

Sample space = every element in relation to the question

Permutations without repeated letters = $n!$

↳ Use Mississippi if there are repeated numbers $\rightarrow \frac{n!}{2!2!1!1! \dots}$

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Raised to the power questions \rightarrow amount of choices raised to number of events

↳ flip a coin 5 times, odds you get 5 heads? $1/2^5$

Mutually Exclusive - the 2 events that are mutually exclusive can not happen at the same time.

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B^c) & P(A \cup B) &= P(A) + P(A^c \cap B) \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) & P(A \cap B^c) &= P(A) - P(A \cap B) \\ & & P(A^c \cap B^c) &= 1 - P(A \cup B) \end{aligned}$$

Discrete = limited options, whole numbers

Continuous = can be fractions, no limit.

CDF table = $f(0) = f(0)$, $f(1) = f(0) + f(1)$, $f(2) = f(0) + f(1) + f(2) + \dots$

Probability density function, $f(x) = \frac{d}{dx} F(x)$

↳ put parameters in integral spots of PDF $\rightarrow \int_0^2 f(x) dx$ \downarrow $f(x)$, NOT $F(x)$

Probability distribution of discrete random variable x

↳ $f(x)$ = some function for x = some group of numbers

$$1 = f(x_1) + f(x_2) + \dots + f(x_n) \rightarrow \text{solve for } c.$$

Joint probability density function \rightarrow this is for x and y

↳ $f(x, y)$ = some function x = numbers, y = numbers

$$1 = f(x_1, y_1) + f(x_1, y_2) + \dots + f(x_n, y_n) \rightarrow \text{solve for } c.$$

Sometimes, the numbers will be limited

Marginal Density Functions

- For both x and y , derive from 0 to ∞

$$\int_0^{\infty} f(x, y) dy \quad \text{Marginal distribution of } x \text{ used } dy, \text{ vice versa.}$$

- To check probability for these, when $(x \geq 2, y \geq 2)$ or something like that, use a double integral to check.

Marginal Distribution - usually on a chart of values

↳ Add up the rows or columns for which we are looking at

$x = x_1, x_2, x_3$ $y = y_1, y_2, y_3$ in a table of $x \cdot x$ values

$$f_x(x_1) = f(x_1, y_1) + f(x_1, y_2) + f(x_1, y_3) \text{ for all values of } x$$

↳ If it's just a graph integrate like for marginal density functions

Bounds depend on function bounds for something like $val_1 < x < y < val_2$

bounds for dy are $\int_{x_1}^{val_2}$, bounds for dx are $\int_{val_1}^y$

Conditional Distribution of Marginal Distribution

↳ say $(X|Y)$, take x value of that index over marginal distribution of that y value.

Conditional Distribution of $X|Y$ Distribution with function

$$\text{↳ say } (X|Y) = \frac{f(x, y)}{f_Y(y)}$$

Independence = x and y are independent, if $f(x, y) = f_X(x) \cdot f_Y(y)$

Probability Distribution Function

↳ Solve probability of all cases, put into table $\{val: prob\}$

Expected value - once we have PDF, we can do this.

↳ multiply val. prob at all entries from PDF and add them.

(Marginal Densities are noted in above chapter notes)

Independence is true if $f_{XY}(0,0) = f_X(0) \cdot f_Y(0)$

Using Marginal Density table

$$E[X] = x_1(x_{row}) + x_2(x_{row}) + \dots + x_n(x_{row})$$

$E[Y] \Rightarrow$ same but just with y .

$$E[XY] = x_1 y_1 (P(x_1, y_1)) + x_1 y_2 (P(x_1, y_2)) + \dots + x_n y_n (P(x_n, y_n)) \text{ for all points in the table.}$$

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$$

$E[X^2] =$ Same as $E[X]$, but x values are squared, x_{row} stay the same though

$$\text{Var}(X) = E[X^2] - (E[X])^2 \quad \text{Var}(Y) = E[Y^2] - (E[Y])^2 \quad \rho_{XY} = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)} \cdot \sqrt{\text{Var}(Y)}}$$

Conditional densities $\rightarrow X|Y \rightarrow$ count up all values when y -val, take individual x values, y -val sum

$$\text{Expected value} = E[X+Y] = E[X] + E[Y]$$

Using Marginal Density Function

$$E[X] = \int_L^U x f_X(x) dx$$

$$E[Y] = \int_L^U y f_Y(y) dy$$

$$E[XY] = \int_L^U \int_L^U xy f_{XY}(x, y) dy dx$$

U, L are upper and lower bounds of the function.

$$\text{Cov}(X, Y) = E[XY] - E[X] \cdot E[Y]$$

$$E[X^2] = \int_L^U x^2 f_X(x) dx$$

$$E[Y^2] = \int_L^U y^2 f_Y(y) dy$$

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

$$\text{Var}(Y) = E[Y^2] - (E[Y])^2$$

$$\rho_{XY} = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)} \cdot \sqrt{\text{Var}(Y)}}$$

Conditional densities $f_{X|Y} = \frac{f_{XY}(x, y)}{f_Y(y)}$

Cumulative Distribution Function in tabular Form

$$f(0) = f(0), f(1) = f(0) + f(1), f(2) = f(0) + f(1) + f(2) + \dots$$

$$\rightarrow \begin{array}{c|c|c|c} x & 1 & 2 & \dots \\ \hline f(x) & f(1) & f(1) & \dots \end{array}$$

When CDF is given, Expected value = $E(X)$, Standard Deviation = $SD(X) = \sqrt{\text{Var}(X)}$

Find expected value at $U = \dots \rightarrow$ multiply everything by E and solve.

Find variance at $V = \dots \rightarrow$ multiply everything by var and solve.