An Aircraft Control Systems Design Example

Modern aircraft consist of many complex subsystems, all of which require robust and reliable control. These subsystems are often multi-variable, consisting of multiple inputs and multiple outputs, and frequently the desired responses of a subsystem (such as overshoot and rise time) are in conflict. Coupling evolutionary multi-objective optimisation techniques with conventional controller design methods such as H_∞ or LQG control can provide the engineer with a powerful tool for addressing such problems.

H_∞ Control of Aircraft Flight Dynamics

The control system for the flight dynamics of an aircraft must provide robust and responsive multi-variable control of the ailerons and the rudder, as well as guaranteeing stability in the presence of modelling uncertainty. H_∞ control theory offers a proven method of designing controllers that are robust to such uncertainty. However, with conventional H_∞ control, the performance of the resulting system can often be unsatisfactory. The following subsections will provide an overview of the flight dynamics of an aircraft and introduce the concept of H_∞ control and loop shaping - a technique that allows the designer to 'shape' the response of a system, and thus improve performance.

Flight Dynamics

The dynamics of an aircraft in flight can be described by the rotational moments around its centre of gravity (CG) in Cartesian space. These are shown in Figure 1 where:

- **L** is the *rolling* moment of the aircraft.
- **M** is the *pitching* moment of the aircraft.
- **N** is the *yawing* moment of the aircraft.

These flight dynamics can be separated into *longitudinal* motions, in which the wings remain level (e.g. pitch), and *lateral* motions such as roll and yaw.

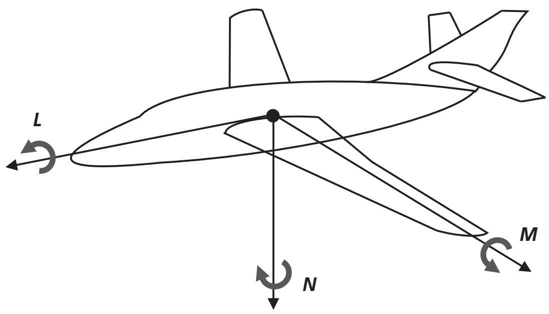


Figure 1: The Rotational Moments of an Aircraft

The equations of motion describing these flight dynamics are non-linear; however, by applying the *small disturbance theory* (Nelson 1998), a linearised model can be found. This linearisation process will only give a good result in cases where the motion of the aircraft can be fully described by small deviations about a steady flight condition (such as in the flight of large commercial aircraft), and therefore should not be used in cases where large amplitude motions are likely to occur.

H_∞ Loop Shaping Controller Design

H_∞ control theory was originally proposed by Zames (1981) to address the problem of uncertainty in the modelling of disturbances and plants and was further developed by Glover and Doyle (1988). H_∞ control theory provides a general framework for the design of optimal controllers, where optimal in this context refers to the minimisation of the H_∞ norm.

A drawback of robust stabilisation using H_∞ control is the inability of the designer to specify performance requirements (Skogestad and Postlethwaite 1996), which can result in compensated systems that, whilst robust, perform unsatisfactorily. To overcome this limitation, McFarlane and Glover (1990) proposed using pre- and post-compensators to 'shape' the open-loop response of the plant (see Figure 2), and then applying robust stabilisation. Selecting the weighting matrices for the pre- and post-compensators is typically challenging since these choices govern the performance of the resulting system.

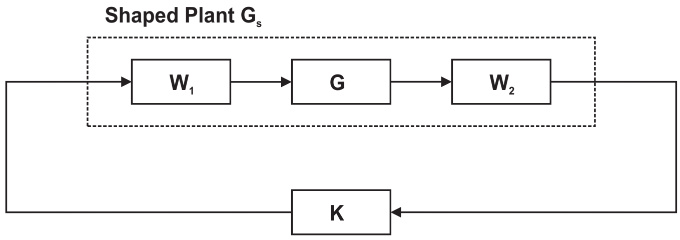


Figure 2: The Shaped and Compensated Plant

The general proceedure for designing an H_∞ controller using loop shaping is (Skogestad and Postlethwaite 1996):

1. Scale the inputs and outputs of the system to improve the condition of the design problem.
2. Select the weighting matrices for the pre- and post-compensators and form the generalised shaped plant model (see Figure 2). As mentioned above, this is typically challenging due the influence this stage has on the performance of the final compensated system.
3. Robustly stabilise the shaped plant, $G_s = W_2 G W_1$, using standard H_∞ control techniques such as those described in Skogestad and Postlethwaite (1996).
4. Test the performance and stability of the compensated system and, if the requirements are not met, alter the weighting matrices formed in step 2.

