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THE FIVE-MINUTE OSCILLATIONS ON THE SOLAR SURFACE*

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ABSTRACT

The acoustic properties of the subphotospheric layers are examined. It is shown that standing acoustic waves may be trapped in a layer below the photosphere. These standing waves may exist only along discrete lines in the diagnostic diagram of horizontal wavenumber versus frequency. The positions of these lines are derived from a modal analysis of the solar envelope The lines for the fundamental mode and the first-overtone mode pass through the centers of the two peaks observed by Frazier. An examination of the energy balance of the oscillations shows that they are overstable. When they are assigned an amplitude of 0.2 km sec^{-1}, they generate about (5–7) $\times$ 10^6 ergs cm^{-2} sec^{-1}. This power output suggests that the dissipation of the 5-minute oscillations above the temperature minimum is responsible for heating the chromosphere and corona.

I. INTRODUCTION

The 5-minute oscillations in the solar photosphere have been studied intensively since their discovery by Leighton and his co-workers (Leighton, Noyes, and Simon 1962; Noyes and Leighton 1963). Athay (1966) has suggested that these oscillations are involved in the process which heats the chromosphere and corona. Unfortunately, the mechanism which generates the oscillatory motion has not been well understood. In particular, the power spectrum of acoustic energy predicted by a theory of generation must be compatible with the observed spectrum. The spectrum derived by Stein (1968) from Lighthill's (1952) turbulence-generation mechanism has a peak of power near periods of 30–60 sec and falls very steeply for periods different from these values. In contrast, most of the power is observed near 300 sec (Leighton *et al.* 1962; Tanenbaum *et al.* 1969). This paper describes a process which may generate the observed 300-sec oscillations and which is essentially different from Lighthill's mechanism.

Moore and Spiegel (1966) pointed out that acoustic waves are overstable in the presence of a superadiabatic temperature gradient and radiative exchange of energy. This paper shows that the 5-minute oscillations are trapped standing acoustic waves, and gives eigenfrequencies for the fundamental and first three overtone modes. Examination of the energy balance of these modes shows that the first three overtones are overstable. In addition, the dispersion relation between frequency and horizontal wavelength may explain the apparently random location of peaks in the power spectrum observed by Howard (1967), Frazier (1968), and Gonczi and Roddier (1969). Tanenbaum *et al.* (1969) showed that the amplitude of the oscillations increases as the wavelength decreases. Consequently, it is reasonable to expect the shortest observable wavelength to dominate the power spectrum. The random locations of the power-spectrum peaks found by Howard (1967) may be interpreted as a result of random variations in the quality of the seeing.

The vertical wavelength of the oscillations is comparable to the horizontal wavelength and is roughly 1000–5000 km. As a result, a correct treatment of the problem must necessarily involve a substantial region below the photosphere. Also, the temperature and rate of radiative cooling change substantially over a distance of 1000 km. Clearly the approximations that the gas is perfect and the atmosphere is isothermal may not be

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used. Indeed, the presence of a superadiabatic temperature gradient is critically important in the discussion of the energy balance. Another consequence of the long vertical wavelength is that the concepts of ray acoustics are useful only as general guides. Finally, although there may be some coupling between the oscillations and the convective motion, this coupling cannot be dominant since Gonczi and Roddier (1969) have found that the oscillations remain in phase for a least 1 hour whereas typical convective cells live 7–8 minutes. This lifetime of 1 hour or more differs considerably from the usually quoted lifetime of 8–10 minutes. This shorter lifetime has prompted many workers to postulate that the oscillations are generated directly by the motions of convective cells. The observations by Gonzci and Roddier show that the shorter lifetime should be interpreted as a beat period rather than a true lifetime. Such a beat period requires there to be two or more natural frequencies for the motion. We shall show in the next two sections that such multiple frequencies are the result of the variable acoustic properties below the surface of the Sun.

