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Drill Comparing Orders Of Growth – 1

	Double The input size	Increase input size by one
1. n^2	$(2n)^2/n^2 = 4n^2/n^2 = \underline{4x \text{ slower}}$	$\lim_{n \rightarrow \infty} (n+1)^2/n^2 = n^2 + 2n + 1 / n^2$ (ignore lower order terms when find the limit at ∞) $\rightarrow n^2/n^2 = \underline{1x \text{ or the same.}}$
2. n^3	$(2n)^3/n^3 = 8n^3/n^3 = \underline{8x \text{ slower}}$	$\lim_{n \rightarrow \infty} (n+1)^3/n^3 = n^3 + 3n^2 + 3n + 1 / n^3 \rightarrow n^3/n^3 = \underline{1x \text{ or the same.}}$
3. $100n^2$	$100(2n)^2/100n^2 = 100(4)(n^2)/100(n^2) = \underline{4x \text{ slower}}$	$\lim_{n \rightarrow \infty} 100(n+1)^2/100n^2 = 100(n^2 + 2n + 1) / 100(n^2) = n^2 + 2n + 1 / n^2 \rightarrow n^2/n^2 = \underline{1x \text{ or the same.}}$
4. $n \log n$	$(2n) \log(2n)/n \log n = 2 \log(2n)/\log n = 2(\log(2n)/n) = 2 \log 2 \sim \underline{2x \text{ slower}}$	$\lim_{n \rightarrow \infty} ((n+1) \log(n+1))/n \log n$ (get rid of lower order terms) $\rightarrow n+1/n \rightarrow n/n = \underline{1x \text{ or the same}}$
5. 2^n	$2^{2n}/2^n$, subtract the exponents of like bases in division ($2n - n = n$) $\rightarrow 2^{2n}/2^n = \underline{(2^n)x \text{ slower}}$	$2^{(n+1)}/2^n = \underline{2x \text{ slower}}$ (subtract exponents $(n+1) - n = 1$)

10^{10} operations per second * 3600 seconds in an hour = **$36 * 10^{13}$ operations a hour.** Since each mathematical description of algorithm performance reflects on operations that are performed for a given input size we can set our algorithms to, **$36 * 10^{13}$** , and solve for n and that will show us for which input size does that algorithm perform, **$36 * 10^{13}$** operations which is the amount of operations the computer can do anyways in an hour.

1. N^2	$N^2 = 36 * 10^{12} \rightarrow \sqrt{n^2} = \sqrt{36 * 10^{12}} \rightarrow n = \underline{60 * 10^5 \text{ or } 6,000,000}$
2. n^3	$N^3 = 36 * 10^{12} \rightarrow \sqrt[3]{n^3} = \sqrt[3]{36 * 10^{12}} \rightarrow n = \underline{\text{floor}(33,019.2725) = 33,019}$
3. $100n^2$	$100n^2 = 36 * 10^{12} \rightarrow \sqrt{n^2} = \sqrt{(36 * 10^{12})/100} \rightarrow n = \underline{600,000}$
4. $n \log n$	$N \log n = 36 * 10^{12} \rightarrow N_{\max} \sim \underline{28.9 * 10^{11}}$ (basically the logn takes away one order of magnitude so its almost constant but not quite – so with calculator tweaking you can get the approximate value of n.)
5. 2^n	$2^n = 36 * 10^{12} \rightarrow \log_2(36 * 10^{12}) = 45.033 \rightarrow \text{floor}(45.033) = \underline{45} = N_{\max}$
6. 2^{2n}	$2^{2n} = 36 * 10^{12} \rightarrow (\log_2(36 * 10^{12}))/2 = 22.5165 \rightarrow \text{floor}(22.5165) = \underline{22} = N_{\max}$

