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Game Theory: Evolutionary Games

*Activities for the Evolutionary Games section of the Game Theory course.

1. Hawk-Dove with Retaliators

Game Description: Being nice but retaliating in face of aggression is a well known (human) behavior. Is there any rationale for it? Retaliator is Dove unless he faces a Hawk in which case he switches to Hawk. Retaliators copy the strategy of the opponent. It can do so because the game has a signaling / intention phase (fight or display) followed by behavior. Payoffs are shown next:

	H	D	R
H	$(v-c)/2$	v	$\frac{v-c}{2}$
D	0	$v/2$	$v/2$
R	$\frac{v-c}{2}$	$v/2$	$v/2$

H-D-R Payoffs

1.1 Experimentation

Case c > v:

a) Game Analysis - What is expected from the point of view of ESS?

We know already that neither H nor D are pure Nash equilibria therefore can't be ESS. R is a Nash equilibrium. R can resist invasion by H when $c > 0$ since $(v - c)/2 < v/2$.
R can't resist invasion by Doves. When hawks are absent Doves and Retaliators have the same fitness and any proportion can drift (It isn't an ESS but is called neutral stability). But then if Doves drift above a given proportion, a mixture of Doves and Retaliators can be invaded by Hawks.
Denoting q the frequency of R : A rare H can invade when $q < v/(v + c)$.
Simply compute the expected payoffs for a single H coming in a population with a proportion q of R and $(1 - q)$ of D. $u(H, q) = q\times(v - c)/2 + (1 - q)v$. A single H can invade if $q\times(v - c)/2 + (1 - q)v > v/2$ i.e $q < v/(v + c)$.

Once H have invaded they in turn are invaded by D and we get back to the proportion $\hat{p} = v/c$ of H and $(1 - \hat{p})$ of D (The ESS of the H-D game without retaliators).

The expected payoff of Doves or Hawk under $\hat{p}$ is given by $(1 - v/c)\times v/2$. For rare R facing the mixed population of Hawks and Doves given by $\hat{p}$ the expected payoff is: $v/c\times(v - c)/2 + (1 - v/c)\times v/2$ which is lower than $(1 - v/c)\times v/2$ provided that $v < c$ which is true by assumption.

We conclude that the most probable long term outcome is the mix $\hat{p} = v/c$ of Hawks and $1 - \hat{p}$ Doves.

b) What is observed from simulations? (please report the parameter space on which simulations where performed, number of simulations and reported statistical indicator(s) used for conclusions)

Experimentation

Parameter Space:

$$p \in \{0.25, 0.45\}$$
$$q \in \{0.25, 0.45\}$$
$$c \in \{0.36, 0.46, 0.56\}$$
$$v \in \{0.15, 0.25, 0.35\}$$

Number of Simulations:

Each parameter configuration was run 10 times (8,000 ticks as time limit), yielding a total number of 360 simulations using NetLogo's multithreading capabilities (12 threads) in a total time of ≈9 min.

Simulation results:

The statistical indicator for every set of parameter configuration results is the average value of all the members of the population for every configuration of parameters. For this particular case, we also include a column computing 'v/c' per parameter configuration since that operation is a good estimator of $\widehat{p}$ Hawks for many cases.

The results are shown in Table 1.

As expected from the theoretical analysis, the population converges in long term to a mix almost exclusive of Hawks and Doves where Hawk approximates in average to the given v/c value for each parameter set. Retaliators are almost completely excluded long term.

