PDE Report: Poisson Equation

Michael Dramitinos Ph4987

March 8, 2022

1 Introduction

We solved the 2D Poisson equation numerically, using finite differences with an iterative relaxation approach.

$$-\vec{\nabla}^2\phi = S(x, y) \tag{1}$$

We used Dirichlet boundary conditions on the following region:

$$\Omega = (0, 1] \otimes [-0.5, 0.5] \quad \phi(x, y) = 0 \ \forall\, x, y \in \partial\Omega$$

And Neumann boundary conditions for $x = 0$

$$\frac{\partial \phi}{\partial x}|_{x=0} = 0$$

The following source term was used:

$$S(x, y) = -\left(\sin(\pi x)\cos(\pi y) + \sin(5\pi x)\cos(5\pi y)\right) \tag{2}$$

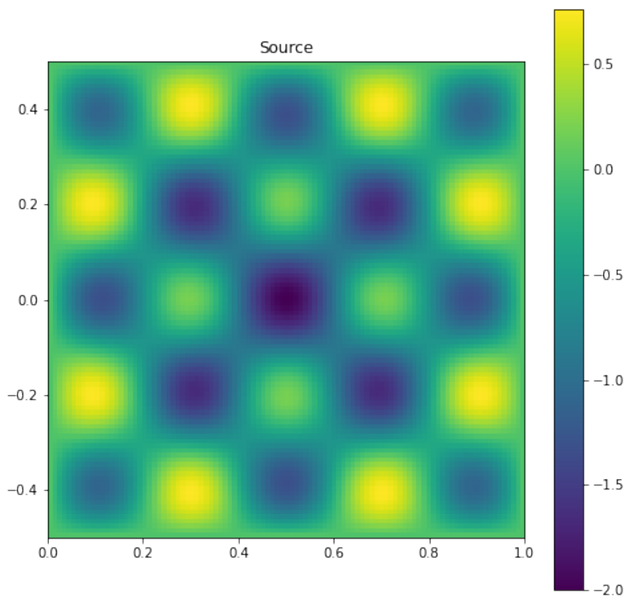

Figure 1: Source: $S(x, y)$

2 Solution

A grid resolution of $(101, 101)$ was used, which gives $h = \frac{1}{N-1} = 0.01$. The resulting potential is:

