

Non-linear Spatio-temporal modelling using the Integro-Difference Equation

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Abstract

Index Terms

Dynamic spatio-temporal modelling, Integro-Difference Equation (IDE), ...

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I. INTRODUCTION

Non-linear spatio-temporal behaviour...

II. NEURAL FIELD MODEL

III. STATE SPACE REPRESENTATION OF THE SPATIO-TEMPORAL NON-LINEAR IDE

We define the p th order non-linear, stochastic, spatio-temporal IDE as

$$\rho_t(\mathbf{r}) = \int_{\Omega} w(\mathbf{r}, \mathbf{r}') f(v_{t-p}(\mathbf{r}')) d\mathbf{r}' + \mathbf{e}_{t-p}(\mathbf{r}) \quad (1)$$

where $\rho_t(\mathbf{r})$ is a lag polynomial defined as

$$\rho_t(\mathbf{r}) = (1 + \sum_{i=1}^p \varphi_i L_i) v_t(\mathbf{r}) \quad (2)$$

where $v_t(\mathbf{r})$ denotes the spatial field at time t and at spatial location \mathbf{r} , $f(\cdot)$ is some nonlinear map, and $w(\cdot)$ is the spatial kernel. The disturbance $\mathbf{e}_{t-p}(\mathbf{r})$ is a zero-mean normally distributed noise process, spatially coloured but temporally independent, with covariance

$$\text{cov}(e_t(\mathbf{r}), e_{t+\tau}(\mathbf{r}')) = \gamma(\mathbf{r} - \mathbf{r}') \delta(t - \tau) \quad (3)$$

where $\delta(\cdot)$ is the Dirac-delta function. The spatial field is observed by

$$\mathbf{y}_t = \int_{\Omega} m(\mathbf{r}_{n_y} - \mathbf{r}') v_t(\mathbf{r}') d\mathbf{r}' + \boldsymbol{\varepsilon}_t, \quad (4)$$

where the observation vector \mathbf{y}_t , compiled at n_y spatial locations and at time t , is corrupted by an i.i.d normally distributed zero-mean white noise $\boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}_{\varepsilon})$ with $\boldsymbol{\Sigma}_{\varepsilon} = \sigma_{\varepsilon}^2 \mathbf{I}_{n_y}$. It is assumed here that $\boldsymbol{\Sigma}_{\varepsilon}$ and $\gamma(\mathbf{r} - \mathbf{r}')$ are known. The output kernel $m(\mathbf{r} - \mathbf{r}')$ governs the sensor pick-up geometry and is defined by

$$m(\mathbf{r} - \mathbf{r}') = \exp\left(-\frac{(\mathbf{r} - \mathbf{r}')^{\top}(\mathbf{r} - \mathbf{r}')}{\sigma_m^2}\right), \quad (5)$$

where σ_m sets the sensor width. The superscript \top denotes the transpose operator.

Here we consider the case where $p = 1$, therefore we have

$$v_t(\mathbf{r}) + \varphi_1 v_{t-1}(\mathbf{r}) = \int_{\Omega} w(\mathbf{r}, \mathbf{r}') f(v_{t-1}(\mathbf{r}')) d\mathbf{r}' + \mathbf{e}_{t-1}(\mathbf{r}) \quad (6)$$

IV. ESTIMATION OF THE NON-LINEAR SPATIO-TEMPORAL IDE MODEL

- A. Unscented Kalman Filter*
- B. Unscented RTS Smoother*
- C. Parameter Estimation*
- D. Computational Complexity*

V. EXAMPLE

VI. CONCLUSION

APPENDIX

Lemma 1:

Proof:

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