config	c	v	p	q	avg_hawks	avg_doves	avg_retaliators	v/c
0	0.36	0.15	0.25	0.25	67.1	132.9	0.0	0.41
1	0.36	0.15	0.25	0.45	66.5	132.8	0.7	0.41
2	0.36	0.15	0.45	0.25	66.9	133.1	0.0	0.41
3	0.36	0.15	0.45	0.45	67.7	132.1	0.2	0.41
4	0.36	0.25	0.25	0.25	104.0	95.5	0.5	0.69
5	0.36	0.25	0.25	0.45	104.7	95.0	0.3	0.69
6	0.36	0.25	0.45	0.25	102.9	96.3	0.8	0.69
7	0.36	0.25	0.45	0.45	103.1	96.0	0.9	0.69
8	0.36	0.35	0.25	0.25	125.3	71.5	3.2	0.97
9	0.36	0.35	0.25	0.45	126.8	70.9	2.3	0.97
10	0.36	0.35	0.45	0.25	126.0	71.5	2.5	0.97
11	0.36	0.35	0.45	0.45	126.9	69.9	3.2	0.97
12	0.46	0.15	0.25	0.25	50.0	142.5	7.5	0.32
13	0.46	0.15	0.25	0.45	55.6	143.9	0.5	0.32
14	0.46	0.15	0.45	0.25	53.4	145.8	0.8	0.326
15	0.46	0.15	0.45	0.45	55.3	144.2	0.5	0.32
16	0.46	0.25	0.25	0.25	76.9	110.2	12.9	0.54
17	0.46	0.25	0.25	0.45	85.9	113.6	0.5	0.54
18	0.46	0.25	0.45	0.25	86.6	113.0	0.4	0.54
19	0.46	0.25	0.45	0.45	87.1	112.8	0.1	0.54
20	0.46	0.35	0.25	0.25	100.0	67.8	32.2	0.76
21	0.46	0.35	0.25	0.45	123.8	75.0	1.2	0.76
22	0.46	0.35	0.45	0.25	124.1	75.0	0.9	0.76
23	0.46	0.35	0.45	0.45	124.9	73.6	1.5	0.76
24	0.56	0.15	0.25	0.25	48.0	151.8	0.2	0.26
25	0.56	0.15	0.25	0.45	46.7	152.9	0.4	0.26
26	0.56	0.15	0.45	0.25	47.3	152.5	0.2	0.26
27	0.56	0.15	0.45	0.45	46.7	153.2	0.1	0.26
28	0.56	0.25	0.25	0.25	65.8	123.1	11.1	0.44
29	0.56	0.25	0.25	0.45	72.0	127.6	0.4	0.44
30	0.56	0.25	0.45	0.25	73.8	125.9	0.3	0.44
31	0.56	0.25	0.45	0.45	72.7	127.3	0.0	0.44
32	0.56	0.35	0.25	0.25	95.0	104.5	0.5	0.625
33	0.56	0.35	0.25	0.45	95.5	104.2	0.3	0.625
34	0.56	0.35	0.45	0.25	95.1	104.3	0.6	0.625
35	0.56	0.35	0.45	0.45	94.2	105.2	0.6	0.625

Table 1. Hawk-Dove-Retaliator game for c > v and a population of 200.

Case v > c:

c) Game Analysis - What if expected from the point of view of ESS?

In the initial Hawk-Dove game without retaliators the unique equilibrium is H (It is a dominant strategy and in fact we are now in a prisoner's dilemma since playing Dove would be Pareto superior). Therefore , a population of Doves can be invaded by Hawks $u(H, D) > u(D, D)$.

Second, in turn in a population of only H , R can invade $u(H, H) = u(R, H)$ and $u(R, R) > u(H, R)$. But note that since Doves have the same fitness as retaliators, Doves could appear by random drift. But any random drift of doves would be replaced by hawks (by our first argument) which would be replaced by R (by our second argument). Therefore the most probable outcome is only R.

d) What is observed from simulations? (please report the parameter space on which simulations where performed, number of simulations and reported statistical indicator(s) used for conclusions)

As mentioned in question 'c', the expected outcome of the combinations proposed by the parameter space presented next where v>c is a population consisting exclusively of a retaliator.

Experimentation

Parameter Space:

p ∈ {0.25, 0.45}
q ∈ {0.25, 0.45}
v ∈ {0.36, 0.46, 0.56}
c ∈ {0.15, 0.25, 0.35}

Number of Simulations:

Each parameter configuration was run 10 times (10,000 ticks as time limit), yielding a total number of 360 simulations using NetLogo's multithreading capabilities (12 threads) in a total time of ≈10 min.

Simulation results:

The statistical indicator for every set of parameter configuration results is the average value of all the members of the population for every configuration of parameters.

The results are shown in Table 2.

In the long term we can confirm that the population converges to almost only retaliators having some spikes of doves and with almost non-existent presence of hawks.

