

## p-Mode Oscillations in Gravitationally Highly Stratified Magnetic Solar Atmospheres

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### ABSTRACT

The aim of the work reported in this paper is to gain understanding of the propagation characteristics of p-mode oscillations in the highly gravitationally stratified magnetic solar atmosphere. An objective is the measurement of the properties of the solar atmosphere and its magnetic structures. We present a comparison of the analysis of results from observations and numerical simulations.

The paper describes 3D numerical magnetohydrodynamic (MHD) simulations of a model solar atmosphere with a uniform vertical cylindrically symmetric magnetic field and employing simulation drivers resulting in oscillations which mimic the behaviour of *p*-mode oscillations. The simulations were run for different values of the magnitude of the magnetic field and a *p*-mode driver with a fixed period of 300 s. For the observational study, a typical active region was selected. We report results for the temporal analysis of the observational data for a region containing a small sunspot (solar pore).

The paper reports the variation of the energy flux and oscillation frequency of the magnetosonic modes and examines their dependence on the magnetic field strength. The comparison with observational data indicate the presence of oscillation signals with a frequency close to that measured for the simulated results.

We conclude that magnetic regions of the solar atmosphere are favourable regions for the propagation of energy by slow magnetosonic modes. The results exhibit a frequency shift, for different values of the magnetic field. The obtained periodic behaviour is confirmed by observational data, featuring similar frequencies based on the intensity times series of images taken by the Solar Dynamics Observatory.

*Keywords:* editorials, notices — miscellaneous — catalogs — surveys

### 1. INTRODUCTION

Theoretical and computational studies coupled with observations from solar telescopes reveal diverse structures and dynamics in the Sun's atmosphere. The culmination of these studies of the Chromosphere and the upper solar atmosphere has enhanced our knowledge of the variety of magnetic field structures. Despite our armoury of observations and the diverse range of computational models it still remains a challenge to make sense of this complex menagerie of dynamical structures, understand solar atmospheric heating (i.e. chromospheric and coronal) and more generally space weather phenomena.

An example of the dynamical complexity are the ubiquitous five-minute oscillations in the solar atmosphere that are referred to as the solar global acoustic oscillations or *p*-modes. These global oscillations are interpreted as trapped

acoustic waves, i.e. standing acoustic oscillations of the solar interior. Earlier models of these oscillations assumed that there was reflection by the photosphere with at most evanescent propagation above the photosphere. It has been realised that with kinetic pressure as the restoring force, acoustic oscillations of the photosphere may be perturbed by the solar  $p$ -modes. There is now increasing evidence for leakage of these modes. The  $p$ -modes were interpreted as resonant modes trapped in a cavity formed from the steep change in density at the solar surface and a lower turning point in the interior caused by the increase in the speed of sound resulting in refraction. The physical characteristics of the solar sub-surface layers can be estimated using observations of the standing modes. The complexity and variety of magnetic structures in the solar atmosphere gives rise to a mixture of waves providing powerful diagnostics to aid our understanding and advance our knowledge.

Early theoretical studies reported in Roberts (1981a) and Roberts (1981b) considered wave propagation in idealised magnetic slab structures, this work demonstrated the fundamental conditions for wave propagation. These models were soon advanced to cylindrical magnetic structures in Edwin & Roberts (1983) which demonstrated the conditions under which different types of waves can be sustained under the conditions of the solar corona. These papers neglected gravitational structuring and focused on magnetic structure, there was also an emphasis on the localised wave phenomena. The analysis of Campbell and Roberts (1989) investigated the role of horizontal Chromospheric magnetic fields in the modification of acoustic modes. As well as demonstrating the shift of frequencies they were also able to predict the circumstances under which different Chromospheric regions would either act as a window permitting the propagation of acoustic energy, a sink where the energy is trapped or reflected back into the Chromosphere. The propagation of acoustic oscillations in an isothermal atmosphere with a vertical magnetic field was investigated by Hindman et al. (1996) although small frequency shifts were demonstrated the analysis was unable to demonstrate the absorption of waves e.g. that is observed in sunspots.

Much of the theoretical analysis has focused on the effect of magnetic flux tubes on wave propagation and neglected gravitational stratification. Hasan & Christensen-Dalsgaard (1992) considered an isothermal stratified atmosphere and investigated the influence of vertical fields on a variety of modes including  $p$ -mode oscillations. In the limit of weak fields they were able to analyse the spectrum of oscillations in using the modes for a non magnetised stratified atmosphere and the slow magnetoacoustic mode. The characteristics of generated waves are dependent on the motions at the footpoints of magnetic field concentrations including those in the intergranular lanes. For example vortex motions have been demonstrated to generate Alfvén waves Fedun et al. (2009b).

The link between oscillations in the solar atmosphere and solar global oscillations which are the topic of the computational modelling work described in this paper is described by Erdélyi (2006). An outward propagating wave is reflected inward from the solar upper surface, or boundary layer, because of a sudden decrease in the plasma density, while the lower boundary of the cavity is formed by the increasing sound speed due to temperature rises. For global oscillation modes which exceed the acoustic cutoff frequency, there is wave leakage out from the cavity into the atmosphere. A model for understanding the behaviour of solar global oscillations based on the Klein-Gordon equation was developed by Taroyan & Erdélyi (2008), the model suggests a global resonance. Waves which are normally trapped by a frequency cut-off barrier are enhanced by a resonance enabling propagation into the solar atmosphere. Enhancements to the models described earlier are presented by Pintér et al. (2007), their work utilised an exponentially decaying horizontal magnetic field as a representation of the magnetic carpet. The inhomogeneous three layer model leads to characteristic frequencies.

For a number of decades intensity oscillations in the solar atmosphere have been the subject of observation and study see Banerjee et al. (2011), De Moortel (2009), Mathioudakis et al. (2013), Ruderman & Erdélyi (2009), Wang (2011). The observed periods of atmospheric intensity oscillations range from some seconds to several hours Auchère et al. (2014). Much earlier, it was proposed by Jensen & Orrall (1963), that the 3- and 5-minute oscillations were the characteristics of the photosphere and the chromosphere. Recently the association of intensity oscillations with oscillations in coronal loops and sunspots has been confirmed for oscillation periods of three minutes. Using magneto-seismology, observations of intensity observations in the solar atmosphere can be used to understand the magnetic structures in the solar atmosphere see Roberts et al. (1984), Banerjee et al. (2007), Zaqarashvili & Murawski (2007), Erdélyi & Taroyan (2008), Verth et al. (2010). Additionally it has been recognised that the 5 minute oscillations in loops are not related to sunspots De Moortel et al. (2002). Evidence for global oscillations is suggested by the observational analysis of Gyenge & Erdélyi (2018) which demonstrates that the frequent recurrence of micro-flare events exhibit fluctuations between 3 minutes and 12 minutes, these fluctuations arise during time intervals before and after of major solar flares.