II. TRAPPED WAVES

A simplified form of the local dispersion relation is that given by Whitaker (1963). We let k_z, k_h, c, ω_0, and N be the vertical and horizontal wavenumbers, the adiabatic sound velocity, long acoustic cutoff frequency, and the Väisälä-Brunt gravity-wave frequency. Whitaker's dispersion relation may then be written

$$k_z^2 = \frac{\omega^2 - \omega_0^2}{c^2} - k_h^2\left(1 - \frac{N^2}{\omega^2}\right). \tag{1}$$

In a nonisothermal atmosphere ω_0 and N are given by

$$\omega_0 = \frac{c}{2H} \quad \text{and} \quad N^2 = -\frac{g}{\rho}\left(\frac{\partial \rho}{\partial S}\right)_P \frac{dS}{dz}, \tag{2}$$

where H, g, ρ, S, and z are the *density* scale height, acceleration of gravity, density, entropy, and altitude. In a stratified atmosphere ω_0, N, and c are functions of altitude so that k_z does not remain constant. Since we are ignoring the coupling with convective motions, the atmosphere is uniform in time and on horizontal planes. Consequently, a mode of oscillation may be assigned a value for ω and k_h. For such a mode, altitudes where $k_z = 0$ are boundaries between regions where waves propagate and where they are attenuated. Ray acoustics suggests that a wave packet moving from the propagating region toward the attenuating region will be reflected from the boundary surface. Although equation (1) does not include the effect of radiative exchange of energy, it shows that for some values of ω and k_h there are two altitudes where $k_z = 0$. Near the photosphere ω_0 increases by about a factor of 1.6 due to the increase in mean molecular weight. For frequencies between the minimum and maximum of ω_0 there is a reflecting surface in the photosphere. As long as this condition on ω is satisfied, it is always possible to choose k_h small enough that $k_z > 0$ below the photosphere. For finite values of k_h a second reflecting layer is present well below the photosphere as a result of the increase in sound velocity. For particular values of ω and k_h between these limits, trapped standing oscillations may be set up. The fact that the oscillations are observed in a region where they are not permitted does not constitute a problem since the decay distance for the energy density $\frac{1}{2}\rho v^2$ is quite long and in fact the velocity amplitude increases with altitude.

In the presence of radiative exchange of energy, k_z is complex rather than purely real or purely imaginary. For this case the analogue of equation (1) with $k_z = 0$ was obtained by Souffrin (1966) by equating the real and imaginary parts of k_z. In a nonisothermal atmosphere we may obtain a similar formula if we neglect the difference between the pressure and density scale heights and the gradients of c^2, ω_R, and N^2. The resulting formula for the critical horizontal wavenumber is

$$k_h = \frac{\omega^2}{c^2} \frac{\omega^2 + \gamma\omega_R^2 - \omega_0^2(1 + \omega_R^2/\omega^2)}{\omega^2 + \omega_R^2 - N^2}, \quad (3)$$

where γ is the ratio of specific heats,

$$\omega_R = \frac{16\sigma T^3 \kappa}{C_P} (1 - \tau_e \cot^{-1} \tau_e), \quad (4)$$

and C_P, κ, and τ_e are, respectively, the heat capacity at constant pressure, the opacity, and the effective optical thickness of the perturbation. The formula for ω_R was derived by Spiegel (1957) for an infinite uniform medium. For this uniform case Spiegel found $\tau_e = \rho\kappa/k$ for a perturbation whose total wavenumber is k. Near the photosphere, however, the optical thickness of a perturbation cannot be greater than the optical depth of the point. The effective vertical wavenumber for any perturbation cannot be less than $\rho\kappa/\tau$. The actual vertical wavenumber for the perturbation is considerably less than the lower limit, so a good estimate for τ_e is

$$\tau_e = \left[\left(\frac{k_h}{\rho\kappa}\right)^2 + \frac{1}{\tau^2}\right]^{-1/2}. \quad (5)$$

While the general characteristics discussed above must be true for any model of the solar envelope, the particular relation between k_h and ω must depend on the structure of the photosphere. The present analysis was performed on a model calculated with the nonlocal mixing-length theory described by Ulrich (1970). The important properties of this model are given in Table 1.[1] The entropy of the adiabatic region may not be correct in this model, but the general nature of the relation between k_h and ω is independent of this uncertainty. In fact, we may hope that improved observations of the 5-minute oscillations will allow an accurate determination of the envelope adiabat.