config	v	c	p	q	avg_hawks	avg_doves	avg_retaliators
0	0.36	0.15	0.25	0.25	0.3	45.4	154.3
1	0.36	0.15	0.25	0.45	0.0	41.6	158.4
2	0.36	0.15	0.45	0.25	0.2	40.6	159.2
3	0.36	0.15	0.45	0.45	0.2	42.9	156.9
4	0.36	0.25	0.25	0.25	0.1	51.6	148.3
5	0.36	0.25	0.25	0.45	0.1	51.9	148.0
6	0.36	0.25	0.45	0.25	0.1	52.5	147.4
7	0.36	0.25	0.45	0.45	0.8	53.6	145.6
8	0.36	0.35	0.25	0.25	0.2	55.5	144.3
9	0.36	0.35	0.25	0.45	0.0	52.2	147.8
10	0.36	0.35	0.45	0.25	3.6	52.9	143.5
11	0.36	0.35	0.45	0.45	1.8	69.2	129.0
12	0.46	0.15	0.25	0.25	0.3	34.8	164.9
13	0.46	0.15	0.25	0.45	0.2	30.2	169.6
14	0.46	0.15	0.45	0.25	0.3	26.4	173.3
15	0.46	0.15	0.45	0.45	0.4	36.9	162.7
16	0.46	0.25	0.25	0.25	0.4	50.3	149.3
17	0.46	0.25	0.25	0.45	0.5	57.9	141.6
18	0.46	0.25	0.45	0.25	1.0	49.3	149.7
19	0.46	0.25	0.45	0.45	1.1	61.3	137.6
20	0.46	0.35	0.25	0.25	0.2	50.0	149.8
21	0.46	0.35	0.25	0.45	0.2	67.3	132.5
22	0.46	0.35	0.45	0.25	0.0	46.5	153.5
23	0.46	0.35	0.45	0.45	0.1	58.2	141.7
24	0.56	0.15	0.25	0.25	1.1	33.1	165.8
25	0.56	0.15	0.25	0.45	0.2	25.9	173.9
26	0.56	0.15	0.45	0.25	0.4	28.9	170.7
27	0.56	0.15	0.45	0.45	3.3	27.7	169.0
28	0.56	0.25	0.25	0.25	0.9	40.5	158.6
29	0.56	0.25	0.25	0.45	0.3	31.3	168.4
30	0.56	0.25	0.45	0.25	0.0	47.1	152.9
31	0.56	0.25	0.45	0.45	0.3	49.7	150.0
32	0.56	0.35	0.25	0.25	0.2	43.7	156.1
33	0.56	0.35	0.25	0.45	0.1	52.3	147.6
34	0.56	0.35	0.45	0.25	0.6	53.1	146.3
35	0.56	0.35	0.45	0.45	0.1	50.9	149.0

Table 2. Hawk-Dove-Retaliator game for v > c population of 200.

2. Hawk-Dove-Retaliators with additional parameter δ

Game Description: Consider the variant of Hawk–Dove in which a third strategy is available: "Retaliator", which fights only if the opponent does so. Assume that a retaliator has a slight advantage over a passive animal against a passive opponent. 'A' is for an aggressive strategy (Hawk), 'P' for the passive strategy (Dove).

Assume δ < 1/2 v and that v<c.

	A	P	R
A	$\frac{1}{2}(v-c), \frac{1}{2}(v-c)$	$v, 0$	$\frac{1}{2}(v-c), \frac{1}{2}(v-c)$
P	$0, v$	$\frac{1}{2}v, \frac{1}{2}v$	$\frac{1}{2}v-\delta, \frac{1}{2}v+\delta$
R	$\frac{1}{2}(v-c), \frac{1}{2}(v-c)$	$\frac{1}{2}v+\delta, \frac{1}{2}v-\delta$	$\frac{1}{2}v, \frac{1}{2}v$

H-D-R with additional parameter δ Payoffs

Game Analysis:

The only symmetric pure strategy equilibrium is (R, R). This equilibrium is strict, so that R is an ESS. We know that the set of ESS is included in the set of Nash equilibria.
Now consider the possibility that the game has a mixed strategy equilibrium (α, α). If α assigns a probability 0 to A then R yields a payoff higher than does P against an opponent who uses α, a contradiction with the assumption that a mixed strategy equilibrium where all strategies in the support (P and R) must yield the same payoffs against α. If α assigns probability 0 to P then R yields a payoff higher than does A against an opponent who uses α, again a contradiction. Thus in any mixed strategy equilibrium (α, α), the strategy α must assign positive probability to both A and P.
If α assigns probability 0 to R then we need $\alpha = (v/c, 1 - v/c)$ (The calculation is the same as for Hawk–Dove). Because R yields a lower payoff against this strategy than do A and P (i.e $u(R, \alpha) < u(A, \alpha) = u(\alpha, \alpha)$ and therefore R can't invade), and the strategy is an ESS in Hawk–Dove, it is an ESS in the present game.
The last possibility is that the game has a mixed strategy equilibrium (α, α) in which α assigns positive probability to all three actions. If so, the expected payoff to this strategy is less than $1/2v$, because the pure strategy P yields an expected payoff less than $1/2v$ against any such strategy (In the support of the equilibrium distribution all pure strategies yield the same payoffs). Notably $u(R, \alpha) = u(\alpha, \alpha)$. But then $u(R, R) = 1/2\,v > u(\alpha, R)$, violating the second condition in the definition of an ESS. In summary both R and the mixed strategy that assigns probability v/c to A and $1 - v/c$ to P are ESSs. Depending on initial condition the replicator dynamics will converge to one or the other.