Optimal Controller Design

Evolutionary optimisation methods have previously been used in finding both optimal controller parameters and optimal controller structures (Schroder 1998). This Section aims to find an optimal H_∞ loop shaping controller for the lateral stability control of a Boeing 747 aircraft[1]. This is a multivariable system with two inputs (the control signals for the aileron and rudder) and two outputs (the roll and sideslip angles), and can be represented by the following transfer function matrix[2]:

[1] For a thorough investigation into the design of an H_∞ lateral stability controller the interested reader is directed to (Giacoman Zarzar 2007).

[2] This can be shown to be a minimal realisation of the system using the *minreal* command in MATLAB.

$$\mathbf{G} = \begin{pmatrix} \frac{0.1845s^2+0.04795s+0.1995}{s^4+0.6807s^3+1.049s^2+0.3373s-0.001979} & \frac{0.06591s^2-0.12s-0.5158}{s^4+0.6807s^3+1.049s^2+0.3373s-0.001979} \\ \frac{-0.01448s^2-0.01962s+0.001359}{s^4+0.6807s^3+1.049s^2+0.3373s-0.001979} & \frac{0.005334s^3+0.4377s^2+0.1884s-0.00432}{s^4+0.6807s^3+1.049s^2+0.3373s-0.001979} \end{pmatrix}$$

In this Section, the Grid-enabled framework for evolutionary optimisation introduced in this Chapter is used to determine the optimal weights (with respect to the performance and stability requirements of the system) for the pre- and post-compensators in the H_∞ loop shaping design process outlined previously. The pre- and post-compensators have the following structures:

$$\mathbf{W_1} = \begin{pmatrix} \frac{s+a}{s} & 0 \\ 0 & \frac{s+b}{s} \end{pmatrix}$$

$$\mathbf{W_2} = \begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix}$$

with the order of these compensators being determined by the order of the plant.

This H_∞ loop shaping controller design process can be formulated as a multi-objective optimisation problem, where each performance requirement is treated as a separate objective, and thus solved using a multi-objective evolutionary algorithm. The decision variables in this optimisation procedure are the weights, a, b, c, d, in the compensators. However, this controller design problem is computationally expensive since, for every candidate solution, an H_∞ controller has to be synthesized and the response of the compensated system obtained by computer simulation.

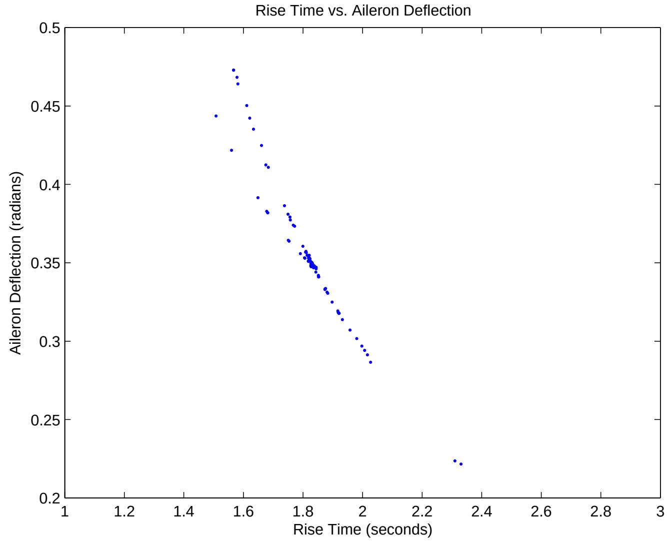
A set of performance requirements arising from domain specific knowledge about the problem have been specified for the response of the compensated system (see Table 1). Some of these requirements are hard constraints and others are simply desired goals. Several of the performance requirements for this controller are in competition which makes achieving all the goals difficult.

Requirements	Type
Minimise the Overshoot in response to a step input	Goal (Overshoot $< 5\%$)
Minimise the Rise Time	Goal ($T_r < 3$ seconds)
Minimise the Settling Time	Goal ($T_s < 4$ seconds)
Prevent Aileron actuator saturation	Constraint (Aileron deflection $<$ 0.349 radians)
Prevent Rudder actuator saturation	Constraint (Rudder deflection $<$ 0.52 radians)
Controller must be robust to 30% multiplicitive uncertainty	Constraint

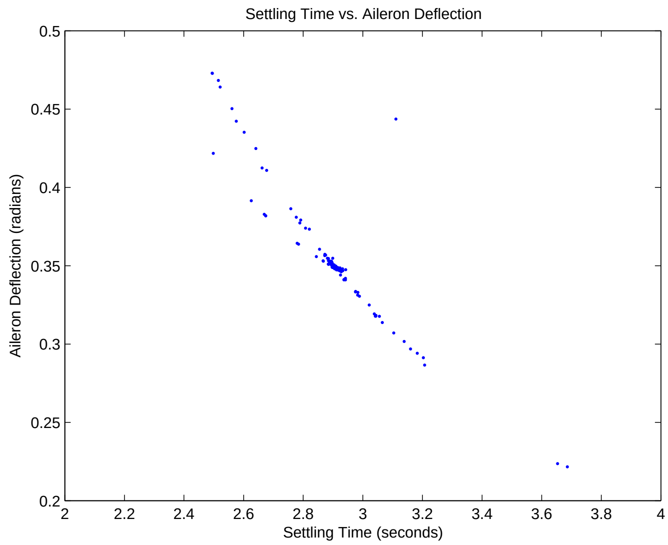
Table 1: Performance Requirements for the H_∞ Controller Design Problem