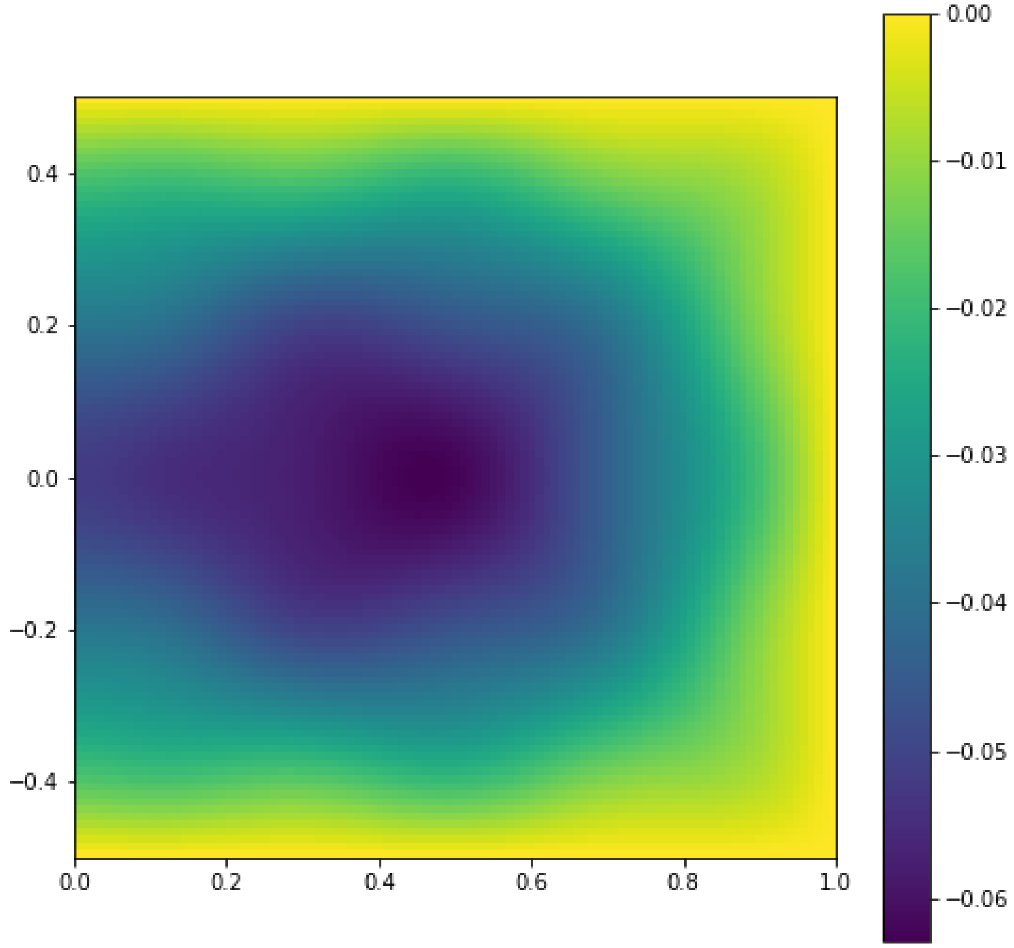

Figure 2: Potential $\phi(x, y)$

We can then calculate the electric field

$$\vec{E}(x, y) = -\vec{\nabla}\phi(x, y) \tag{3}$$

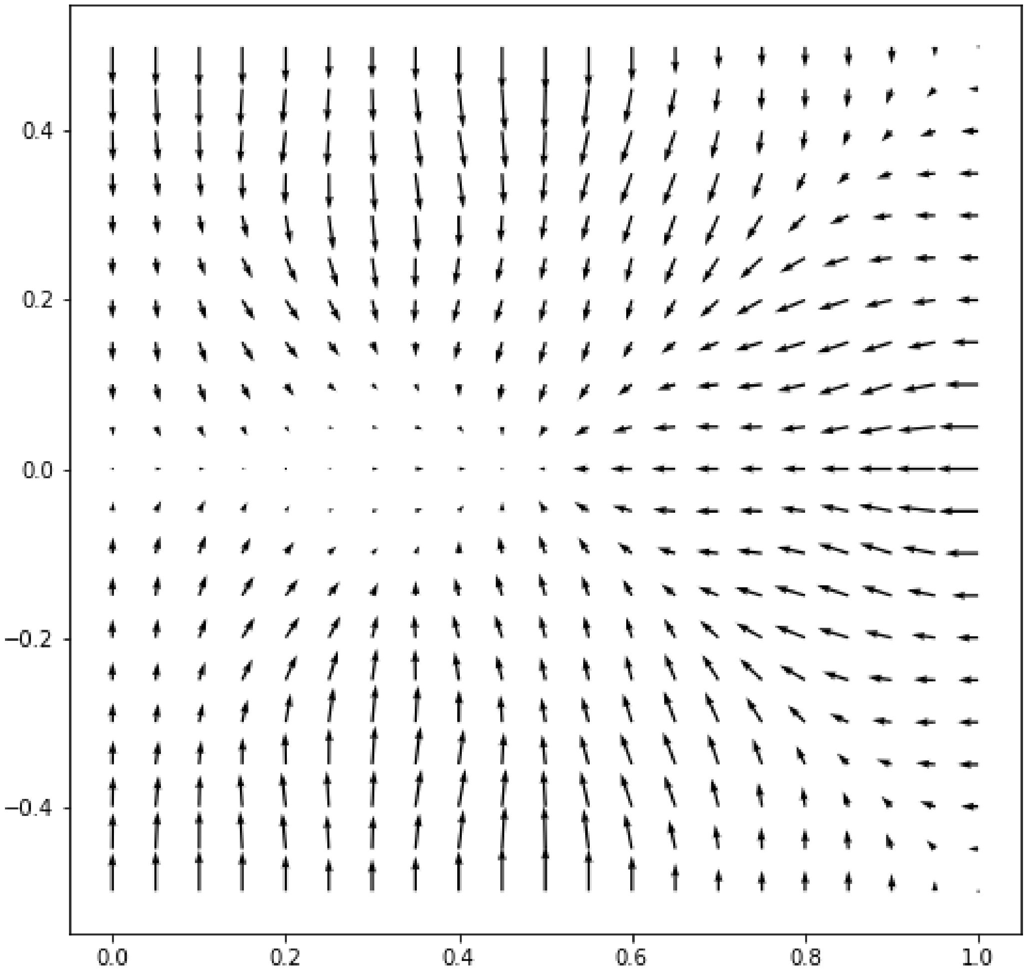

Figure 3: Electric field $\vec{E}(x, y)$

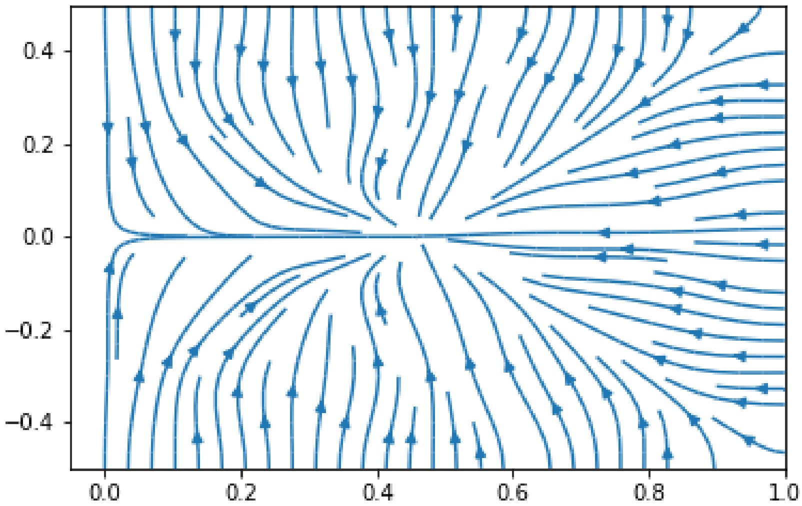

Figure 4: Stream plot of the electric field $\vec{E}(x, y)$