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Professor Leff

Algorithms 2545

AsymptoticsDrill-2

determine whether $f = O(g)$ (write "1"), or $f = \Omega(g)$ (write "2"), or $f = \Theta(g)$ (write "3").

ROW	F(n)	G(n)	Your Answer (1/2/3)
1	$\log n^2$	$\log n + 5$	3
2	$\text{Sqrt}(n)$	$\log n^2$	2
3	$\log^2 n$	$\log n$	2
4	n	$\log^2 n$	2
5	$n \log n + n$	$\log n$	2
6	10^{10}	$\log 42$	3
7	2^n	$10 n^2$	2
8	2^n	1.5^n	2
9	$2(\log n)^2$	$\log n + 1$	2?
10	2^n	2^{2n}	1

1. No, The bound can be tighter, $O(n^2)$ simply states that worst case the algorithm performs n^2 , this leaves $O(n)$ performance a possibility for some inputs and can not be ruled out.
2. No, the upper bound can be looser, namely up to n^2 behavior; Therefore some inputs may have n^2 performance.
3. Yes, Theta means its bounded above and below by a function/behavior, in this case n^2 . That means worst case is n^2 and best case is n^2 , therefore it is impossible for any inputs to be bounded from above(worst case) by n because that would be a contradiction to the assertion that the functions best case is n^2 . Therefore it is impossible for the algorithm to be $O(n)$ on any inputs .
4. Yes, one thing Theta tells us is that its bounded from below(best possible performance for this algorithm) by (in this case) n^2 performance. Therefore it must be impossible for any input to achieve an $O(n)$ performance.
5. Yes, because in both instances the highest order term is n^2 which characterizes the behavior for both upper and lower bound.