Evolutionary Algorithm Implementation

The Grid-enabled optimisation framework propose in this Chapter was used here in a multi-objective genetic algorithm architecture, with Fonseca and Fleming's (1998) modified Pareto ranking scheme used to incorporate the preference information shown in Table 1 into the search. A real-valued decision variable representation was used, with selection being performed using Stochastic Universal Sampling (Baker 1987) and variation introduced into the population using extended intermediate recombination and BGA mutation (Mühlenbein and Schlierkamp-Voosen 1993). The population size was 100 individuals, and the MOEA was run for 100 generations.



(a)



(b)

Figure 3: Trade-offs Between Objectives and the Aileron Control Signals

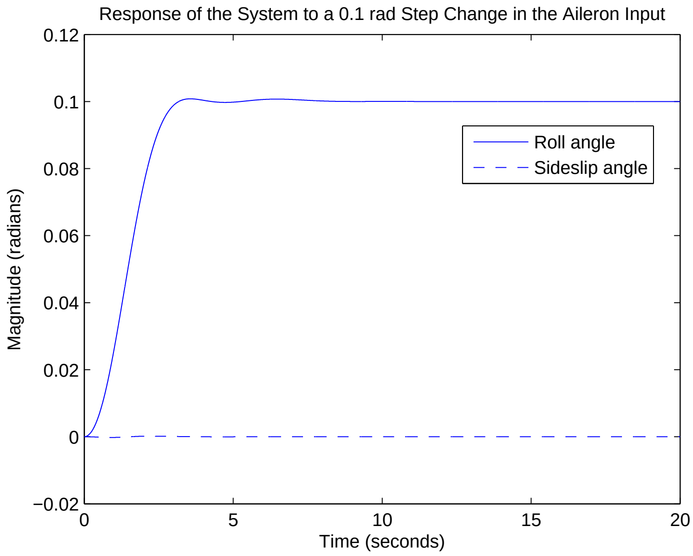
Results

It can be seen from Figure 3 that there is a strong trade-off between the control signal for the aileron actuator and the other requirements in Table 1 (such as the the rise time and settling time). This shows that the response of the final compensated system is limited by the maximum range of the aileron actuator.

Table 2 shows the achieved performance of the compensated system and Figure 4 shows the response of the system to a 0.1 radians step change in the aileron control signal. It can be seen from Table 2 that all the performance requirements specified in Table 1 are satisfied, and significant improvements over all the goal values have been achieved.

Goals	Achieved Value
Minimise the Overshoot in response to a step input (Goal: Overshoot $< 5\%$)	0.87%
Minimise the Rise Time (Goal: $T_r < 3$ seconds)	1.83 seconds
Minimise the Settling Time (Goal: $T_s < 4$ seconds)	2.9 seconds
Prevent Aileron actuator saturation (Constraint: Aileron deflection < 0.349 radians)	0.3488 radians
Prevent Rudder actuator saturation (Constraint: Rudder deflection < 0.52 radians)	0.0188 radians
Controller must be robust to 30% multiplicitive uncertainty (Constraint)	Robust to 35.9% multiplicitive uncertainty

Table 2: Achieved Performance of the Compensated System



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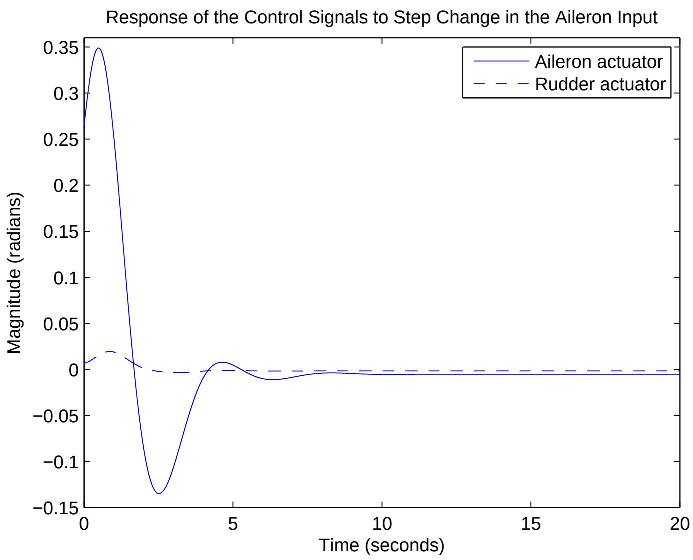

(b)

Figure 4: Step Response of the Compensated System to Aileron Deflection

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