Model-Based Estimation of Intracortical Spatial Properties

Dean R. Freestone*, Parham Aram†, Kenneth Scerri‡, Michael Dewar §,
Visakan Kadirkamanathan† and David B. Grayden*
*Department of Electrical and Electronic Engineering
The University of Melbourne, Parkville, VIC, Australia
Email: see http://www.neuroeng.unimelb.edu.au/
†Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield, UK
‡Department of Systems and Control Engineering, University of Malta, Msida, MSD, Malta
§Department of Applied Physics and Applied Mathematics, Columbia University, New York, NY, USA

***Abstract*—**

I. Introduction

Discuss model-based estimation

II. Method

A. Stochastic Neural Field Model

$$v\left(\mathbf{r},t\right) = e^{-\zeta t} v\left(\mathbf{r},0\right) + \int_0^t h\left(t-t'\right) g\left(\mathbf{r},t'\right) \mathrm{d}t', \tag{1}$$

where $v\left(\mathbf{r},0\right)$ is the membrane voltage at the initial time $t=0$, $g(\cdot)$ describes the mean firing rate, $h(\cdot)$ is the first-order post-synaptic response kernel of the form

$$h(t) = \eta(t)\exp\left(-\zeta t\right), \tag{2}$$

where $\zeta = \tau^{-1}$, τ is the synaptic time constant and $\eta(t)$ is the Heaviside step function. The firing rate is modeled by having both a deterministic component and stochastic component, modeled by a spatially coloured space-time Wiener process $W(\mathbf{r},t)$, and is described by

$$g(\mathbf{r},t)\mathrm{d}t = \tilde{g}(\mathbf{r},t)\mathrm{d}t + \sigma_W \mathrm{d}W(\mathbf{r},t), \tag{3}$$

where $\sigma_W \geq 0$ is a measure of the introduced space-time randomness. For $\sigma_W = 0$ a pure deterministic process is recovered. Alternatively if $\sigma_W > 0$, the membrane voltage $v(\mathbf{r},t)$ is itself a stochastic process. Incorporating the stochastic process is a variation of the typical WC or Amari formulations accounting for model uncertainty and unknown inputs. and is modeled by a spatially coloured space-time Wiener process $W(\mathbf{r},t)$.

Assuming an infinite propagation velocity for action potentials within the field (intracortical patch), the deterministic incoming firing rate is described by the spatial convolution

$$\tilde{g}\left(\mathbf{r},t\right) = \int_\Omega w\left(\mathbf{r},\mathbf{r}'\right) f\left(v\left(\mathbf{r}',t\right)\right) \mathrm{d}\mathbf{r}', \tag{4}$$

where $w(\cdot)$ is the spatial connectivity kernel and Ω is the spatial domain representing the cortical sheet or surface. The function $f(\cdot)$ relates mean post-synaptic potentials to mean firing rates and follows a sigmoid described by

$$f\left(v\left(\mathbf{r}',t\right)\right) = \frac{1}{1+\exp\left(\varsigma\left(v_0 - v\left(\mathbf{r}',t\right)\right)\right)}, \tag{5}$$

where v_0 and ς describe the firing threshold and the slope of the sigmoid respectively. By substituting equation 3 into equation 1 we obtain the stochastic integral equation model

$$v\left(\mathbf{r},t\right) = e^{-\zeta t} v\left(\mathbf{r},0\right) + \int_0^t h\left(t-t'\right) \tilde{g}\left(\mathbf{r},t'\right)\mathrm{d}t' + \sigma_W \int_0^t h\left(t-t'\right)\mathrm{d}W\left(\mathbf{r},\right. \tag{6}$$

Here we assume that $v\left(\mathbf{r},0\right)$ is independent of the disturbance on the action potentials and assumed to be generated by a known distribution. We stress that σ_W does not depend on the field $v(\mathbf{r},t)$ and hence the noise in equation 6 is strictly additive. The general integral-differential equation is given by

$$\mathrm{d}v(\mathbf{r},t) + \zeta v(\mathbf{r},t)\mathrm{d}t = \tilde{g}(\mathbf{r},t)\mathrm{d}t + \mathrm{d}W(\mathbf{r},t) \quad t \geq 0, v(\mathbf{r},0) = GP? \tag{7}$$

