Non-linear Spatio-temporal modelling using the Integro-Difference Equation

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Abstract

Index Terms

Dynamic spatio-temporal modelling, Integro-Difference Equation (IDE), ...

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I. Introduction

Non-linear spatio-temporal behaviour...

II. Neural Field Model

III. State space Representation of the Spatio-temporal Non-linear IDE

The dynamics of the pth order non-linear, stochastic, spatio-temporal IDE is described by

$$\rho_t(\mathbf{r}) = \int_\Omega w(\mathbf{r},\mathbf{r}')f(v_{t-p}(\mathbf{r}'))d\mathbf{r}' + \mathbf{e}_{t-p}(\mathbf{r}) \tag{1}$$

where $\rho_t(\mathbf{r})$ is a lag polynomial defined as

$$\rho_t(\mathbf{r}) = (1+\sum_{i=1}^{p}\varphi_i L_i)v_t\left(\mathbf{r}\right) \tag{2}$$

where $v_t(\mathbf{r})$ denotes the spatial field at time t and at spatial location $\mathbf{r}$, $f(.)$ is some nonlinear map, and $w(.)$ is the spatial kernel. The disturbance $\mathbf{e}_{t-p}(\mathbf{r})$ is a zero-mean normally distributed noise process, spatially coloured but temporally independent, with covariance

$$cov\left(e_t\left(\mathbf{r}\right), e_{t+\tau}\left(\mathbf{r}'\right)\right) = \gamma\left(\mathbf{r}-\mathbf{r}'\right)\delta(t-\tau) \tag{3}$$

where $\delta(.)$ is the Dirac-delta function. The spatial field is observed by

$$\mathbf{y}_t = \int_\Omega m\left(\mathbf{r}_{n_y}-\mathbf{r}'\right)v_t\left(\mathbf{r}'\right)d\mathbf{r}' + \boldsymbol{\varepsilon}_t, \tag{4}$$

where the observation vector $\mathbf{y}_t$, compiled at n_y spatial locations and at time t, is corrupted by an i.i.d normally distributed zero-mean white noise $\boldsymbol{\varepsilon}_t \sim \mathcal{N}\left(\mathbf{0},\boldsymbol{\Sigma}_\varepsilon\right)$ with $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} = \sigma_\varepsilon^2\mathbf{I}_{n_y}$. It is assumed here that $\boldsymbol{\Sigma}_\varepsilon$ and $\gamma\left(\mathbf{r}-\mathbf{r}'\right)$ are known. The output kernel $m(\mathbf{r}-\mathbf{r}')$ governs the sensor pick-up geometry and is defined by

$$m\left(\mathbf{r}-\mathbf{r}'\right) = \exp\left(-\frac{(\mathbf{r}-\mathbf{r}')^\top(\mathbf{r}-\mathbf{r}')}{\sigma_m^2}\right), \tag{5}$$

where σ_m sets the sensor width. The superscript $\top$ denotes the transpose operator.

Here we consider the case where $p=1$, therefore we have

$$v_t(\mathbf{r}) + \varphi_1 v_{t-1}(\mathbf{r}) = \int_\Omega w(\mathbf{r},\mathbf{r}')f(v_{t-1}(\mathbf{r}'))d\mathbf{r}' + \mathbf{e}_{t-1}(\mathbf{r}) \tag{6}$$

IV. Estimation of the Non-linear Spatio-temporal IDE Model

A. Unscented Kalman Filter

B. Unscented RTS Smoother

C. Parameter Estimation

D. Computational Complexity

V. Example

VI. Conclusion

Appendix

Lemma 1:

Proof:

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