Figure 1 shows the altitude of the reflecting layer as a function of the horizontal wavelength for several periods of oscillation. Altitude zero is at optical depth unity. The shortest-period oscillation which is reflected in the photosphere is about 180 sec. Oscillations with shorter periods will penetrate to the chromosphere before reflection and have not been studied. They may be reflected at that level by the increase in sound velocity; however, the relevant physical phenomena may be quite different. The cutoff frequency is the value of ω_0 at the temperature minimum. In fact, the power observed by Tanenbaum *et al.* (1969) drops sharply as the cutoff period is approached. Evidently, reflection above the temperature minimum is much less effective than reflection from the photosphere.

The principal conclusions of this section are that the 5-minute oscillations are acoustic waves trapped below the solar photosphere and that power in the (k_h, ω)-diagram should be observed only along discrete lines.

III. MODAL ANALYSIS

The arbitrary variation of the atmospheric parameters requires the calculation to be performed numerically. Some simplification of the momentum and continuity equations results from using the mass flux

$$j = \rho v \quad (6)$$

[1] The presence of negative values for ω_0 in this table is due to a density inversion just below the photosphere. This density inversion is unavoidable in models computed with a nonlocal convective theory. An extensive discussion of this problem is not appropriate here since the density inversion plays only a minor role in determining the acoustic properties of the atmosphere.

TABLE 1

IMPORTANT PROPERTIES OF THE AVERAGE MODEL

$-z$ (km)	T (° K)	c (km sec^{-1})	$\omega_0 \times 10^2$ (sec^{-1})	$\omega_R \times 10^2$ (sec^{-1})	$N^2 \times 10^4$ (sec^{-2})
239	4640	7.23	+3.23	0.33	+10.1
387	4680	7.23	+3.17	1.10	+ 9.5
474	4820	7.30	+3.00	2.11	+ 8.7
570	5130	7.56	+2.71	3.57	+ 6.2
685	5940	8.16	+2.40	6.36	+ 4.5
719	6330	8.41	+1.78	9.14	+ 2.3
750	6830	8.70	+1.13	7.61	− 3.9
793	7960	9.12	−1.76	4.47	−18.6
821	9280	9.39	−3.69	1.94	−30.4
837	10000	9.61	−1.67	0.72	−15.0
863	10700	9.92	+0.08	0.20	− 8.0
908	11300	10.3	+0.77	0.04	− 3.1
1040	12300	10.9	+1.06	0.00	− 1.0
1220	13200	11.6	+1.10	0.00	− 0.47
1410	14100	12.2	+1.07	0.00	− 0.27
2730	19300	15.8	+0.87	0.00	− 0.03
4640	27600	20.9	+0.67	0.00	0.00
7770	45000	29.6	+0.48	0.00	0.00
17900	119000	50.8	+0.29	0.00	0.00
49800	755000	95.2	+0.17	0.00	0.00