To show that this is indeed the case consider the function $\kappa(v(\mathbf{r},t),t) = v(\mathbf{r},t)e^{\zeta t}$. We note that $\kappa(v(\mathbf{r},t),t)$ is twice differentiable so that we can apply Ito's formula to obtain

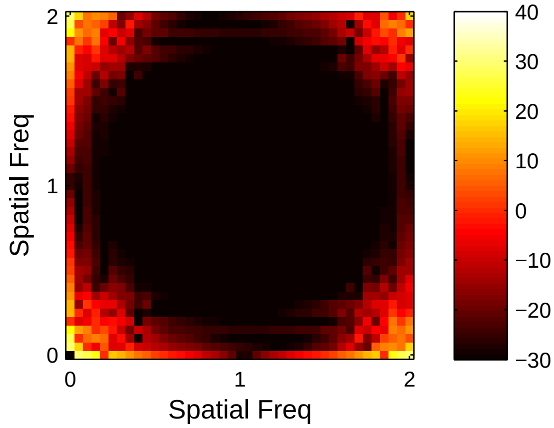
$$\mathrm{d}\kappa = e^{\zeta t} g(\mathbf{r},t)\mathrm{d}t + e^{\zeta t}\sigma \mathrm{d}W(\mathbf{r},t) \tag{8}$$

Integrating over $[0,t]$ and multiplying throughout by $e^{-\zeta t}$ gives the required result. The neural field equations must be written as a discrete-time finite-dimensional model in order to relate it to patient-specific data. The discrete-time model is found applying the a first-order Euler-Maruyama method on equation 7 giving

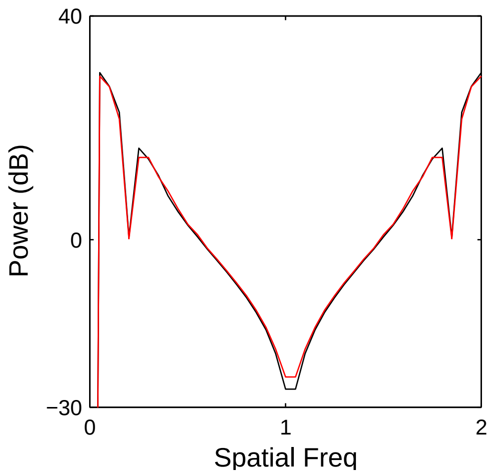
$$v\left(\mathbf{r},t+T_s\right) - v\left(\mathbf{r},t\right) + \zeta T_s v\left(\mathbf{r},t\right) = T_s \int_\Omega w\left(\mathbf{r},\mathbf{r}'\right) f\left(v\left(\mathbf{r}',t\right)\right)\mathrm{d}\mathbf{r}' + \sigma_W [W \tag{9}$$

where T_s is the time step or sampling period. To simplify the notation, the sample at the the current time step shall be indexed by t and the future time step by $t+1$ for the rest of the paper. Rearranging equation 9 gives the integro-difference equation (IDE) form

$$v_{t+1}\left(\mathbf{r}\right) = \xi v_t\left(\mathbf{r}\right) + T_s \int_\Omega w\left(\mathbf{r},\mathbf{r}'\right) f\left(v_t\left(\mathbf{r}'\right)\right)\mathrm{d}\mathbf{r}' + e_t\left(\mathbf{r}\right), \tag{10}$$



(a)



(b)

Fig. 1. a) Field spatial frequency in dB. b) Spatial frequency cross-section showing cutoff frequency. The black line shows the spatial frequency in the x direction and the red line shows the frequency in the y direction.

where $e_t(\mathbf{r}) = \sigma_W[W(\mathbf{r}, t + T_s) - W(\mathbf{r}, t)]$ is the increment of a space-time Wiener process, $i.i.d.$ with zero spatial mean such that $e_t(\mathbf{r}) \sim \mathcal{GP}(\mathbf{0}, T_s\sigma_W^2\gamma(\mathbf{r}-\mathbf{r}'))$. Here $\mathcal{GP}(\mathbf{0}, T_s\sigma_W^2\gamma(\mathbf{r}-\mathbf{r}'))$ denotes a zero mean spatial Gaussian process with covariance function $\gamma(\mathbf{r}-\mathbf{r}')$ [**?**].