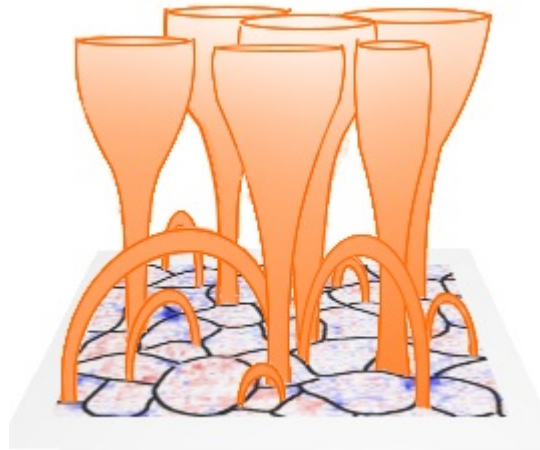
The novelty of this work is a comparison of numerical simulations with observations, demonstrating the link between ubiquitous oscillations and global oscillations modelled using our numerical representation which is of sufficient complexity to represent the dominant dynamical processes in the magnetic solar atmosphere. Comparison of the reported observational work with the results of the modelling suggests that the sampling process is sufficient to draw such inferences.

We report the results of numerical simulations of photospheric  $p$ -mode oscillations in a model solar atmosphere with a uniform and vertical magnetic field. We also present the results of a temporal analysis based on observations of intensity oscillations with the objective of providing evidence of the existence of the temporal behaviour exhibited by our simultaneous. The work contributes to our understanding of the propagation and modification of global oscillations in the highly gravitationally stratified solar atmosphere.

First, we consider the variety of magnetic structures in the solar atmosphere and address the wave motions that are observed in these regions. This is followed with a description of the solar atmospheric model, magnetic field configuration and the simulation method we used to model  $p$ -mode oscillations in a model magnetic solar atmosphere.

## 2. STRUCTURES IN THE SOLAR ATMOSPHERE

There are many different kinds of structures in the highly dynamic solar chromosphere. Bright spots which form in the trenches between solar granules are known as faculae, these features which are formed near magnetic field concentrations constantly form and dissipate over time scales of several minutes. Pores are a few Mm across and they are the smaller counterparts of sunspots, they are the bright areas near to and around sunspots or faculae. The bright areas that extend away from active regions are called plage regions. The magnetic fields in this area diffuse away into the quiet Sun regions. The magnetic network is a network of lines which outline super-granules. Super-granules are convective regions about 30 Mm across, and they possess strong horizontal flows. The motions within the super-granules result in the concentration of bundles of magnetic field lines. The mean photospheric field in the inter-network region is 100-300 G. On the other hand, solar active regions contain sunspots which have sizes between 1 - 50 Mm.



**Figure 1.** The schematic solar magnetic network.

Active regions producing flares may have fields which easily exceed the normal range of 100-500 G. As well as these massive concentrations, solar magnetograms reveal many north poles in the quiet photosphere, this is known as the magnetic carpet. These structures can be observed in figure 1. Along with the coronal funnels arising from grain boundaries, the picture shows a range of network loops with temperatures which can be in the range  $10^5\text{K}$  to  $10^6\text{K}$ . The dynamical phenomena of concern in this paper result in waves and oscillations in the solar atmosphere. The upward propagation of waves through the solar atmosphere can result in coronal heating, frequency shifts and other wave phenomena. A range of wave transformations may occur including reflection and refraction by atmospheric structures.

Our initial computational studies were applicable to quiet sun regions with magnetic fluxes in the range 5-10G this includes the non magnetic solar chromosphere and the quiet inter-network regions between the magnetic flux concentrations. Given the variety of solar atmosphere regions for example the network, inter network, plage and faculae regions it is recognised that the modes of oscillation with periods of 3 and 5 minutes exhibit varying behaviour. In particular, the variation observed for different magnetic structures and reflecting layers such as the transition layer influences propagation in the upward and downward direction.

The power spectra presented in the first figure of Griffiths et al. (2018b) exhibit a variation of propagation characteristics at different levels within the solar atmosphere and within different regions such as coronal holes, the quiet sun and active regions. In general the power spectra indicate a preponderance of long period 5 minute waves with frequencies in the range 1.5-5mHz. Also observed are the distinctive peaks for the short period 3 minute waves (with frequencies in the range 5-8mHz). The power spectra exhibit peaks in much longer period ranges for example 12 minute waves (frequencies in the range 1.1-1.5mHz) and 16 minute waves (frequencies 1-1.1mHz). For the quiet sun regions the 5 minute modes are stronger at photospheric levels and diminished higher up in the corona, but note a small peak for the data from the AIA 211 corresponding to 2.0MK.

The review of Khomenko and Calvo Santamaria (2013) summarises this multifaceted picture. In the close proximity of the magnetic network elements, the longer 5 minute modes propagate efficiently to the chromosphere. The 3 minute modes propagate from the photosphere to the chromosphere in the network cell interiors for restricted regions of the network and internetwork. Although these long-period halos are present in the chromosphere they are most prominent in the photosphere. With their more complex magnetic structures, a more complex pattern is exhibited in plage and faculae regions. Observations show that the 3 minute modes exhibit enhancement in both photosphere and chromosphere whereas the power of the 5 minute modes increase significantly in the chromosphere. These power enhancements are known as “halos” and have been widely reported Kontogiannis et al. (2010).

### 3. MOTIVATION

Given the complexity of the dynamics and the diversity of structures in the solar atmosphere, it is understood that a truly realistic model is challenging and requires a hybrid multi-disciplined approach. In order to develop a model providing a representation of the solar atmosphere it is necessary to establish that the modelling tools give a consistent behaviour in idealised test cases and that there is a consistency between the computational and theoretical models. Many computational MHD simulations of the sun have been undertaken, some of the approaches have resulted in an encouraging degree of realism Vögler et al. (2005), Gudiksen et al. (2011). Computational MHD simulations of the propagation of waves in 3D solar atmospheres were undertaken by Fedun et al. (2009a). Initially they considered hydrodynamical models, in later simulations Fedun et al. (2009b) Vigeesh et al. (2012) reported results for magnetized solar atmospheres featuring an idealized flux tube. These models with point drivers demonstrated the leakage of magneto-acoustic energy into the solar atmosphere. The work of Khomenko and Calvo Santamaria (2013) and Calvo Santamaria, Khomenko and Collados (2015) reviewed and presented 2D computational MHD modelling of wave propagation in magnetic features such as sunspots and arcades.

The models reveal that vertical magnetic fields enable energy to reach the corona. Our initial models of a realistically stratified model of the solar atmosphere Griffiths et al. (2018b) were hydrodynamic simulations. In these simulations atmospheric perturbations caused by photospheric global oscillations are represented using drivers located in the photosphere so as to mimic the influence of the solar p-modes. The results of the hydrodynamic modelling exhibited agreement with the energy flux predictions from a 2 layer Klein-Gordon model. This agreement supported the interpretation of the interaction between the solar atmosphere and the global oscillations. Also revealed by the simulations was a consistency between power flux measurements from SDO and frequency dependent energy flux measurements from the numerical simulations. This observed propagation of energy into the mid to upper atmospheric regions of the quiet sun occurred for a range of frequencies. Such observations may explain observed intensity oscillations for periods greater than the well known 5-minute and 3-minute oscillations. It was also found that energy flux propagation into the lower solar corona is strongly dependent on the particular wave modes.