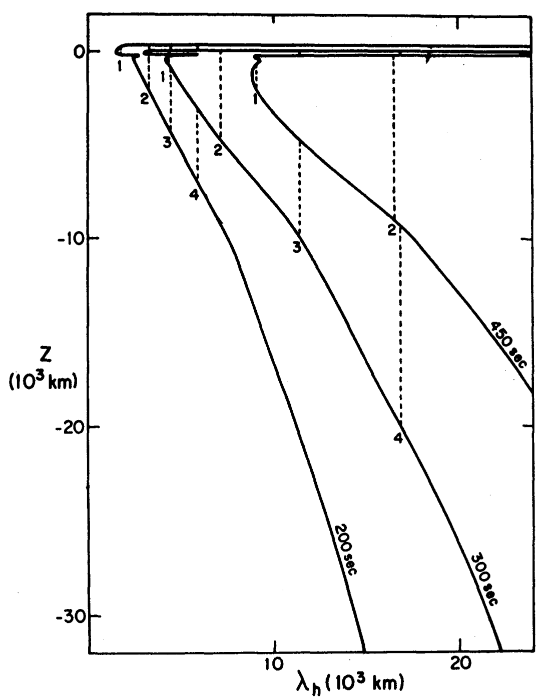

FIG. 1.—Altitude z of the reflecting layer versus horizontal wave length λ_h for three periods of oscillation. Permitted oscillations are found to the right of each line. Dashed lines indicate positions of the eigensolutions found in § III. These solutions are labeled by their modal numbers defined as the number of nodes between the reflecting layers plus one.

in place of the velocity. In terms of this variable, only the coefficients in the energy equation change significantly with depth. The second-order term dropped, $\nabla\cdot(\rho \boldsymbol{v}\boldsymbol{v})$, is somewhat different from the usual term dropped, $\rho(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}$; however, in a linearized analysis the results should not be affected. We denote fluctuations in the thermodynamic variables with primes, and subtract the hydrostatic equation from the momentum equation. The linearized equations of motion are

$$\frac{\partial \boldsymbol{j}}{\partial t} = -\nabla P' + \boldsymbol{g}\rho' , \tag{7}$$

$$\frac{\partial \rho'}{\partial t} = -\nabla\cdot\boldsymbol{j} , \tag{8}$$

$$\frac{T\partial S'}{\partial t} = -\frac{T}{\rho} j_z \frac{dS}{dz} - C_P\omega_R T' , \tag{9}$$

where $\boldsymbol{g} = (0, 0, -g)$. We may express S' and T' in terms of P' and ρ' by use of the equation of state.

The modal equations are obtained by setting $\partial/\partial t = i\omega$ and $\partial^2/\partial x^2 + \partial^2/\partial y = -k_h^2$. The z-dependence of all quantities may then be obtained from a numerical integration. A common procedure in problems such as this is to allow ω to be complex. The imaginary part of ω then indicates whether the mode grows or decays. Unfortunately, this method cannot be used at present because of the difficulties with the outer boundary condition discussed below. Consequently, these calculations have been done with real ω, and the growth or decay of the oscillations has been determined from energy balance in § IV.

The main result of the arbitrary temperature stratification is that $\partial/\partial z$ may not be replaced by $ik_z - 1/(2H)$. We may eliminate the horizontal component of $\boldsymbol{j}$ by dividing equation (7) by $i\omega$ and multiplying the x- and y-components by $\partial/\partial x$ and $\partial/\partial y$, respectively. After the results are used in equation (8), we obtain

$$\frac{\partial P'}{\partial z} = -i\omega j_z - g\rho' , \quad \frac{\partial j_z}{\partial z} = -i\omega\rho' - \frac{k_h^2 P'}{i\omega} , \tag{10}$$

$$\rho' - \frac{P'}{c^2} = \frac{N^2 j_z}{i\omega g} - \frac{\omega_R}{i\omega}\left(\rho' - \gamma\frac{P'}{c^2}\right) . \tag{11}$$

These equations give the altitude dependence of the complex amplitude of the oscillations. They must be supplemented by boundary conditions. The interior boundary condition is given by equation (1). In that equation k_z gives the z-dependence of $j_z/\rho^{1/2}$, and we may set $N^2 \sim 0$. The boundary condition is then

$$\frac{1}{j_z}\frac{dj_z}{dz} = \left(\frac{\omega_0^2 - \omega^2}{c^2} + k_h^2\right)^{1/2} - \frac{\omega_0}{c} . \tag{12}$$