B. Maybe: Basis Function Decomp

C. Estimation of Connectivity Kernel Support

III. Results

A. Linear Neural Field Model

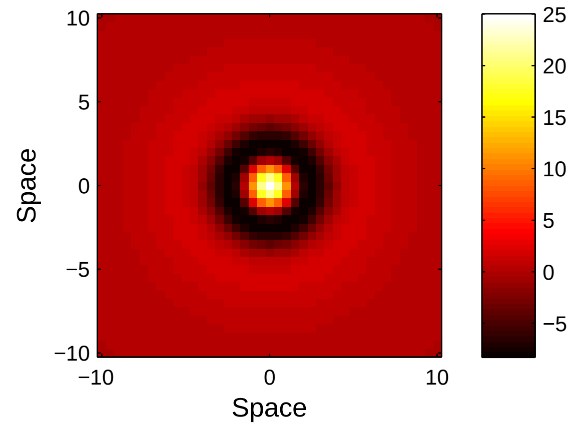
Show the exact estimation of the kernel.

B. Nonlinear Neural Field Model

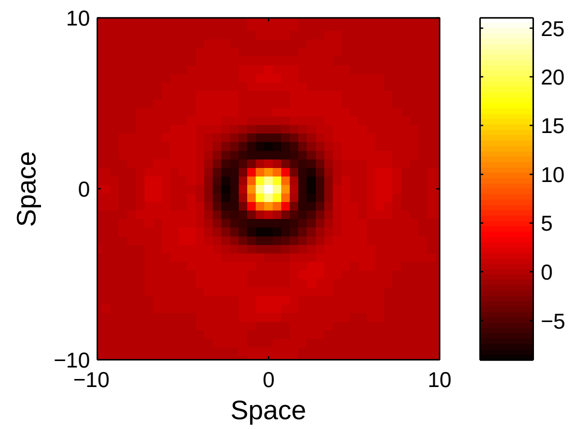
IV. Discussion

V. Conclusion

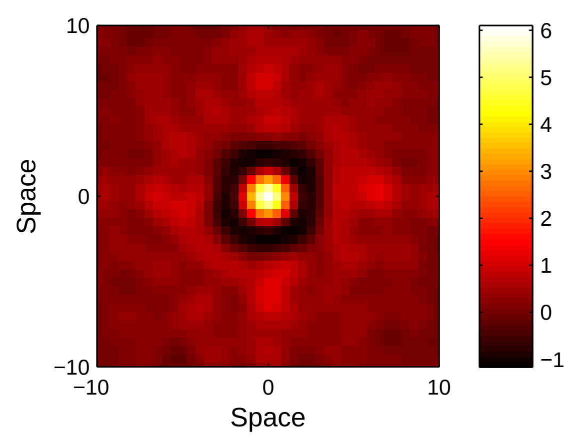
The conclusion goes here.



(a)



(b)



(c)

Fig. 3. a) The actual kernel. b) Estimate of kernel when generating data using the linear model. b) Estimate of the kernel when using the nonlinear model.