In this paper we present results for 3D numerical MHD simulations with an extended driver representing photospheric p-mode oscillations in a magnetic solar atmosphere, the objective is to gain understanding of the propagation characteristics of the p-mode oscillations.

### 4. NUMERICAL COMPUTATION METHODS

The simulations described in this paper were undertaken using the SMAUG code, a GPU implementation of the Sheffield Advanced Code (SAC) [Shelyag et al. \(2008\)](#). The Sheffield MHD Accelerated Using GPUs (SMAUG) [Griffiths et al. \(2015\)](#) and SAC are derived from the versatile advection code (VAC) developed by [Tóth \(1996\)](#). SAC and SMAUG are numerical MHD solvers which can be used to model the time-dependent evolution of photospheric oscillations in the solar atmosphere. The SMAUG code can simulate linear and non-linear wave propagation in strongly magnetised plasma with structuring and stratification.

We use the same general system of ideal MHD equations applicable to an ideal compressible plasma and used for our initial hydrodynamic simulations, [Griffiths et al. \(2018b\)](#).

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{\partial(\rho \mathbf{v})}{\partial t} + \nabla \cdot (\mathbf{v} \rho \mathbf{v} - \mathbf{B} \mathbf{B}) + \nabla p_t = \rho \mathbf{g}, \quad (2)$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (\mathbf{v} e - \mathbf{B} \mathbf{B} \cdot \mathbf{v} + \mathbf{v} p_t) + \nabla p_t = \rho \mathbf{g} \cdot \mathbf{v}, \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{v} \mathbf{B} - \mathbf{B} \mathbf{v}) = 0. \quad (4)$$

The total pressure  $p_t$  is given by

$$p_t = p_k + \frac{\mathbf{B}^2}{2}, \quad (5)$$

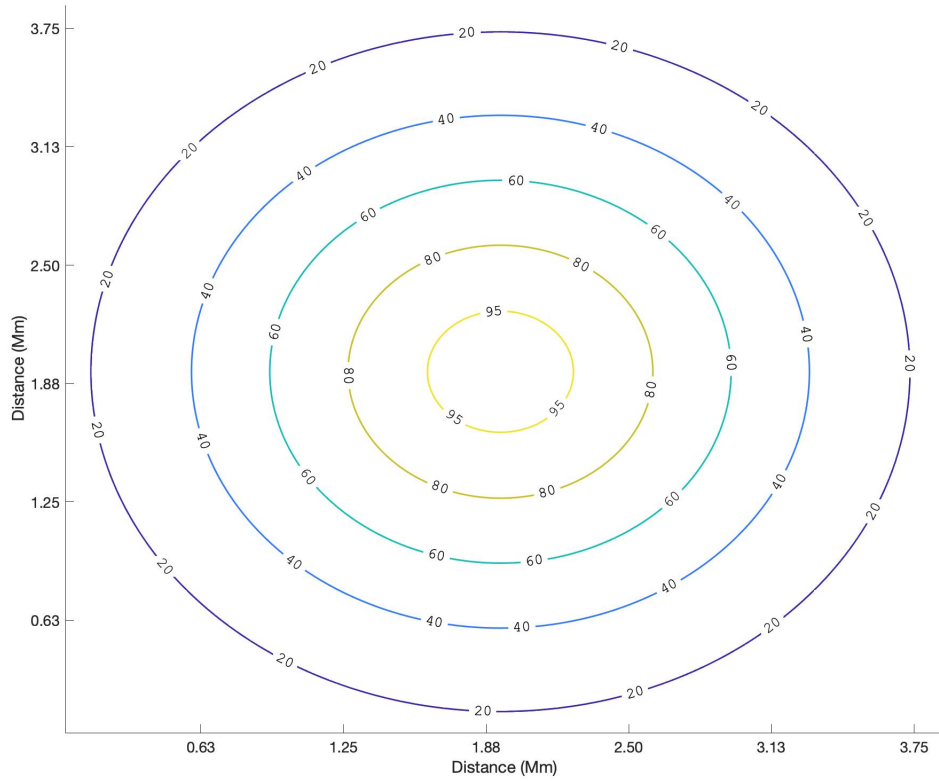
and the kinetic pressure,  $p_k$ , is written as

$$p_k = (\gamma - 1) \left( e - \frac{\rho \mathbf{v}^2}{2} - \frac{\mathbf{B}^2}{2} \right). \quad (6)$$

In the system of equations above,  $\mathbf{B}$  is the magnetic field,  $\mathbf{v}$  is the velocity,  $\rho$  is the mass density,  $\mathbf{g}$  is the gravitational acceleration vector and  $e$  is the energy density. The SMAUG code used for the simulations reported here employs perturbed versions of the general set of MHD equations given above. For the perturbed versions the density, energy density and magnetic field are expressed in terms of perturbed and background quantities as follows

$$\begin{aligned} \rho &= \tilde{\rho} + \rho_b, \\ e &= \tilde{e} + e_b, \\ \mathbf{B} &= \tilde{\mathbf{B}} + \mathbf{B}_b. \end{aligned}$$

Assuming a magneto-hydrostatic equilibrium of the background plasma, the background quantities which do not change in time have a subscript  $b$ . The time varying perturbed quantities do not have a subscript. The fully non-linear MHD numerical finite element solver employs hyper-diffusion and hyper-resistivity to achieve numerical stability of the computed solution of the MHD equations [Caunt and Korpi \(2001\)](#). A more detailed description of the full set of MHD equations, including the hyper-diffusion source terms are given in [Griffiths et al. \(2015\)](#) and [Shelyag et al. \(2008\)](#).



**Figure 2.** Initial Magnetic Field Configuration, radial field distribution, uniform in the vertical direction with a maximum value of 100G

## 5. COMPUTATIONAL MODEL

The hydrodynamical studies reported in [Griffiths et al. \(2018b\)](#) employed simulation drivers with physical characteristics representing p-mode oscillations with varying modes and periods. The MHD simulations reported here use the same driver and model of the solar atmosphere as used in our initial hydrodynamical study. The model is modified by the inclusion of a uniform, vertical and cylindrically symmetric magnetic field.

The dimensions of the simulation box are  $L_x = 4$  Mm,  $L_y = 4$  Mm and with a height of  $L_z = 6$  Mm in the gravitationally stratified  $z$ -direction. The computational box is an array of elements  $128 \times 128 \times 128$ . The upper boundary of the model is in the solar corona whilst the lower boundary is coincident with the photosphere. The perturbed MHD code used here is suited to studying the propagation of wave energy from the photosphere, across the transition layer and leaking into the solar corona. The time scales relevant to our study are determined by the 5-minute  $p$ -mode oscillations the model employs open boundary conditions thereby allowing us to model the wave propagation. To generate these oscillations we use vertical velocity drivers which are extended across the base of the model. The following sections describe the model solar atmosphere and the driver.

