Tensors in General Relativity

Inverse metric

In[ ]:=
```
InverseMetric[g_] := Simplify[Inverse[g]]
```

Christoffel Symbol

In[ ]:=
```
ChristoffelSymbol[g_, xx_] := Block[{n, ig, res}, n = 4;
  ig = InverseMetric[g];
  res = Table[(1/2) * Sum[
      ig[[i, s]] * (-D[g[[j, k]], xx[[s]]] + D[g[[j, s]], xx[[k]]] + D[g[[s, k]], xx[[j]]]),
      {s, 1, n}], {j, 1, n}, {k, 1, n}, {i, 1, n}];
  Simplify[
   res]]
```

Curvature Tensor

In[ ]:=
```
CurvatureTensor[g_, xx_] := Block[{n, Chr, res}, n = 4;
  Chr = ChristoffelSymbol[g, xx];
  res = Table[D[Chr[[j, k, l]], xx[[i]]] - D[Chr[[i, k, l]], xx[[j]]] +
     Sum[Chr[[j, k, m]] * Chr[[i, m, l]], {m, 1, n}] - Sum[Chr[[i, k, m]] * Chr[[j, m, l]],
      {m, 1, n}], {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}];
  Simplify[
   res]]
```

Riemann Tensor

In[ ]:=
```
RiemannTensor[g_, xx_] :=
 Block[{n, Curv, gg, res}, n = 4; Curv = CurvatureTensor[g, xx]; gg = g;
  res = Table[Sum[Curv[[i, j, k, m]] × gg[m, l], {m, 1, n}],
    {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}];

  res]
```

Ricci Tensor

```
RicciTensor[g_, xx_] := Block[{Rie, res, n}, n = 4;
  Rie = CurvatureTensor[g, xx];
  res = Table[Sum[Rie[[j, i, k, j]] , {j, 1, n}], {i, 1, n}, {k, 1, n}];
  Simplify[res]]
```

Ricci Scalar

```
RicciScalar[g_, xx_] := Block[{Ricc, ig, res, n}, n = 4;
  Ricc = RicciTensor[g, xx];
  ig = InverseMetric[g];
  res = Sum[ig[[j, i]] × Ricc[[j, i]], {j, 1, n}, {i, 1, n}];
  Simplify[res]]
```

Local Coordinates and Metric

```
xx = {u, v, x, y};

g = {{a[u, x, y], 1 , b1[u, x, y], b2[u, x, y]}, {1, 0, 0, 0},
    {b1[u, x, y], 0, c[u, x, y], 0}, {b2[u, x, y], 0, 0, c[u, x, y]}};
```

Example

In[ ]:=
```
RicciTensor[g, xx]
```

Out[ ]=

$$\left\{\left\{\frac{1}{2\,c[u,x,y]^2}\left(b1^{(0,0,1)}[u,x,y]^2-2\,b1^{(0,0,1)}[u,x,y]\,b2^{(0,1,0)}[u,x,y]+b2^{(0,1,0)}[u,x,y]^2+c^{(1,0,0)}[u,x,y]^2-c[u,x,y]\left(a^{(0,0,2)}[u,x,y]+a^{(0,2,0)}[u,x,y]-2\left(b2^{(1,0,1)}[u,x,y]+b1^{(1,1,0)}[u,x,y]-c^{(2,0,0)}[u,x,y]\right)\right)\right),\ 0,\right.$$

$$\frac{1}{2\,c[u,x,y]^2}\left(b1^{(0,0,1)}[u,x,y]\,c^{(0,0,1)}[u,x,y]-c^{(0,0,1)}[u,x,y]\,b2^{(0,1,0)}[u,x,y]+c^{(0,1,0)}[u,x,y]\,c^{(1,0,0)}[u,x,y]-c[u,x,y]\left(b1^{(0,0,2)}[u,x,y]-b2^{(0,1,1)}[u,x,y]+c^{(1,1,0)}[u,x,y]\right)\right),$$

$$\frac{1}{2\,c[u,x,y]^2}\left(-b1^{(0,0,1)}[u,x,y]\,c^{(0,1,0)}[u,x,y]+b2^{(0,1,0)}[u,x,y]\,c^{(0,1,0)}[u,x,y]+c[u,x,y]\,b1^{(0,1,1)}[u,x,y]-c[u,x,y]\,b2^{(0,2,0)}[u,x,y]+c^{(0,0,1)}[u,x,y]\,c^{(1,0,0)}[u,x,y]-c[u,x,y]\,c^{(1,0,1)}[u,x,y]\right)\Big\},$$

$$\{0,0,0,0\},\ \left\{\frac{1}{2\,c[u,x,y]^2}\left(b1^{(0,0,1)}[u,x,y]\,c^{(0,0,1)}[u,x,y]-c^{(0,0,1)}[u,x,y]\,b2^{(0,1,0)}[u,x,y]+c^{(0,1,0)}[u,x,y]\,c^{(1,0,0)}[u,x,y]-c[u,x,y]\left(b1^{(0,0,2)}[u,x,y]-b2^{(0,1,1)}[u,x,y]+c^{(1,1,0)}[u,x,y]\right)\right),\ 0,\right.$$

$$\left.\frac{c^{(0,0,1)}[u,x,y]^2+c^{(0,1,0)}[u,x,y]^2-c[u,x,y]\left(c^{(0,0,2)}[u,x,y]+c^{(0,2,0)}[u,x,y]\right)}{2\,c[u,x,y]^2},\ 0\right\},$$

$$\left\{\frac{1}{2\,c[u,x,y]^2}\left(-b1^{(0,0,1)}[u,x,y]\,c^{(0,1,0)}[u,x,y]+b2^{(0,1,0)}[u,x,y]\,c^{(0,1,0)}[u,x,y]+c[u,x,y]\,b1^{(0,1,1)}[u,x,y]-c[u,x,y]\,b2^{(0,2,0)}[u,x,y]+c^{(0,0,1)}[u,x,y]\,c^{(1,0,0)}[u,x,y]-c[u,x,y]\,c^{(1,0,1)}[u,x,y]\right),\ 0,\ 0,\right.$$

$$\left.\left.\frac{c^{(0,0,1)}[u,x,y]^2+c^{(0,1,0)}[u,x,y]^2-c[u,x,y]\left(c^{(0,0,2)}[u,x,y]+c^{(0,2,0)}[u,x,y]\right)}{2\,c[u,x,y]^2}\right\}\right\}$$