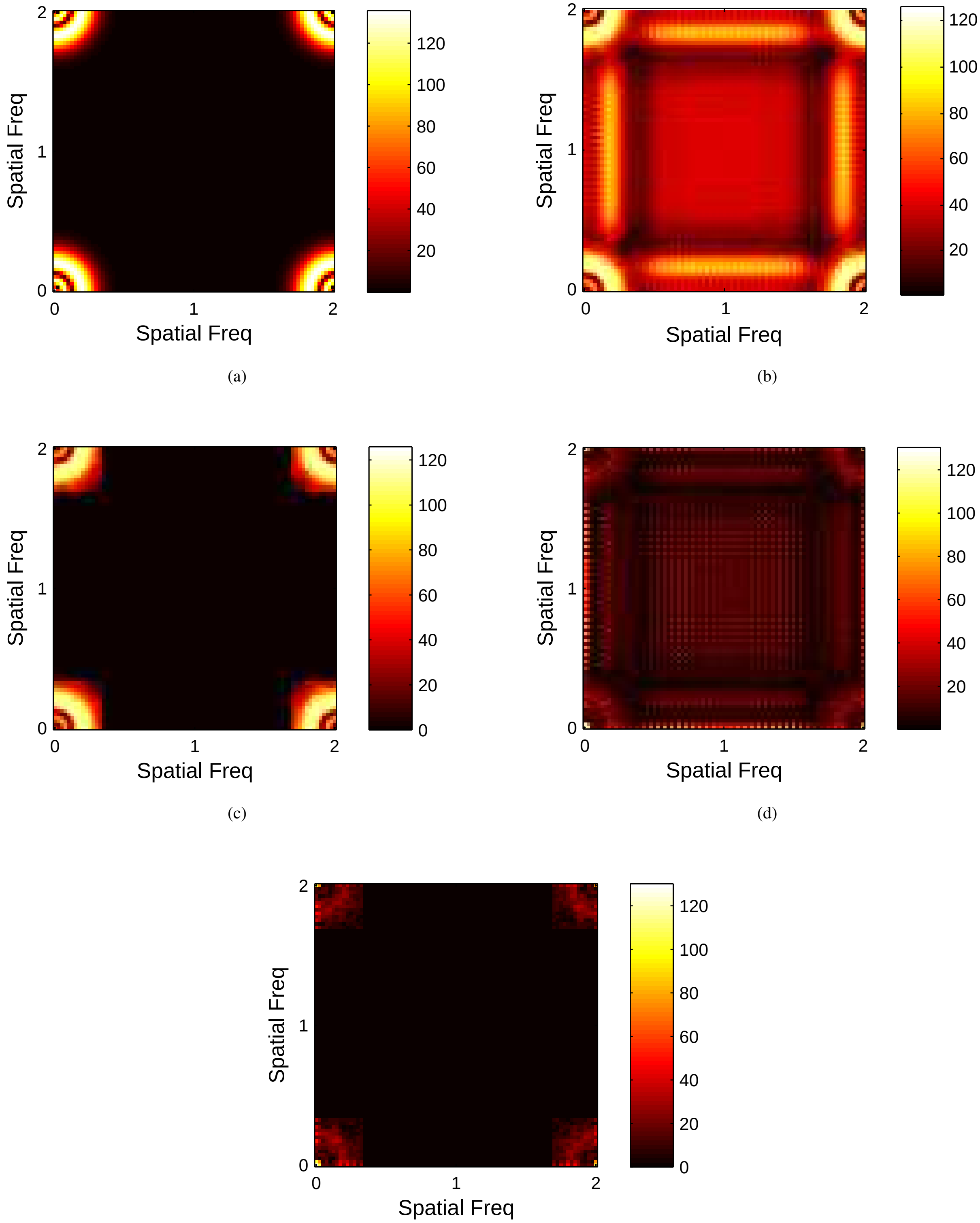
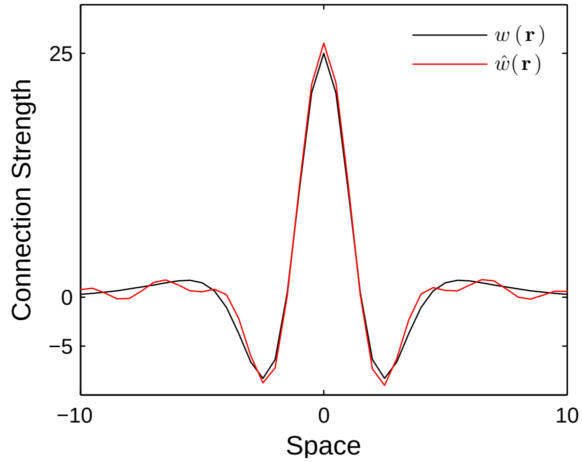
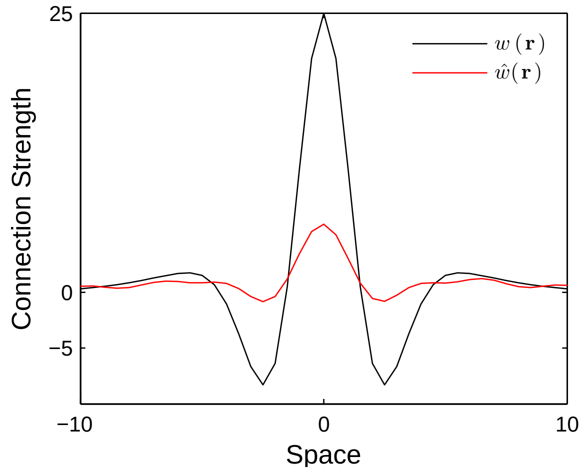