Data obtained from solar observations was used to construct a semi-empirical model solar atmosphere, the resulting model is a representation of the quiet sun. Employing the fundamental assumption of hydrostatic equilibrium, the VALIIIc model [Vernazza et al. \(1981\)](#) was used to construct a model of the chromosphere in equilibrium. For atmospheric heights greater than 2.5 Mm the results from a model of solar coronal heating were used [McWhirter et al. \(1975\)](#). The atmospheric profiles for temperature, density, sound speed and frequency cut-off for this model are shown in figure 5. A further possibility for a model solar atmosphere is the use of parametric models, the smoothed step function used by [Murawski and Zaqarashvili \(2010\)](#) is an example. Discussion of the validity of model solar atmospheres and realistic models of the chromosphere, indicate the need for observationally derived semi-empirical models,

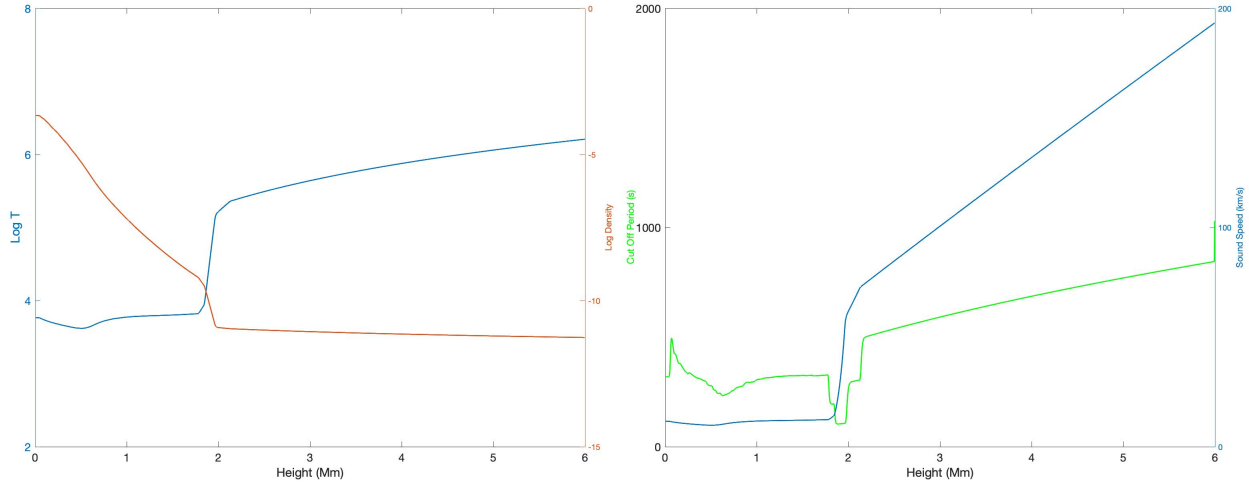


see Carlsson and Stein (1995), Kalkofen (2012). It has been suggested that local dynamo action and joule heating in the dynamical solar chromosphere make the construction of models particularly challenging Leenaarts et al. (2011).

For the simulations described here we use a simplistic model which is uniform in the vertical ( $z$ ) direction. The cylindrically symmetric field was constructed using the parametrisation in equation 7, the effective cylinder radius was fixed at  $R = 0.14\text{Mm}$ . Simulations were run for different values of  $B_{max}$ .

$$B_z = B_{max} e^{-\frac{x^2+y^2}{R^2}}, \quad (7)$$

Since the field is uniform in the vertical direction the model atmosphere is in magnetohydrostatic equilibrium. Details of the construction procedure for the model atmosphere, the resulting density profiles and temperature profiles are provided in Griffiths et al. (2018b).



**Figure 3.** The left hand panel shows the temperature and density profile used for the simulations and based on the VALIIIc model. The right hand panel shows the computed soundspeed and frequency cutoff for a solar model atmosphere based on the VALIIIc model.

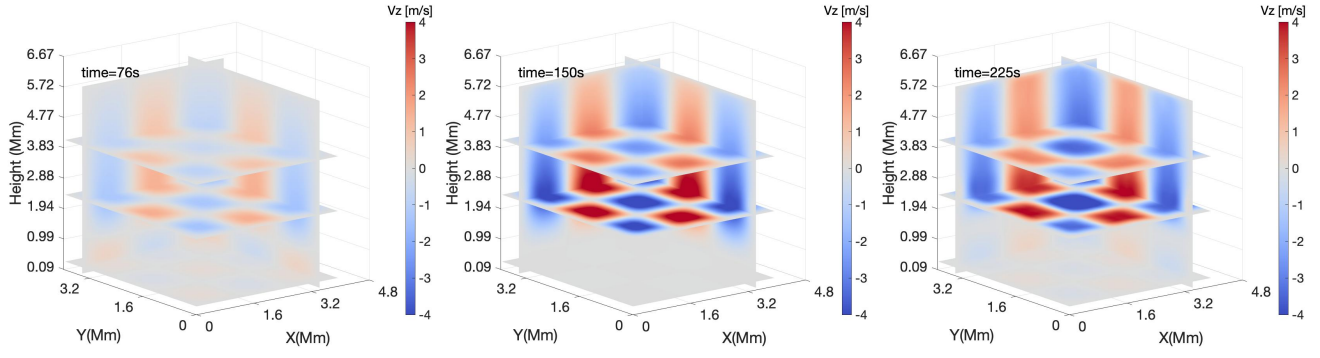
## 6. NUMERICAL DRIVERS FOR $P$ -MODE OSCILLATIONS

The work reported here is an extension of the earlier work of Malins (2007), their study represented photospheric buffeting motion using different point drivers. These studies demonstrated surface waves, structures in the transition zone and identified the effect of cut-offs induced by the stratified solar atmosphere. The overview in Section 2 identified physical phenomena delivering energy into the solar atmosphere and resulting in oscillatory behaviour.

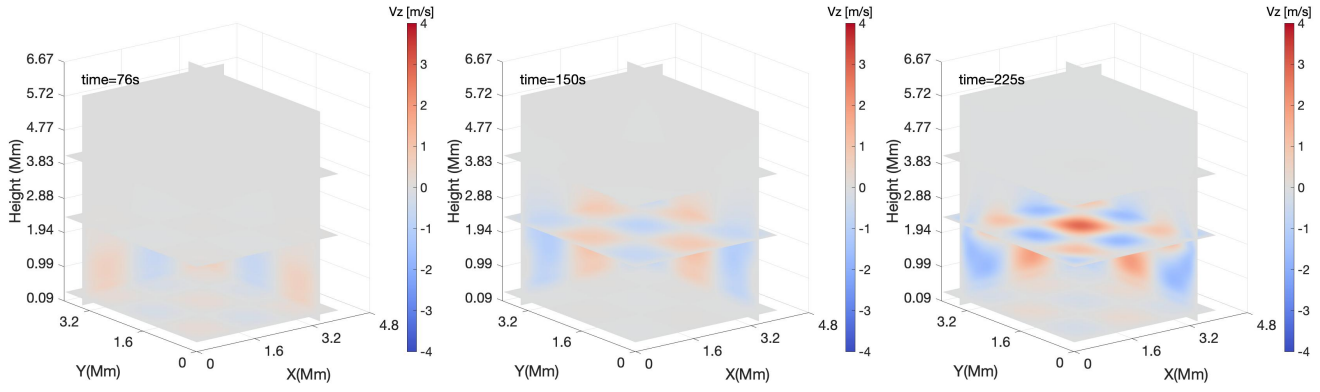
The simulations presented in this paper, employ an extended driver resulting in the perturbation of the entire lower boundary of the model. Photospheric  $p$ -mode oscillations for the real sun have a horizontal wavelength and coherence. The vertical velocity driver used here is an acoustic  $p$ -mode driver located at the photosphere and exciting waves which propagate into our realistic 3D model of the solar atmosphere. An extended driver with a sinusoidal dependence and a wavelength of 8 Mm applied along the middle of the base of a computational domain of dimension 4 Mm represents a *fundamental mode*. Drivers may be constructed as an ensemble of these solar global eigenmodes. The driver is represented by the expression shown in equation (8)