The outer boundary condition is less simple. For frequencies less than the maximum value of ω_0 at the temperature minimum there is a layer about the temperature minimum through which the amplitude decreases. The simultaneous decrease in density maintains the velocity amplitude roughly constant. Beyond the temperature minimum at some point the oscillations are again permitted. Since the thickness of the attenuating layer is not great, the two propagating regions are coupled together. Ultimately, the temperature rise in the chromosphere will cause the oscillations to be attenuated. For modes whose horizontal wavelength is roughly 10^4 km, this second reflecting layer is located where the temperature is roughly 10^5 ° K. For most chromosphere models the upper permitted region is about 2000 km thick. Since the density scale height is about 150 km through this region, the velocity amplitude must increase by about a factor of exp

(2000/300) ≈ 1000. The observed velocity amplitude of 0.3 km sec^{-1} in the photosphere would be increased to 300 km sec^{-1}. Clearly some dissipation mechanism must operate to keep the velocity small.

The mechanism discussed by Osterbrock (1961) involves the conversion of acoustic waves into magnetoacoustic waves. These waves then are converted to shocks by the decreasing density, and the energy is dissipated rapidly. One formulation of the outer boundary condition would be to couple the mode to an outgoing wave where propagation becomes permitted beyond the temperature minimum. This is not an acceptable solution, however, since the vertical wavelength is about 1000–2000 km—about the distance between the temperature minimum and the point where shock formation is supposed to take place. The procedure used in the present calculation has been to find that

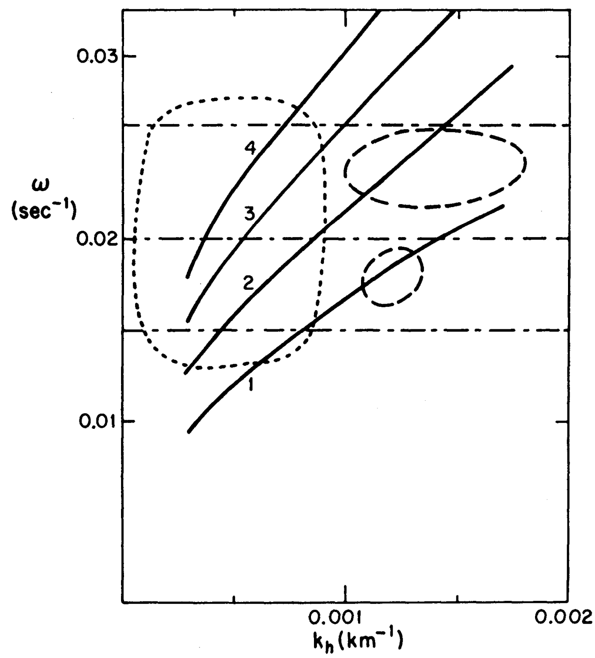

FIG. 2.—Diagnostic (k_h, ω)-diagram. Loci of the eigensolutions are shown as solid lines labeled by their modal numbers. Short dashed line indicates region where the oscillations were found by Tanenbaum *et al.* (1969). The two long dashed curves indicate the frequencies found by Frazier (1968). The three long-short dashed lines indicate the frequencies found by Gonczi and Roddier (1969). These workers did not determine the spatial wavelengths of the observed oscillations, but their results may be interpreted as modes 1–3 at about the resolution limit of Tanenbaum *et al.* (1969).

mode which has the smallest velocity amplitude above the temperature minimum. This mode may be expected to be damped least by the shock formation. The simple condition of smallest velocity amplitude does not provide any phase relation between the pressure perturbation and the velocity. Consequently, the growth or damping of the oscillations has been determined indirectly by the method described in § IV. The simple boundary condition does determine the eigenfrequencies to an accuracy of about 1 percent.