Fig. 2. a) FFT of kernel. b) FFT of kernel estimate when generating data with linear model showing numerical issues. c) Thresholded FFT of kernel estimate when generating data with linear model. d) FFT of kernel estimate when generating data with nonlinear model. e) Thresholded FFT of kernel estimate when generating data with nonlinear linear model.



(a)



(b)

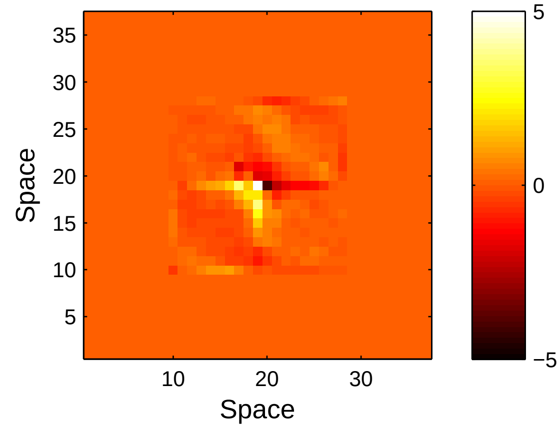
Fig. 4. a) From the linear model. b) from the nonlinear model.

ACKNOWLEDGMENT

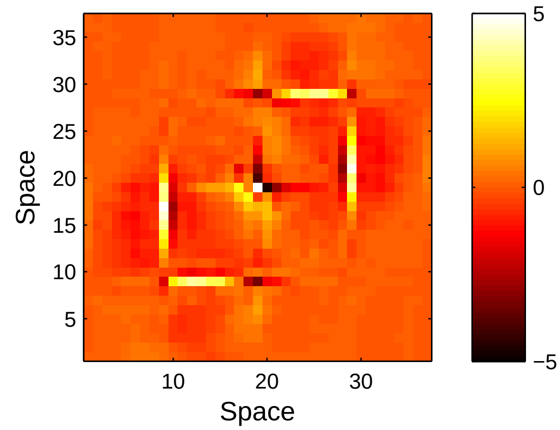
The authors would like to thank...

REFERENCES

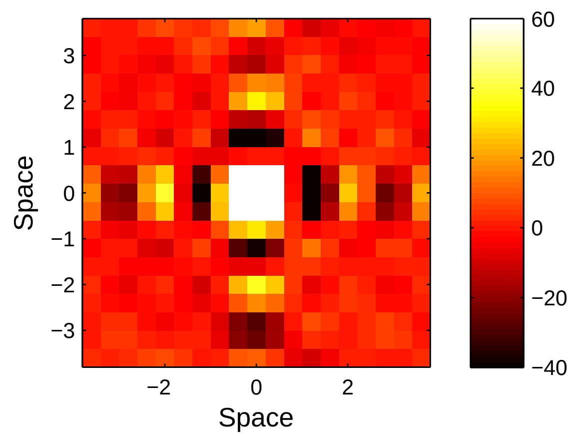
[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.



(a)



(b)



(c)

Fig. 5. Kernel estimates from data. a) Using linsolv. b) Using psuedo inverse of convolution matrix. b) Using FFT.