$$V_z = A_{nm} \sin\left(\frac{2\pi t}{T_s}\right) \sin\left(\frac{(n+1)\pi x}{L_x}\right) \sin\left(\frac{(m+1)\pi y}{L_y}\right) \exp\left(-\frac{(z-z_0)^2}{\Delta z^2}\right), \quad (8)$$

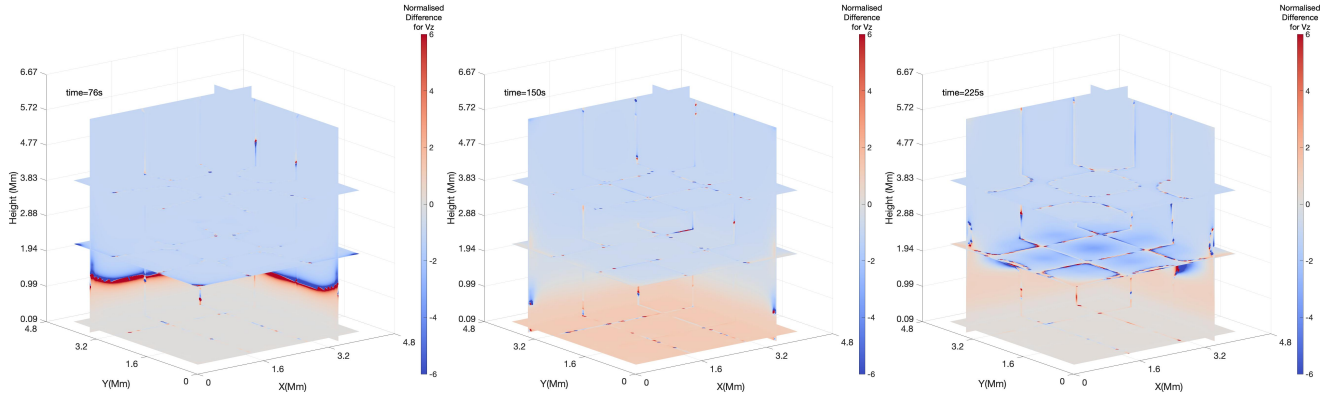
For the simulations here, a  $p$ -mode driver corresponding to the 5 minute mode, was used with period 300s and mode (2,2). Earlier studies demonstrated the effectiveness of this mode with energy propagation. Simulations were run for different values of the magnitude of the magnetic field. The mode numbers identified here are the  $n$  and  $m$  values in the expression for the driver shown in equation (8).



**Figure 4.** Vertical Component of the Velocity for Different Sections of the Simulation for 76s, 150s and 225s for a vertical field with maximum field of 100G.



**Figure 5.** Vertical Component of the Velocity for Different Sections of the Simulation for 76s, 150s and 225s for a magnetic field of 0G.



**Figure 6.** Normalised Difference of the Vertical Component of the Velocity for Different Sections of the Simulation for 76s, 150s and 225s for a magnetic field of 100G.

For the driver equation given in 8,  $T_s$  is the period,  $A_{nm}$  is the amplitude, the indices  $n$  and  $m$  define the mode, the lengths of the base of the simulation box in the  $x$  and  $y$  directions are  $L_x$  and  $L_y$  are respectively. The driver width,  $\Delta z$  is set to 4km, the parameter  $z_0$  was set so that the vertical location of the driver is in the photosphere, coincident with the location of the temperature minimum and 0.5 Mm above the lower boundary of the model. The simulations presented use the parameter  $A_{nm}=500 \text{ ms}^{-1}$  with the mode indices set to  $n, m = 2$ .

## 7. MAGNETOACOUSTIC WAVES IN UNIFORM VERTICAL MAGNETIC FIELD CONFIGURATIONS



Wave Speed (km/s)	0G	50G	75G	100G
2Mm	12.6	96.5	47.7	25.2
1Mm	10.1	64.1	44.4	45.4
0.5Mm	8.7	45.4	37.8	32.3

**Table 1.** The table shows wave speeds obtained from the distance-time plots for the 300s period driver with magnetic fields of 0G, 50G, 75G and 100G.

Magnetic Field (G)	1Mm	2Mm	4Mm	5.5Mm
0G	0.155	$-1.771 \times 10^{-5}$	$1.227 \times 10^{-6}$	$8.194 \times 10^{-7}$
50G	0.270	-0.399	0.040	0.021
75G	-0.507	-0.126	0.015	0.007
100G	-0.255	-0.226	0.019	0.006

**Table 2.** The table shows the time averaged and integrated energy flux ratio obtained for the 300s period driver with magnetic fields of 0G, 50G, 75G and 100G.

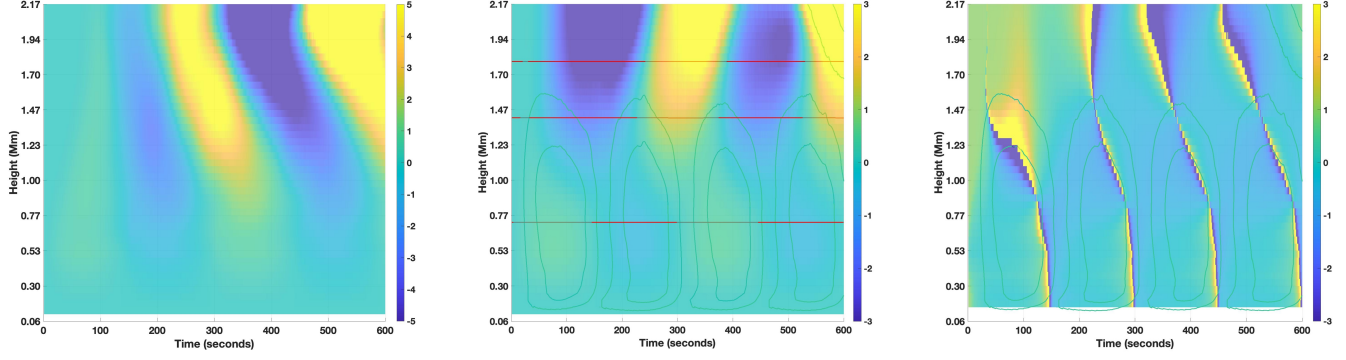
Magnetohydrodynamic simulations have been performed with p-mode oscillations of the photospheric layer and for magnetic field strengths of 0G, 50G, 75G and 100G. The plasma  $\beta$  for the model decreases rapidly from a value of 50 at 0.7Mm above the lower boundary of the simulation domain,  $\beta$  is 1 at a height of 1.39Mm. It is anticipated that for the region with  $\beta \approx 1$ , mode conversion occurs with full or partial conversion to magnetohydrodynamic modes.