The dispersion relation between k_h and ω is shown in Figure 2, together with a rough representation of the relevant observational material. It is clear from this diagram that poor resolution in horizontal wave number will smear out the distinct frequencies. In particular, observational efforts which do not attempt to identify the wavelength of the oscillations are bound to lead to more or less random frequencies in the range $0.015 < \omega < 0.032$. Consistent observations may be obtained only in cases where a narrow range of wavelengths is selected. Such selection appears to have been achieved by Frazier (1968) by virtue of this high spatial resolution and small field of view. The work by Tanenbaum *et al.* (1969) also included a resolution of the power in one spatial dimension. Their results do not show the distinct lines predicted here, although there is a hint of diagonal ridges roughly parallel to the dispersion lines.

IV. ENERGY BALANCE

a) The Fundamental Equation of Conservation

The present incomplete understanding of the outer boundary condition eliminates the possibility of determining the growth or decay of the oscillations from the imaginary part of the frequency. A physically meaningful alternative is to compare the energy change in a fixed volume of space with the energy of the oscillation. Three processes act to change the energy of the oscillations. First, there is the work done on the oscillating fluid during one cycle. This is

$$\text{work} = -\int_0^{2\pi/\omega} v_z P' dt\,, \tag{13}$$

where v_z, P', and all quantities below associated with the oscillation are the real parts of the complex modal solutions found in § III. The second process is the flow of thermal energy in through the surface. This is

$$\text{thermal flux} = -\int_0^{2\pi/\omega} \rho v_z E dt\,, \tag{14}$$

where E is the internal energy of the fluid. Finally, there is the second-order part of the radiative flux which escapes into space. As long as the optical-depth scale does not chance significantly during the cycle, we may evaluate the radiative flux from

$$\text{radiative flux} = -\int_0^{2\pi/\omega} \int_\tau^\infty 12\sigma T^2 T'^2 E_2(\tau')d\tau' dt\,, \tag{15}$$

where σ is the Stefan-Boltzmann constant and E_2 is the second exponential integral. The minus sign appears in all these formulae because we wish to determine the rate of change of the energy in the volume of space. We shall neglect the flux of kinetic energy $\frac{1}{2}\rho v^2 v_z$ which is third order.

The energy density of the oscillations is also composed of two parts—thermal and kinetic. These are

$$\text{thermal-energy density} = \int_0^{2\pi/\omega} [\rho E - (\rho E)_0] dt\,, \tag{16}$$

$$\text{kinetic-energy density} = \int_0^{2\pi/\omega} \tfrac{1}{2}\rho v^2 dt\,. \tag{17}$$

Landau and Lifshitz (1959, § 49) show that

$$\frac{d}{dt}\int_{z_1}^{z_2} (\text{energy density})\, dz = \text{flux}\,(z_1) - \text{flux}\,(z_2) \tag{18}$$

when the energy density and flux (flux in this context includes work) are the sums of the parts given in equations (13)–(17). It is worth emphasizing that equation (18) is a rigorous result to second order.

The energy flux is virtually zero in the deep interior where radiative exchange is negligible. This is a result of our boundary-condition equation (12), which sets the velocity and thermodynamic fluctuations 90° out of phase. Equation (11) clearly shows that this phase relationship is altered when $\omega_R \sim \omega$. For most of the eigensolutions found in § III the flux at the temperature minimum was negative, a result which indicates that the oscillations are overstable. The cause for the overstability is the same as the mechanism discussed by Souffrin and Spiegel (1967) in connection with gravity modes. A comparison of the energy of the oscillations to the flux showed that the amplitude should increase by a factor of e in from twenty to thirty periods. This is a rather significant result since it eliminates the necessity of coupling the convective velocity field to the acoustic modes. It is likely that interaction between the convective and oscillatory velocity fields will take place and that this interaction may even contribute to the energy of the oscillatory field. Present observational evidence does not indicate that this interaction is dominant. Frazier (1968) has concluded from a direct inspection of his observations that the two velocity fields are uncorrelated.

