Part I.1 Eigenvalues and eigenvectors

**Part I.1: Eigenvalues and eigenvectors**

**(hand)**: Calculate the eigenvalues for each of the following matrices:

$$\boldsymbol{A}_1 = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$$

$$\boldsymbol{A}_2 = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix}$$

$$\boldsymbol{A}_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\boldsymbol{A}_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\boldsymbol{A}_5 = \begin{bmatrix} 25 & 0 & 20 \\ 0 & 1 & 0 \\ 20 & 0 & 41 \end{bmatrix}$$

1) $A_1 = \begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix} \rightarrow \det(A_1 - \lambda I) = 0$

$$\det(A_1 - \lambda I) = \begin{vmatrix} 5-\lambda & 2 \\ 4 & 3-\lambda \end{vmatrix} \Rightarrow (5-\lambda)(3-\lambda) - 8 = 0$$

$$\Rightarrow 15 - 8\lambda + \lambda^2 - 8 = 0$$

$$\Rightarrow \lambda^2 - 8\lambda + 7 = 0$$

$$\Rightarrow (\lambda - 7)(\lambda - 1) = 0$$

$$\lambda_1 = 7, \quad \lambda_2 = 1$$

2.) $A_2 = \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix} \rightarrow \det(A_2 - \lambda I) = 0$

$$\det(A_2 - \lambda I) = \begin{vmatrix} 4-\lambda & 3 \\ 3 & 4-\lambda \end{vmatrix} \Rightarrow (4-\lambda)^2 - 9 = 0$$

$$\Rightarrow 16 - 8\lambda + \lambda^2 - 9 = 0$$

$$\Rightarrow \lambda^2 - 8\lambda + 7 = 0$$

$$\Rightarrow (\lambda - 7)(\lambda - 1) = 0$$

$$\Rightarrow \lambda_1 = 7 \quad \lambda_2 = 1$$

3-) $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow \det(A_3 - \lambda I) = 0$

$\det(A_3 - \lambda I) = \begin{vmatrix} 1-\lambda & 1 \\ 1 & 1-\lambda \end{vmatrix} \Rightarrow (1-\lambda)^2 - 1 = 0$

$\Rightarrow \lambda^2 - 2\lambda + 1 - 1 = 0$

$\Rightarrow \lambda^2 - 2\lambda = 0$

$\Rightarrow \lambda(\lambda - 2) = 0$

$\Rightarrow \lambda_1 = 2, \lambda_2 = 0$

4-) $A_4 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow \det(A_4 - \lambda I) = 0$

$\det(A_4 - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} \Rightarrow \lambda^2 - 1 = 0$

$\Rightarrow (\lambda - 1)(\lambda + 1) = 0$

$\Rightarrow \lambda_1 = 1, \lambda_2 = -1$

5-) $A_5 = \begin{bmatrix} 25 & 0 & 20 \\ 0 & 1 & 0 \\ 20 & 0 & 41 \end{bmatrix} \Rightarrow \det(A_5 - \lambda I) = 0$

$$\det(A_5 - \lambda I) = \begin{vmatrix} 25-\lambda & 0 & 20 \\ 0 & 1-\lambda & 0 \\ 20 & 0 & 41-\lambda \\ 25-\lambda & 0 & 20 \\ 0 & 1-\lambda & 0 \end{vmatrix} = 0$$

$\Rightarrow ((25-\lambda).(1-\lambda)(41-\lambda) + 0 + 0) - (20.20.(1-\lambda) + 0 + 0)$

$[25 - 26\lambda + \lambda^2](41 - \lambda) - (400 - 400\lambda) = 0$

$\Rightarrow 1025 - 25\lambda - 1066\lambda + 26\lambda^2 + 41\lambda^2 - \lambda^3 - 400 + 400\lambda = 0$

$\Rightarrow -\lambda^3 + 67\lambda^2 - 691\lambda + 625 = 0$ from eqn (1);

$\Rightarrow -(\lambda - 1)(\lambda^2 - 66\lambda + 625) = 0$ $\lambda_1 = 1$

$\Rightarrow -(\lambda - 1)((\lambda - 66)\lambda + 625) = 0$ $\lambda_2 = 33 - 4\sqrt{29}$

$\Rightarrow$ (1) $\lambda((67 - \lambda)\lambda - 691) + 625 = 0$ $\lambda_3 = 33 + 4\sqrt{29}$