Figure 4 shows the vertical component of the velocity at various times for different sections through the simulation box. Each plot in figure 4 corresponds to a vertical field configuration with a maximum field strength ( $B_{max}$ ) of 100G. Comparison with the 0G case in figure 5, illustrates a clear difference between the purely hydrodynamic and the MHD case. The figures for the MHD case exhibit evidence of a fast moving magneto-acoustic wave mode. The measured propagation speed is consistent with that of a fast magneto-acoustic mode. Figures 4 and 5 compare the wave modes at a quarter, half and three-quarters of a cycle. Figure 6 shows the normalised difference between the results for the 100G case and the 0G case. The normalisation is achieved by dividing the difference value by the sum of the values for the 0G and 100G case. The largest difference arises at the height where the velocity reverses direction. Our results indicated that even a small magnetic field appears to enhance the motion of plasma in the corona and there is an apparent difference in phase between the magnetic field cases. As well as an increase in the velocity amplitude with increasing magnetic field there is a small shift in the frequency of the oscillation. For magnetic fields with strengths between 1kG and 50G, the theoretical prediction of Hindman et al. (1996) resulted in frequency shifts in the microhertz and nanohertz range, although this was a prediction of helioseismology their result provides insight into the mechanism of frequency shifts of waves in atmospheric magnetic structures.

A set of videos of all the simulations that were performed can be obtained from the online research data archive hosted by The University of Sheffield Griffiths et al. (2018a). The videos display the evolution of the  $z$ -component of the plasma velocity along different layers of our model solar atmosphere. Each video shows the value of the vertical component of the plasma velocity ( $z$ -component in m/s) along different slices through the simulation box. Each video is labelled using the magnetic field strength in Gauss.

A distance-time plot for the 300s period driver, with the 100G field is shown in figure 7. The wavespeeds computed from this distance-time plot are shown in table 1. The speeds for the 0G field are consistent with the speed of sound in the solar atmosphere, whilst the speeds for the non zero magnetic field are consistent with propagation speeds for magnetosonic modes.

Investigation of figure 6 reveals excitation from the driver along with standing oscillations in the chromospheric cavity. There is also partial reflection of the signal at the boundary of the chromosphere and the transition layer. The results also display evidence for signals propagating horizontally at the transition region and with frequencies similar to that of the driver. The distance-time plot in figure 7, for the 100G case exhibits reduced reflection at the transition layer and indicates evidence for energy leakage into the solar Corona. Figure 7 shows an initial fast mode pulse followed by slow mode oscillations above the line which corresponds to a plasma  $\beta$  value of one. Below an height of 1Mm we observe the standing mode oscillations along with oscillations of the magnetic field perturbations. Below 0.3Mm the driver oscillations can be observed in conjunction with possible magneto-acoustic slow mode oscillations.



**Figure 7.** Distance-time plot of the vertical component of the velocity in the mid chromosphere, for the case of the difference between 0G and 100G, the central redline is the line for beta equal to 1, the lower and upper redline correspond to beta equal to 50 and 0.5 respectively.

Since the source terms perturb only the vertical component of the velocity and the model is cylindrically symmetric, pure Alfvénic modes are not expected.

The distance-time plots and our sections displaying the vertical component of the plasma velocity indicate significant differences in the propagation behaviour for the hydrodynamic and non-zero magnetic field cases. It is necessary to understand and quantify the extent of the energy leakage into the solar atmosphere. To investigate the influence of the magnetic field on the propagation of wave energy we employ an expression for the energy flux which was used by Bogdan et al. (2003). The wave energy flux  $\mathbf{F}_{\text{wave}}$  is given by

$$\mathbf{F}_{\text{wave}} = \tilde{p}_k \mathbf{v} + \tilde{\mathbf{B}} \cdot \mathbf{B}_b \mathbf{v} + \mathbf{v} \cdot \tilde{\mathbf{B}} \mathbf{B}_b.$$

We compute the time averaged energy flux integrated over different cross sections of the simulation box.

$$F_{\text{int}} = \frac{1}{t_{\text{max}}} \int_0^{t_{\text{max}}} \int \mathbf{F}_{\text{wave}} \cdot d\mathbf{A} dt, \quad (9)$$

These expressions are dependent on the perturbed kinetic pressure  $\tilde{p}_k$ .

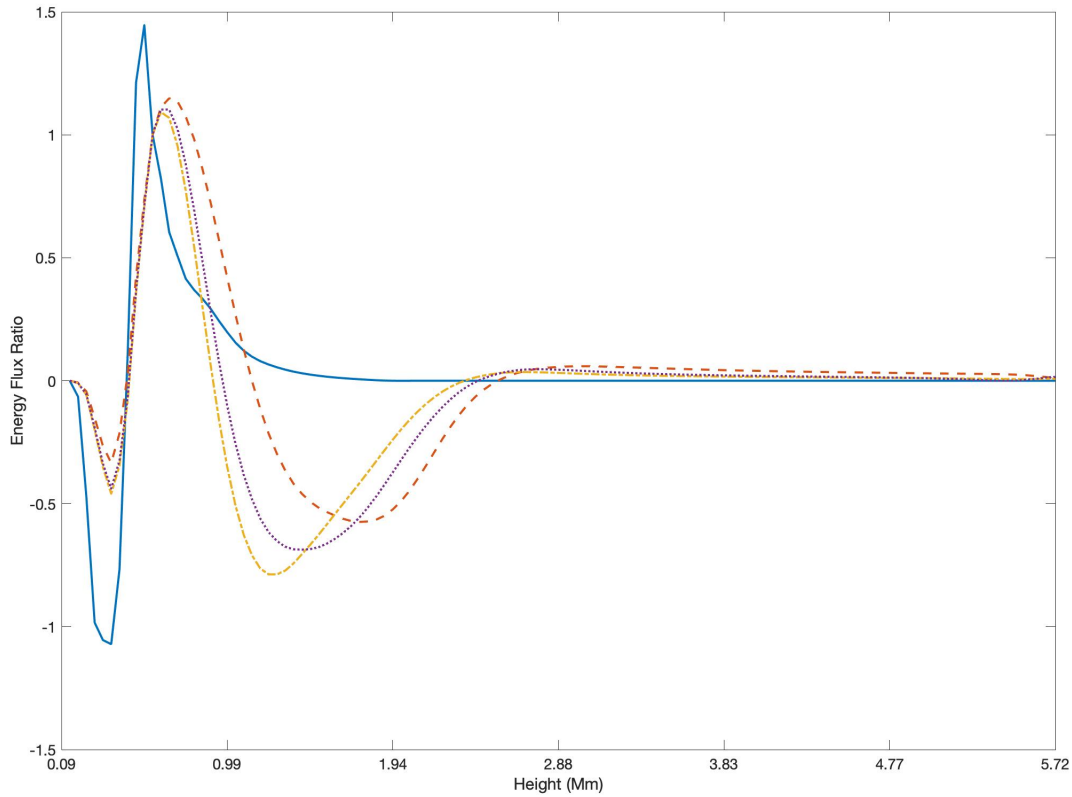
$$\tilde{p}_k = (\gamma - 1) \left( \tilde{e} - \frac{(\tilde{\rho} + \rho_b) \mathbf{v}^2}{2} - \frac{\mathbf{B}^2}{2} \right).$$