b) *Dissipation above the Photosphere*

The discussion of the energy balance to this point has been incomplete in that the region above the temperature minimum has been ignored. As indicated in § III, the motion above the temperature minimum is coupled to that below in a rather direct fashion. In the upper region the motion is generally thought to involve conversion of mechanical energy to heat through some hydromagnetic interaction (see, for example, the discussions by Osterbrock 1962 and Athay 1966). Since the oscillations are not observed to grow in time, we may equate the nonradiative heat input to the chromosphere to the energy produced below the photosphere by the overstable oscillation. Clearly the energy production is proportional to the square of the amplitude of the oscillation. Without an explicit treatment of the dissipation, we must assign the amplitude on the basis of the observations. The summary by Tanenbaum *et al.* (1969) suggests $v_{\rm rms} = 0.2$ km sec^{-1} for wavelengths between 500 and 4000 km. Table 2 gives the required dissipation per unit area per unit time if the entire observed velocity is assigned to each of the modes and wavelengths indicated. The modal number is defined as one plus the number of nodes between the reflecting layers. The actual nonradiative heat input to the chromosphere and corona is a weighted average of these numbers and depends on the detailed distribution of the velocity amplitude on the (k_h, ω)-plane. The available evidence suggests that the modes which produce the most energy (those with periods about 300 sec) are also the modes with the largest amplitudes. Clearly this energy-production mechanism is in good agreement with the rate of energy loss by radiation of 5.6×10^6 ergs cm^{-2} sec^{-1} found by Athay (1966) for all layers above the temperature minimum (Table 2).

c) *Uncertainty in ω_R*

An important uncertainty in these calculations is the radiative-interaction rate ω_R. The formula we have used may be quite inaccurate in optically thick regions. In general, we may determine ω_R from

$$q = \int_0^\infty \kappa_\nu (J_\nu - B_\nu) d\nu \,, \tag{19}$$

where J_ν is the mean intensity. We expect that $q \neq 0$ in the unperturbed atmosphere because the convective flux varies with depth. This radiative exchange of heat, q_0, must be statistically balanced by the convective motions. The heat exchange due to the oscillatory motion is then the fluctuation in q. Unfortunately, we must know q_0 before we can determine the fluctuation in q since variations in κ interact directly with q_0. Nonetheless, in principle for a particular mode of oscillation we could determine ω_R from

$$\omega_R = -\frac{q - q_0}{C_P T'} \,. \tag{20}$$

For optical depths smaller than unity, the mean intensity should be unaffected by the perturbation and ω_R should reduce to the form given by Spiegel. At large depths, ω_R is quite small and may be neglected. The crucial superadiabatic layer is thus the only region where ω_R is difficult to determine. Since the superadiabatic layer is quite thin, we may explore the importance of this uncertainty by multiplying ω_R by arbitrary scale factor f in optically thick regions. Because of phase differences in the temperature fluctuation at adjacent layers we must also check complex values of f. Calculations were done with f between 0.1 and 10.0 and with phase angles between $-90°$ and $+90°$. The results showed that the power output is roughly proportional to $f^{-0.3}$ and is virtually unaffected for phase angles between $\pm 45°$. At phase angles of $\pm 90°$ the oscillations were no longer overstable. The result that the power output is inversely proportional to f is due to the fact that when T' is kept very small, the loops in the (T, S)- and (P, V)-diagrams are also very small. The dissipating loops in the optically thin region are unaffected and eventually dominate. Nonetheless, we would have to increase f by about a factor of 50 to eliminate the overstability. It seems unlikely that Spiegel's formula could be wrong by such a large factor. Also, the phase difference between the temperature fluctuations at different layers generally is less than 45°.

