(1000 - exclude)
                                    return include
                                  else
                                    return exclude

2a.   The subproblems are walrus(i, sum) where 0 <= i <= length(X) and 0 <= sum.  Sum doesn't really have a set endpoint
2b.   So we need a 2d array [i, j] where i is 1 -> length(X) and j is 0 -> total sum of all values in X
2c.   Each entry in the memoized array needs either the answer done for either [i + 1, sum + X[i]] or [i + 1, sum]
```

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2d.   This means we must solve the problem i = length(x) to 1 and j = sum all X to 0
2e.   Kattis Solution
2f.   O(i*1000) = O(i)
```

(b) Implement your dynamic programming solution using your preferred language and submit your implementation via Kattis.

6. [Optional] Week 6 problem *spiderman* on Kattis. Submit your solution via Kattis.