Using equation (9), we computed the energy flux integral for each of the drivers at different atmospheric heights and averaged over the total time. We compute the ratio of this integrated energy flux to the integrated energy flux at the location of the driver, the resulting values are shown in table 2. It appears that for heights greater than 4Mm, the energy flux is enhanced for increasing values for the vertical magnetic field. In figure 7, we plot the ratio of the integrated energy flux ratio for different values of the field at different heights and for the different vertical field values (the blue, orange, purple and red are for field values of 0G, 50G, 75G and 100G respectively). This plot demonstrates that for higher B-field magnitudes there is a small leakage of energy propagation, this is interesting because this is consistent with observations and other simulation results which demonstrate an enhancement for inclined fields.

## 8. FREQUENCY ANALYSIS

The top panel of Figure 9 shows a vertical slice of  $B_z$  over time, this is based on the simulation with the initial magnetic field configuration with a maximum value of 100G. The vertical axis represents the height in Mm and the horizontal axis is the time dimension, measured in seconds. From this 2-dimensional plane, 5 layers were selected, indicated by the horizontal grey lines. The middle panel of Figure 9 displays the temporal variation of the selected layers, indicated by different grey shade colours. The time series do not feature non-stationary behaviour, therefore, further transformations (such as detrending, smoothing or differentiation) are not needed. The Hanning-window function is still applied to avoid leakage effects when performing the Fast Fourier Transformation (FFT).

An FFT was applied (the lower panel) for investigating any oscillatory behaviour in the analysed signal. A significant oscillatory pattern was found with frequency range of 3.75 - 4mHz, corresponding to a period range of 4.2 - 4.4 minutes.



**Figure 8.** Shows the Ratio of the Integrated Energy Flux ratio for different values of the field, Blue 0G, Orange 50G, Purple 75 G and Red 100G.

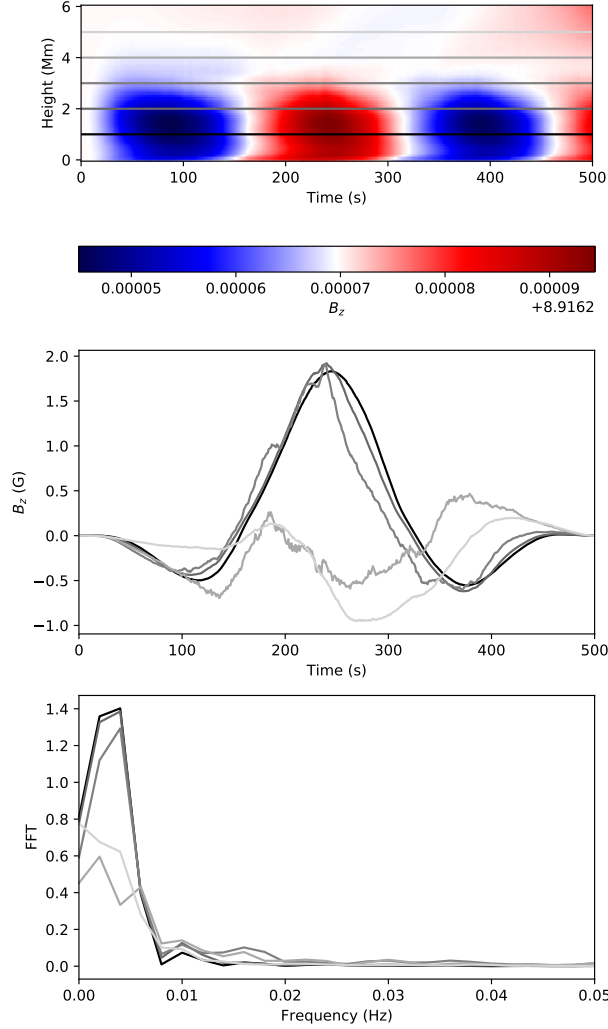
An FFT was also performed based on the other simulations with different initial magnetic field configurations (with a maximum value of 0G, 50G, 75G and 100G). These investigations all showed similar oscillatory behaviour, therefore, we present one FFT as a representative example. Figure 10 features the same analysis as Figure 9, however, the data is now the sample from the B=0 case.

With the objective of confirming the obtained oscillatory behaviour, temporal analysis for the observational data was performed. We investigated intensity oscillations in the solar atmosphere observed by SDO/AIA. The passband 1600 Å was selected because our simulation mainly focused on the lower atmospheric regions, i.e. photosphere and chromosphere. The cadence of 1600 Å images is 24 seconds, therefore it was suitable for studying relatively high-frequency oscillations such as the obtained 4mHz.

The initial magnetic field configuration of our model is a standing magnetic tube, passing through the chromosphere and the lower corona. Therefore, We chose to sample a typical active region. The selected area contains a small sunspot (solar pore), presumably featuring similar magnetic structure as our simulation. The size of the investigated area covers 50 pixels in total, demonstrated by Figure 12. The obtained time series shows non-stationary behaviour, therefore, the observed linear trend is removed by taking the first difference  $\Delta y_t$  of the data. The first difference is defined as the difference between consecutive observations  $y_t$  and  $y_{t-1}$ . Furthermore, the times series is also normalised by applying standard scores (Z-scores), defined by:

$$Z_i = \frac{T_i - \bar{T}}{\sigma(T)}, \quad (10)$$

where, the parameter  $\bar{T}$  is the mean of the time series and the parameter  $\sigma(T)$  is defined as the standard deviation of the data. The top panel of Figure 11 demonstrates the trend removed and normalised time series. The lower panel of Figure 11 shows the result of the applied FFT technique. The dashed line is the significance level ( $3\sigma$ ) which was



**Figure 9.** Temporal analysis of  $B_z$  vertical slices at 2 Mm. The top panel shows the selected vertical slices, indicated by gray colours. The middle panel demonstrates the obtained signal after applying a Hanning window function. The bottom panel shows the result of the FFT analysis based on the 5 selected vertical  $B_z$  slices.

