

BasicAlgebraRevision

November 7, 2018

1 Basic Algebra Revision

1. Expressions, equations, variables and constants
2. Functions
3. Substitution
4. Re-arranging an expression

1.1 Expressions, equations, variables and constants

An expression is a collection of numbers, letters and symbols such as +,-,* and division. Example expressions are shown below.

1.1.1 Addition

a plus 5

$$a + 5$$

Subtraction a minus 5

$$a - 5$$

Multiplication 4 times a (note we forgot the times symbol)!

$$4a$$

Division a divided by 2

$$a/2$$

The examples are shown below `a=var('a')` `b=var('b')`

`a+5` `a-5` `4*a` `a/2` `a-b` `a-x` The letters in the above expressions are known as variables. When the expression is written as shown above we do not know what number that letter is equal to.

We evaluate an equation when we set a variable to a given value for example

```
In [2]: a=1  
        b=2
```

```
In [4]: print(a+5)  
        print(a-5)  
        print(4*a)
```

```
print(a/2)
print(a-b)
print(a-x)
```

6
-4
4
1/2
-1
-x + 1

We use substitution to replace the letters in the expression with the value given by the equation.
An equation has contains an equals sign and has an expression on each side of the sign. Example expressions are shown below.

```
In [5]: b=a+5
        b=a-x
        y=5*a-x
```

1.1.2 Exercise

In the examples below which examples are expressions and which examples are equations? 1.

$$a + 1$$

2.

$$5 * a$$

3.

$$a = x + 1$$

4.

$$y = x - 1$$

5.

$$x + y - 5 * a$$

1.1.3 Exercise

In the examples can you count the number of variables, write each of the variables down? 1.

$$a + 1$$

2.

$$5 * a$$

3.

$$a = x + 1$$

4.

$$y = x - 1$$

5.

$$x + y - 5 * a$$

1.2 Functions

A function is a mathematical expression which when provided with inputs produces an output. We call the inputs the variables. To calculate the value which is output by the we substitute the input values into the expression.

```
In [6]: f(x)=2*x+1
        g(y)=2*y+1
        h(x,y)=x+y
```

1.3 Substitution

1.3.1 Function of One Variable

Lets try some examples of substituting value into an expression which has one variable. Here, we will use SageMath to compute the result of the substitution

First we tell SageMath the variables we are using.

```
In [7]: x=var('x')
```

Define an expression using our newly defined SageMath variable, we give this function the name f

```
In [8]: f=5*x
        print(f)
```

5*x

```
In [9]: f=5*x+3
        print(f)
```

5*x + 3

```
In [10]: f=5*x+6
         print(f)
```

5*x + 6

Now we shall actually substitute a value into our expression. We substitute the value

$$x = 1$$

into the equation

$$f = 5x + 6$$

We use the SageMath substitute command note how we use the syntax f.substitute also note the double equal sign (==)

```
In [11]: print(f.substitute(x==1))
         print(f)
```

```
11
5*x + 6
```

```
In [12]: g(x)=2*x+1
         print(g(1))
```

```
3
```

1.3.2 Now try some substitutions for yourself

For each problem substitute the value provided for the variable

1. Substitute

$$x = 1$$

into

$$(x - 1)$$

2. Substitute

$$x = 2$$

into

$$(x + 2)$$

3. Substitute

$$x = 2$$

into

$$2x$$

4. Substitute

$$x = 3$$

into

$$5x + 3$$

5. Substitute

$$x = 3$$

into

$$2(6x + 2)$$

1.3.3 Function of Two Variables

We can introduce more variables into our equations and expressions. When we use SageMath we use the command `var('variablename')` to introduce another variable.