TABLE 2

REQUIRED DISSIPATION AS A FUNCTION OF HORIZONTAL WAVELENGTH

Modal Number	$\lambda_h = 4.83 \times 10^3$ km		$\lambda_h = 6.98 \times 10^3$ km		$\lambda_h = 12.57 \times 10^3$ km	
	Flux	Period	Flux	Period	Flux	Period
1	−0.3	311	0.0	399	−3.5	532
2	+7.4	256	6.5	313	−2.0	412
3	+7.4	211	6.3	256	+1.2	335
4	...	...	5.0	219	+3.7	282

NOTE.—Flux is in units of 10^6 ergs cm^{-2} sec^{-1}; period is in seconds.

V. CONCLUSIONS

The most important finding of this study has been that the 5-minute oscillations are overstable and are capable of supplying the energy lost through radiation in the chromosphere and corona. Also important is the result that the oscillations should be confined to distinct lines on the diagnostic (k_h, ω)-plane. These lines have not yet been found because of poor resolution in k_h and ω. The double peak observed by Frazier (1968) and the multiple peaks found by Gonczi and Roddier (1969) are compatible with the positions of the dispersion lines. More precise observations at longer wavelengths should permit an accurate determination of the entropy of the convective envelope.

We may set minimal conditions which will permit the observation of the dispersion lines. First, the spatial analysis must be two-dimensional. Unless very high wavenumbers are observed in one dimension, the total wavenumber is not well established by one-dimensional observations such as are made with a magnetograph. Second, the observations must be long enough, and of a large enough region, to resolve adjacent lines. At a wavelength of 8000 km the region must be roughly 60000 km in diameter and must be observed for roughly 1 hour. Third, these periods and wavelengths must be detectable. This condition requires velocity differences at points separated by 3000 km (4″ of arc) to be measurable and successive observations to be no more than 1 minute apart. Although the shorter-wavelength oscillations should be easiest to resolve, the longer wavelengths will provide the most information about the structure of the solar envelope.

I would like to thank Dr. E. N. Frazier for suggesting that the 5-minute oscillations might be dependent on the subphotospheric layers. I am grateful to Professor R. F. Christy for suggesting that my unexpected result that the oscillations are self-exciting might be more than a numerical error. I wish to thank the Kellogg Laboratory at Caltech for their hospitality and support during the initial phases of this work.

REFERENCES

Athay, R. G. 1966, *Ap. J.*, **146**, 223.
Frazier, E. 1968, *Zs. f. Ap.*, **68**, 345.
Gonczi, G., and Roddier, F. 1969, *Solar Phys.*, **8**, 225.
Howard, R. 1967, *Solar Phys.*, **2**, 3.
Landau, L. D., and Lifshitz, E. M. 1959, *Fluid Mechanics* (London: Pergamon Press).
Leighton, R. B., Noyes, R. W., and Simon, G. W. 1962, *Ap. J.*, **135**, 474.
Lighthill, M. J. 1952, *Proc. Roy. Soc. London, A*, **211**, 564.
Moore, D. W., and Spiegel, E. A. 1966, *Ap. J.*, **143**, 871.
Noyes, R. W., and Leighton, R. B. 1963, *Ap. J.*, **138**, 631.
Osterbrock, D. E. 1961, *Ap. J.*, **134**, 347.
Souffrin, P. 1966, *Ann. d'ap.*, **29**, 55.
Souffrin, P., and Spiegel, E. A. 1967, *Ann. d'ap.*, **30**, 985.
Spiegel, E. A. 1957, *Ap. J.*, **126**, 202.
Stein, R. F. 1968, *Ap. J.*, **154**, 297.
Tanenbaum, A. S., Wilcox, J. M., Frazier, E. N., and Howard, R. 1969, *Solar Phys.*, **9**, 328.
Ulrich, R. K. 1970, *Ap. and Space Sci.*, **7**, 183.
Whitaker, W. A. 1963, *Ap. J.*, **137**, 914.