Experimentation:

Parameter Space:

$$p \in \{0.25, 0.45\}$$
$$q \in \{0.25, 0.45\}$$
$$c \in \{0.16, 0.26\}$$
$$v \in \{0.1\ 0.15\}$$
$$\delta \in \{0.02, 0.04\}$$

Number of Simulations:

Each parameter configuration was run 10 times (10,000 ticks as time limit), yielding a total number of 360 simulations using NetLogo's multithreading capabilities (12 threads) in a total time of ≈10 min.

Simulation results:

The statistical indicator for every set of parameter configuration results is the average value of all the members of the population for every configuration of parameters.

The results are shown in Table 3.

There are 2 clear possible outcomes: either the retaliators absolutely dominate the population, or some few cases were retaliators are dominated and the population consists of a mixed strategy that follows probability v/c to Hawks and $1 - v/c$ to Doves which happens when the values assigned to p and q are both equal to 0.45 in our proposed experiment.

config	c	v	d	p	q	avg_hawks	avg_doves	avg_retaliators
0	0.16	0.1	0.02	0.25	0.25	0.0	0.2	199.8
1	0.16	0.1	0.02	0.25	0.45	0.1	0.0	199.9
2	0.16	0.1	0.02	0.45	0.25	0.1	0.1	199.8
3	0.16	0.1	0.02	0.45	0.45	95.0	103.7	1.3
4	0.16	0.1	0.04	0.25	0.25	0.3	0.1	199.6
5	0.16	0.1	0.04	0.25	0.45	0.1	0.1	199.8
6	0.16	0.1	0.04	0.45	0.25	0.2	0.0	199.8
7	0.16	0.1	0.04	0.45	0.45	0.1	0.1	199.8
8	0.16	0.15	0.02	0.25	0.25	0.0	0.1	199.9
9	0.16	0.15	0.02	0.25	0.45	0.2	0.0	199.8
10	0.16	0.15	0.02	0.45	0.25	0.0	0.4	199.6
11	0.16	0.15	0.02	0.45	0.45	127.5	70.4	2.1
12	0.16	0.15	0.04	0.25	0.25	0.0	0.0	200.0
13	0.16	0.15	0.04	0.25	0.45	0.0	0.2	199.8
14	0.16	0.15	0.04	0.45	0.25	0.1	0.1	199.8
15	0.16	0.15	0.04	0.45	0.45	63.7	34.8	101.5
16	0.26	0.1	0.02	0.25	0.25	0.2	0.1	199.7
17	0.26	0.1	0.02	0.25	0.45	0.1	0.0	199.9
18	0.26	0.1	0.02	0.45	0.25	0.0	0.2	199.8
19	0.26	0.1	0.02	0.45	0.45	64.4	134.9	0.7
20	0.26	0.1	0.04	0.25	0.25	0.2	0.0	199.8
21	0.26	0.1	0.04	0.25	0.45	0.0	0.0	200.0
22	0.26	0.1	0.04	0.45	0.25	0.1	0.0	199.9
23	0.26	0.1	0.04	0.45	0.45	0.0	0.1	199.9
24	0.26	0.15	0.02	0.25	0.25	0.0	0.4	199.6
25	0.26	0.15	0.02	0.25	0.45	9.3	10.7	180.0
26	0.26	0.15	0.02	0.45	0.25	0.0	0.3	199.7
27	0.26	0.15	0.02	0.45	0.45	92.8	106.2	1.0
28	0.26	0.15	0.04	0.25	0.25	0.1	0.0	199.9
29	0.26	0.15	0.04	0.25	0.45	0.1	0.1	199.8
30	0.26	0.15	0.04	0.45	0.25	0.2	0.0	199.8
31	0.26	0.15	0.04	0.45	0.45	72.6	85.9	41.5

Table 3. Hawk-Dove-Retaliator with $\delta < 1/2$ v and v<c with a population of 200.