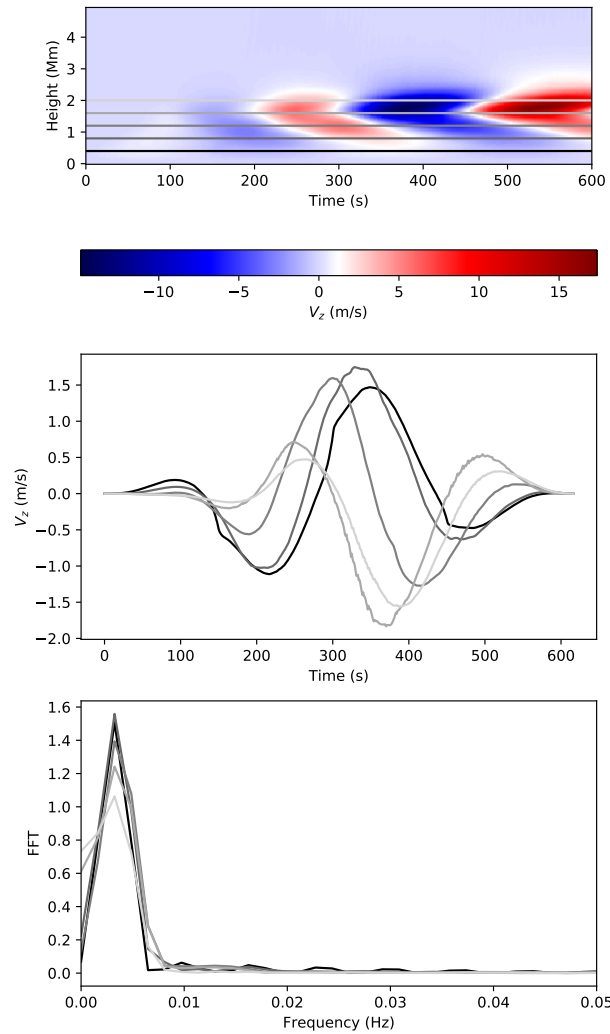
calculated using a Monte-Carlo method. The original data showed red noise signature which transformed to blue noise after differentiating the data. We have generated 1 million blue noise signatures  $N_b$  and calculated the standard deviation  $\sigma(N_b)$  and the mean  $\overline{N_b}$  of the simulated noise, providing our significance level  $S$ :

$$S = \overline{N_b} + 3\sigma(N_b). \quad (11)$$

A significant period is found with frequency range of 3.5 – 4.2 mHz, corresponding a period range of 4 - 4.7 minutes, which is close to the period found in simulation data. Another significant peak (5 mHz) is found with period around 3 minutes which may be an indication of another global oscillation.

## 9. CONCLUSION

In this paper we have presented results for a series of MHD simulations of an extended oscillator at the base of a model solar atmosphere. We have shown that energy is propagated by magnetosonic modes. Slow and fast magnetosonic modes are responsible for carrying some energy back to the chromosphere and the photosphere. The FFT analysis demonstrated oscillatory patterns with periods in the range 4.2-4.4 minutes. This arose for both the hydrodynamic case and the case with a magnetic field of 100G, in this case there was some additional structure for the FFT analysis. These results compare favourably with the FFT analysis observations from carefully selected solar features, for example

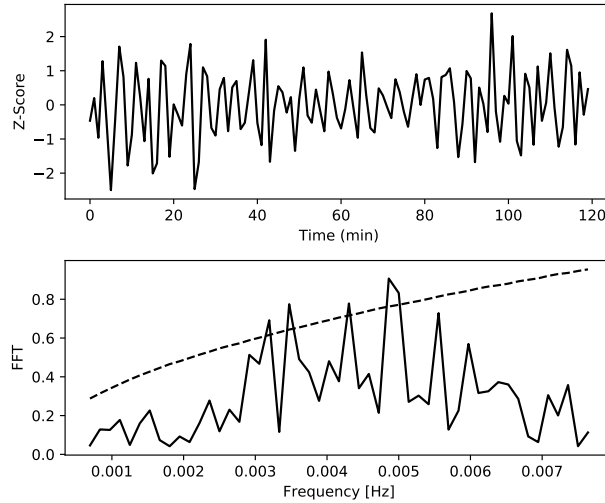


**Figure 10.** Temporal analysis of  $V_z$  vertical slices at 2 Mm. The top panel shows the selected vertical slices, indicated by gray colours. The middle panel demonstrates the obtained signal after applying a Hanning-window function. The bottom panel shows the result of the FFT analysis based on the 5 selected vertical  $V_z$  slices.

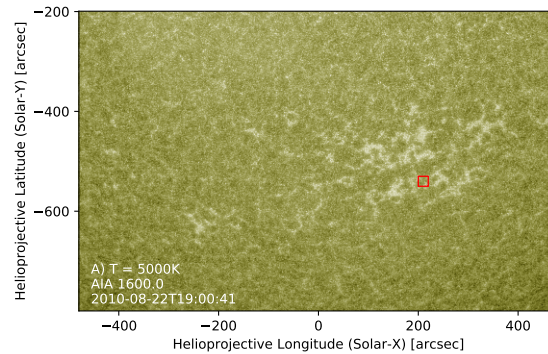
regions with pores. The results exhibit a significant shift of the frequency of these intensity oscillations from the period of the global p-mode oscillation represented in our simulations with a driver of period 5 minutes. Such shifts may be attributed to additional elasticity in the solar atmosphere resulting from the presence of a magnetic field.

Closer inspection of the energy flux propagation results are indicative of enhanced energy flux propagation for inclined magnetic fields. The obtained periodic behaviour is confirmed by observational data, featuring similar frequencies based on the intensity times series of SDO images. The results of the FFT analysis exhibit a difference with the driver frequency. The frequency difference measured from the temporal analysis of the observational and simulation data is larger than would be expected from the analysis of [Hindman et al. \(1996\)](#). This can be understood in part by referring to the work of [Campbell and Roberts \(1989\)](#).

It is encouraging that the results presented here are consistent with the behaviour exhibited by earlier work. There is an issue that due to the extended nature of the driver the amplitudes used may be responsible for delivering vast quantities of energy into the solar atmosphere and for driving a highly numerically unstable system and inducing extremely large shocks [Calvo Santamaria, Khomenko and Collados \(2015\)](#). Significant improvements to the comparisons presented in this paper are expected with the recent work of [Kostogryz et al. \(2021\)](#). Here modelling of intensity perturbations using a VALIIC solar atmosphere model with radiative transfer the intensity oscillations perturbed by the solar global oscillations modelled using the ADIPLS code of [Christensen-Dalsgaard \(2008\)](#). Also, as described



**Figure 11.** Temporal analysis of the intensity of a 50-pixel large area based on AIA 1600Å between 18:00 UT to 20:00 UT on 22 August 2010. The upper panel shows the temporal variation of the Z-Score (detrended and normalised pixel intensity data). The lower panel shows the FFT of the analysed observational data.



**Figure 12.** The selected area from the start of the investigate time series at 18:00 UT on 22 August 2010. The area is indicated by red rectangle. The size of the selected area is 50 pixels.

in [Rast & Martinez Pillet \(2016\)](#), more sensitive observations with instruments such as DKIST may enable further constraints and the range of theoretical models. Future work will address simulation runs over longer time periods and for inclined fields.

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*Software:* SMAUG [Griffiths et al. \(2015\)](#), SAC [Shelyag et al. \(2008\)](#), VAC [Tóth \(1996\)](#)

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