Order Of Growth drill

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Part 3

<u>*Proving* 32n^2 + 17n + 1 is big O(n^2)</u>

By the definition of "Big O" order of growth: T(n) = O(f (n)) if and only there exists positive constants, c and n0 such that T(n) ≤ c × f (n) for all n ≥ n0;

F(n) = 32n^2 + 17n + 1

We need to show that F(n) < = c * n^2 for all n >= n(initial) in order to prove that F(n) = O(n^2)

Let n(initial) = 1

Let c = the factored out sum of the constants (32 + 17 + 1) = 50, (we do this to bring out the c *n^2 form)

In order to do this we note that n^2 >= n^1 >= n^0 and since 17n^2 > 17n & n^2 >= 1(n(initial) is 1):

F(n) <= 32n^2 + 17n^2 + n^2 = (32 + 17 + 1) * n^2 = 50 * n^2 for all n >= n(initial)

F(n) <= 50 * n^2 and by the Definition of Big O → **F(n) = 32n^2 + 17n + 1** = O(n^2).

<u>Showing F(n) is not O(n) or O(nlogn)</u>

Proof by contradiction lets assume F(n) <= c * n then we can assume that 32n^2 <= c * n

If we divide n on both sides we get 32n<= c, this is a contradiction if we choose an n such that 32n = c +1 therefore F(n) is not O(n).

Proof by contradiction lets assume F(n) <= c * nlogn

32n^2 <= c * nlogn

If we divide n on both sides we get 32n<= c * logn

If we divide logn on both sides we get 32n/logn <= c. the lim of 32n/logn is infinity since n> logn.

Therefore we need only choose an n such that 32n/logn = c + x (such that x is a member of positive integers/natural numbers) which is doable since the lim as n → infinity is infinity. That is a contradiction to the notion F(n) <= c * nlogn for all n>= n(initial). Thus F(n) is NOT = O(nlogn).

Part 4

<u>*Proving* 32n^2 + 17n + 1 is big Ω (n^2)</u>

f (n) = Ω(g(n)) if and only if there exist positive constants c and n0 such that f (n) ≥ c × g(n) for all n ≥ n0

we need to show that **F(n) = 32n^2 + 17n + 1** >= c * n^2 let n(initial) = 1

let c = the factored out sum of the negation of the constants, ((-32) + (-17) + (-1)) = -50

because since +a > -a and an^k > -an^k where k > 0 therefore **F(n)** >= (-32)n^2 + (-17)n + (-1)

in order to factor out we note that -(n^2)<= -(n^1) <= -n^0. (n(initial) = 1)

F(n) >= (-32)n^2 + (-17)n^2 + (-1)n^2 = ((-32) + (-17) + (-1)) * n^2 = -50 * n^2 for all n>= n(initial).

And therefore by the definition of Big Omega **F(n) = 32n^2 + 17n + 1** = Ω(n^2)

As an corollary we also see that since **F(n) >=** -50 * n^2 for all n>= n(initial) and -50 * n^2 >= -50n it follows that **F(n) >= -50(n)** which by definition of Big Omega **F(n) = 32n^2 + 17n + 1** = Ω(n)

<u>Showing F(n) is not Ω (n^3)</u>

Proof by contradiction assume that **F(n) >= c * n^3.** lets raise all n terms to n^2 which is even greater than F(n) thus if this isn't greater than F(n) is Chal Vachomer not greater.

So 50n^2 >= c * n^3, divide both sides by n^3.

50/n >= c as n → infinity (Big O talks about asymptotic behavior) 0 >= c is impossible since precondition is that c is positive. Therefore F(n) != Ω(n^3)

Part 5

<u>*Proving* 32n^2 + 17n + 1 is big Θ (n^2)</u>

f (n) = Θ(g(n)) if and only if there exist positive constants c1, c2 and n0 such that c1 × g(n) ≤ f (n) ≤ c2 × g(n) for all n ≥ n0

And as a corollary from part 3 and part 4 we see that

-50 * n^2 <= **F(n) = 32n^2 + 17n + 1** <= 50 * n^2

And by the definition of Big Theta **F(n) = 32n^2 + 17n + 1** = Θ(n^2).

<u>Showing F(n) is not Θ (n^3) or Θ(n)</u>

Proof by contradiction we know from above that its impossible for F(n) >= c * N^3, since this is one of the prerequisites for F(n) = Θ(n^3) it follows that F(n) != Θ(n^3)

Proof by contradiction we know from above that it is impossible for F(n) <= c * n, since this is is one of the prerequisites for F(n) = Θ(n) it follows that F(n) != Θ(n)