Note below that we use functions to substitute or input the variables into the expressions.

```
In [13]: y=var('y')
```

```
h(x,y)=x+y
print(h(1,1))
```

2

```
In [14]: f(x,y)=5*x+y
print(f(x,y))
```

5*x + y

```
In [15]: f(x,y)=5*x+3*y
print(f(x,y))
```

5*x + 3*y

```
In [16]: f(x,y)=5*x+6*y
print(f(x,y))
```

5*x + 6*y

```
In [17]: f(x,y)=5*x+6*y
print(f(x,1))
```

5*x + 6

Substitutions with SageMath To substitute values into our expressions we can use the SageMath `substitute` command note the use of the double equals sign and the use of the `f.substitute` syntax.

```
In [47]: f=5*x+y
print(f.substitute(x==1))
```

y + 5

```
In [48]: print(f.substitute(x==1,y==1))
```

6

```
In [49]: f=5*x+6*y  
        print(f.substitute(x==1,y==1))
```

11

```
In [50]: print(f.substitute(x==1,y==2))
```

17

Exercise Now try some substitutions for yourself. Try the substitutions using pen and paper and also use SageMath.

Hint: For each of these problems use the function definition to substitute the values. e.g.

$$f(x, y) = x + y + 1$$

to substitute $x=1$ $y=1$ use the sage math print command: `print(f(1,1))`

For each problem substitute the value provided for the variable.

1. Substitute

$$x = 1, y = 1$$

into

$$(x + y - 1)$$

2. Substitute

$$x = 2, y = 2$$

into

$$(x - y + 2)$$

3. Substitute

$$x = 2, y = 2$$

into

$$2x + y$$

4. Substitute

$$x = 3, y = 1$$

into

$$5x + 5y + 3$$

5. Substitute

$$x = 3, y = 1$$

into

$$5(x + y) + 3$$

6. Substitute

$$x = 3, y = 1$$

into

$$2(x + y + 3)$$

7. Substitute

$$x = 3, y = 2$$

into

$$(x + y + 3)/y$$

```
In [ ]: #Test your substitutions here
```

1.4 Re-Arranging Expressions

Balancing an equation

What ever you do on the left hand side of an equation you must do on the right

Consider the equation

$$x - 5 = 0$$

What value must x take so that this equation is true, try and guess?

The process of finding what value of the variable x makes the equation true is called solving the equation

We can use the Sage math solve command to find the solution, note how we use the double equal sign, we also specify that want to determine the value x which solves the equation

```
In [12]: print(solve(x-5==0,x))
```

```
[  
x == 1  
]
```

We can also solve this equation by defining a function. We can guess values for x and substitute these into the function we keep on making guesses until our equation

$$x - 5 = 0$$

is satisfied.

```
In [ ]: f(x)=x-5  
        print(f(10))  
        print(f(8))  
        print(f(6))  
        print(f(4))
```

```
In [ ]: print(solve(f(x)==0,x))
```

Lets practice with some further examples. As an exercise try finding the solution by defining a function and using substitution to guess the solution

```
In [13]: print(solve(x-2==0,x))

[
x == 2
]
```

```
In [ ]: print(solve(2*x==2,x))
```

```
In [ ]: print(solve(2*x==6,x))
```

Lets try someting slightly harder

```
In [14]: print(solve(2*x+2==2,x))

[
x == 0
]
```

```
In [10]: f1(x)=5*x+3
          f2(x)=6*x
          eq=(f1(x)==f2(x))
          print(solve(eq,x))

[
x == 3
]
```

1.4.1 Exercise

Go on try the exercise !

Write down functions for the equations we have used above, then use substitution to guess the solutions, for example

```
In [ ]: f1(x)=5*x+3
          f2(x)=6*x

          print(f1(1))
          print(f2(1))
```

Instead of guessing, a much better way of finding the solution is to rearrange the equation. Here is what we can do. Consider the example above

$$5x + 3 = 6x$$

This equation has a left hand side (LHS) and a right hand side (RHS). Now we can add numbers subtract numbers. We can also multiply and divide each side of the equation by a number.

We must obey the rule, whatever we do to the left hand side then we must also do the right hand side. We call this keeping the equation balanced.

Consider our example above

$$f2(x) = f1(x)$$

$$f2(x) - 5x = f1(x) - 5x$$

The step above is called re-arranging and if we actually work this out we'd get

$$5x + 3 - 5x = 6x - 5x$$

Can yo work this out?

Mathematicians frequently use the term identity... An Identity is an equation which is two expressions on each side of the equal sign. These are frequently used by mathematicians when they prove the truth of the equations which are used.

1.5 Further Help

1. [Math Planet - Algebra1](#)
2. [Khan Academy - Algebra Basics](#)